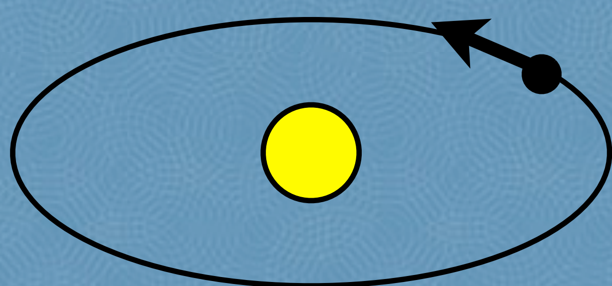
Random-matrix theory of quantum transport

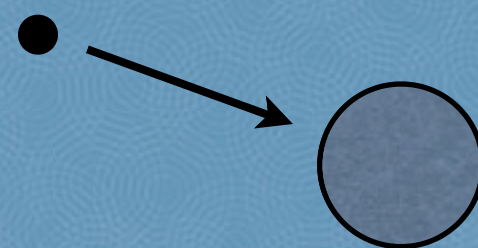
- I. random Hamiltonians
- II. random scattering matrices
- III. localization & superconductivity
- IV. Andreev reflection & topological superconductors

further reading: *arXiv:0904.1432*
slides: *www.iLorentz.org/RMT*

what do these systems have in common?



