Quantum Theory
lecture 5: semiclassics

where classical and quantum mechanics meet

- Bohr-Sommerfeld quantization  
  KP 11.2.0–11.2.1
- WKB approximation  
  KP 11.1.0
- resonant tunneling  
  KP 11.3.0
- Landau levels  
  KP 14.3

KP = Konishi & Paffuti (primary text)
Ballantine (secondary text): § 11.3, 14.4
Bohr-Sommerfeld quantization

Before the arrival of the Schrödinger equation, Bohr and Sommerfeld had found a way to quantize periodic motion by demanding that the phase accumulated in one period should be an integer multiple of $2\pi$:

$$\frac{1}{\hbar} \int p \, dq + \gamma = 2\pi n, \quad n = 0, 1, 2, \ldots$$

- $p = mv + eA$ is the canonical momentum (sum of mechanical momentum $mv$ and electromagnetic momentum $eA$).
- $\gamma$ is a phase shift picked up at the two turning points
  - $\gamma = \pm \pi$ at a hard wall,
  - $\gamma = -\pi/2$ at a smooth turning point.
- Particle in a box: $2pL/\hbar - 2\pi = 2\pi n \Rightarrow p = \pi n \hbar / L$, $E = \frac{1}{2m} (\pi n \hbar / L)^2$ (exact).
- Triangular potential well: approximate (see exercise 5.1).
- More and more accurate in the large-$n$ limit ("semiclassical")
**WKB approximation**

Wenzel-Kramers-Brillouin (Schrödinger eq.) + Jeffreys (more general diff.eq.)

probability amplitude $\psi$ that a particle from $\vec{r}_i$ reaches $\vec{r}_f$ is the sum of the amplitudes along all *classical* paths connecting $\vec{r}_i$ to $\vec{r}_f$

$$\psi = \sum_{\text{paths}} \frac{1}{\sqrt{v(\vec{r}_f)}} \exp \left( i \int_{\vec{r}_i}^{\vec{r}_f} \vec{p} \cdot d\vec{l} + i\gamma \right)$$

- factor $1/\sqrt{v} \Rightarrow$ current density $j = v|\psi|^2$ constant along path
- phase shift $\gamma$ (a.k.a. Maslov index) is $\pi$ at a hard wall (infinite potential) where $\psi = 0$, so that incident and reflected waves cancel
- at a smooth turning point, $v$ changes sign
  \[ e^{i\gamma} = 1/\sqrt{-1} = -i \Rightarrow \gamma = -\pi/2 \]
- sum over paths would be *exact* if we would include also nonclassical paths (Feynman’s path integral)
resonant tunneling

see exercise 5.3

Landau levels

see exercise 5.2