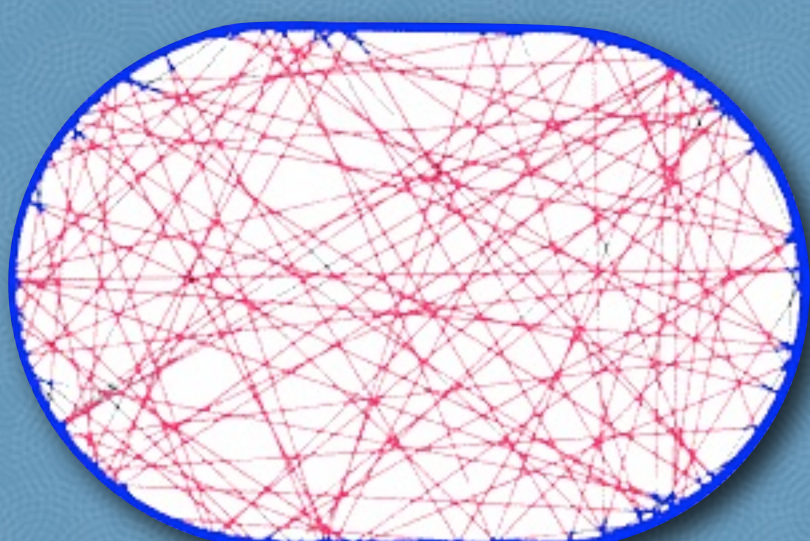
H in B-field



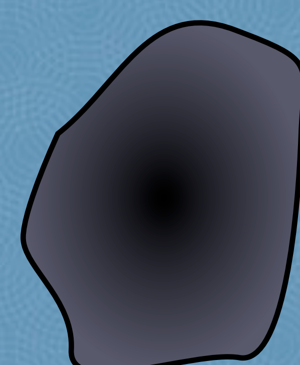
n + ^{166}Er

$$\zeta(\tfrac{1}{2} + is) = 0$$

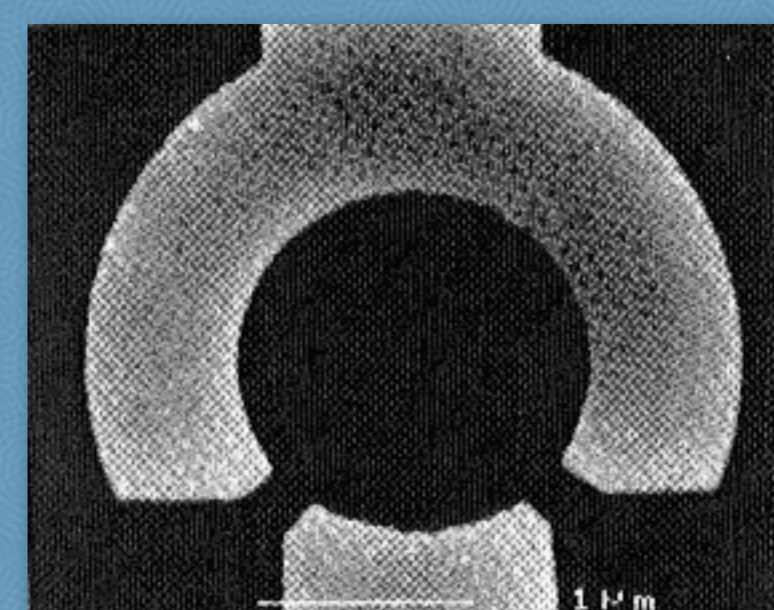
Riemann's hypothesis



chaotic billiard

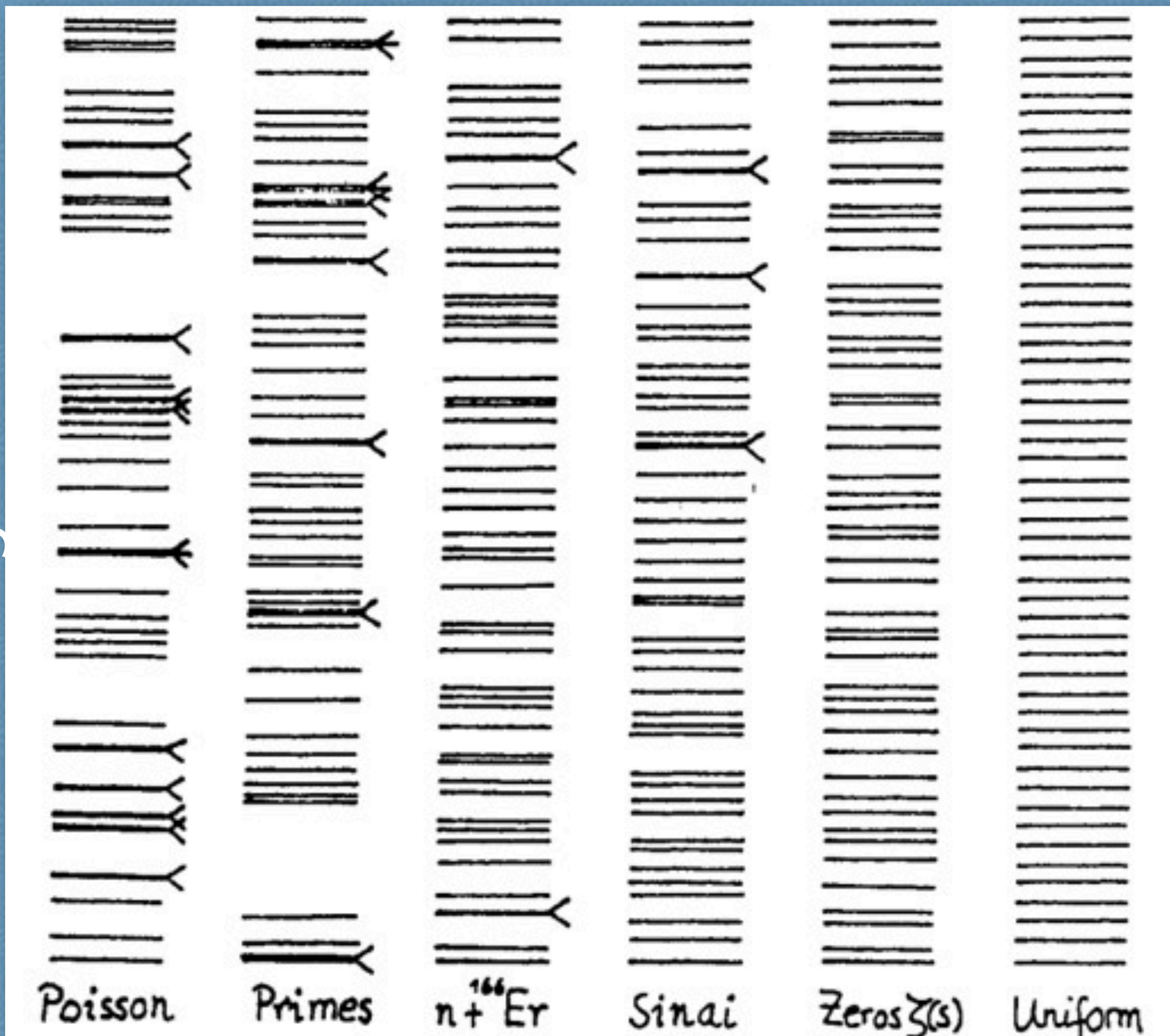


metal grain



quantum dot

universal level spacing



©Bohigas & Giannoni

β=1: B=0
(time-reversal symmetry)

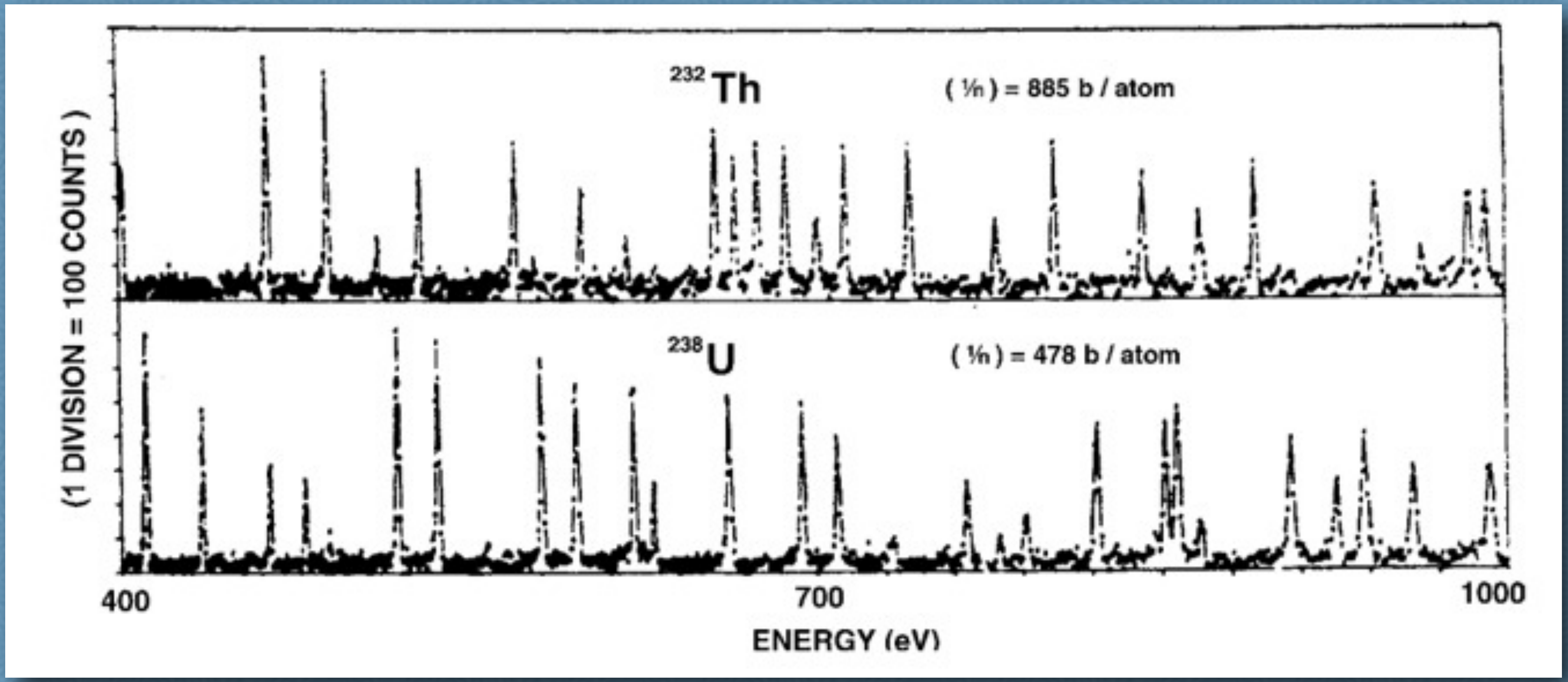
β=2: B≠0
(no time-reversal symmetry)

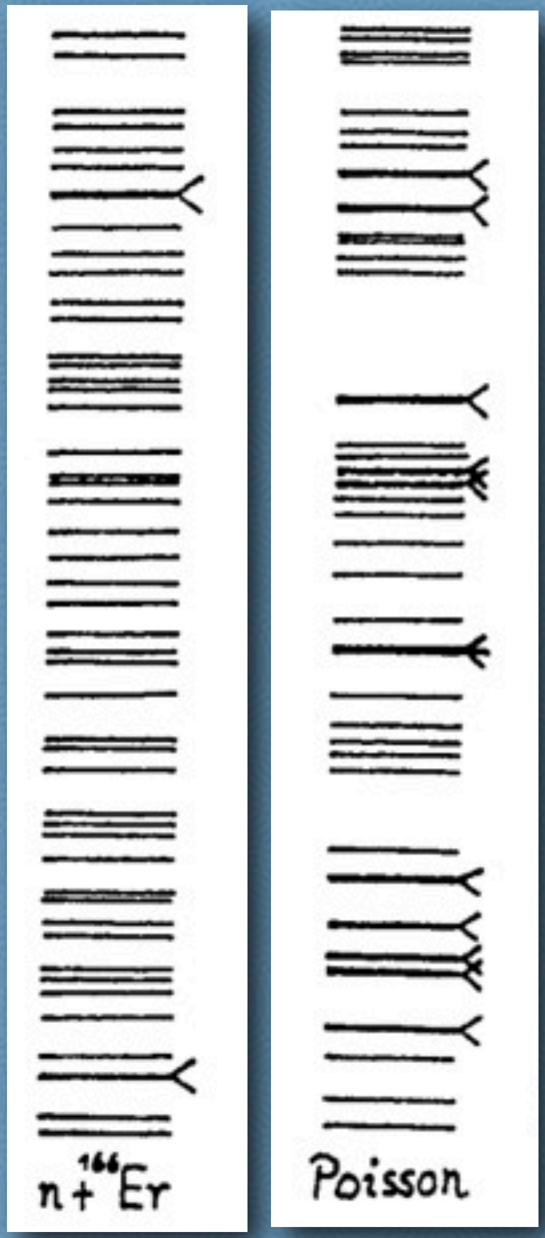
β=4: B=0
(time-reversal symmetry, without spin-rotation symmetry)

$$P(s) \propto s^{\beta} e^{-s^2}$$

Wigner (1957)

RMT & nuclear physics





statistical theory of energy levels
Wigner, Dyson, Mehta,.... (1960-1970)
ensemble of Hamiltonians

random H → correlated E's

level repulsion

geometric origin of level repulsion

$$P(H) = f(\{E\}) \to P(\{E\}) = f(\{E\}) \prod_{i<j} |E_i - E_j|^\beta$$

compare with

Jacobian

$$P(\vec{r}) = f(r) \to P(r) = f(r) r^2$$

(volume element)

β=1 if H is real (orthogonal ensemble)

β=2 if H is complex (unitary ensemble)

minimal model: $f(\{E\}) = \prod_i f(E_i)$

all correlations from volume element

example: Gaussian ensembles

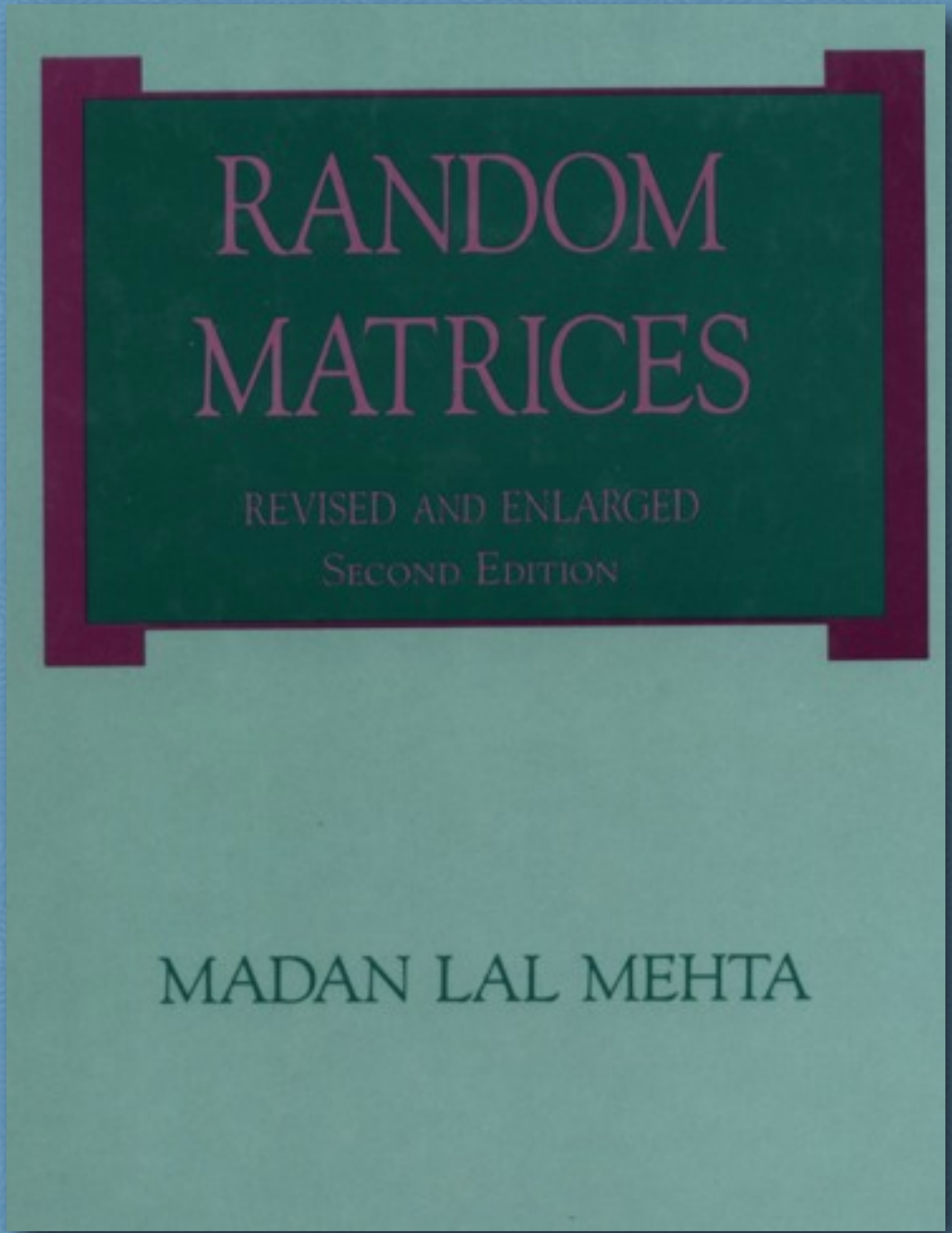
$$f(E) = e^{-cE^2} \to P(H) = \exp(-c\,\mathrm{Tr}\,H^2)$$

GOE: β=1 (*orthogonal*)

GUE: β=2 (*unitary*)

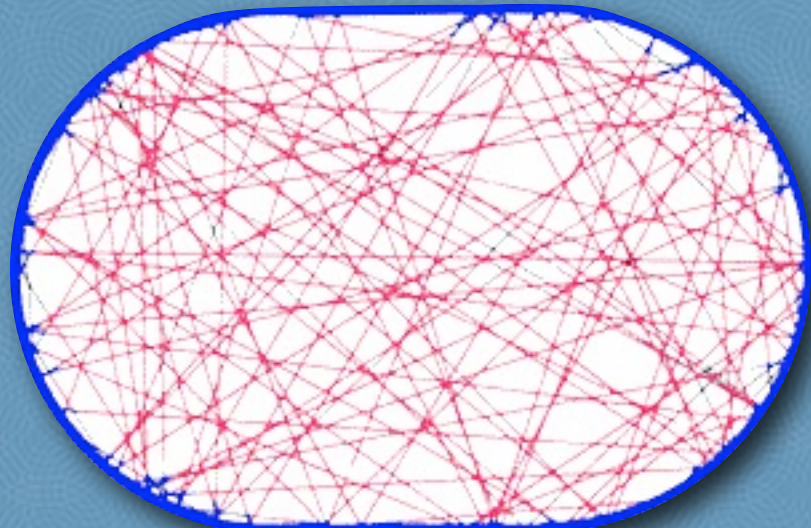
GSE: β=4 (*symplectic*)

Gaussian chosen for mathematical convenience, not fundamental



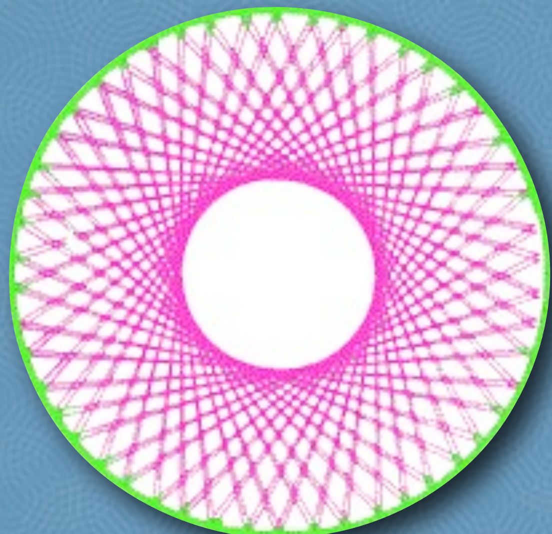
why does it work?

quantization of chaotic motion only leaves geometric level correlations



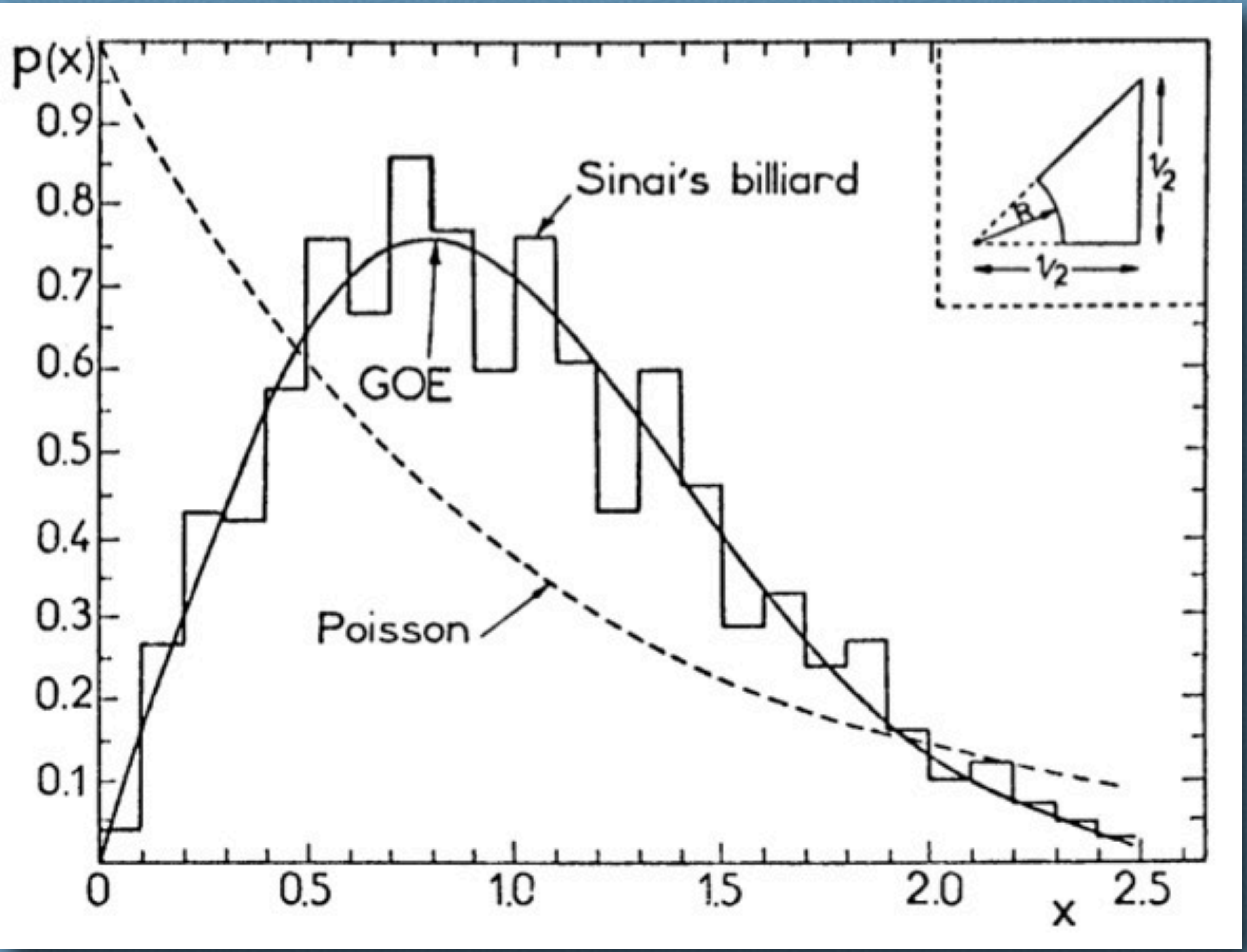
stadium (chaotic)

Wigner ($\beta=1$):
level repulsion



circle (regular)

Poisson:
no repulsion



level repulsion is a quantum signature of chaos

spacing of energy levels

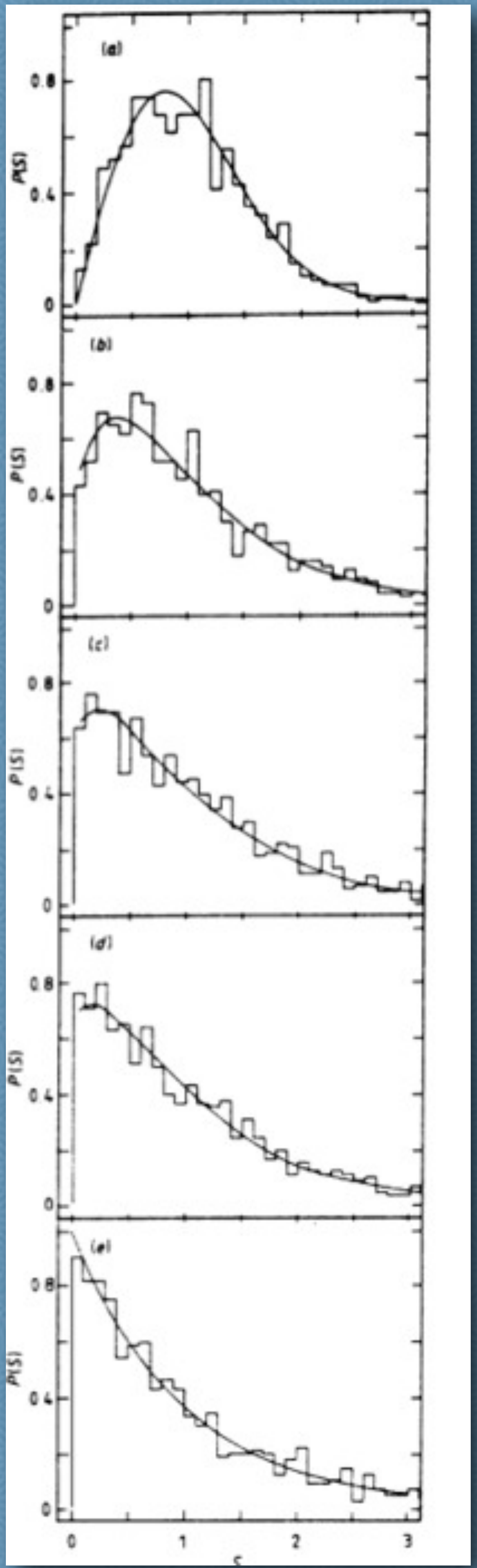
Berry & Tabor (1977)
Bohigas, Giannoni, Schmidt (1984)
very difficult mathematical problem….

Time scales for validity of RMT

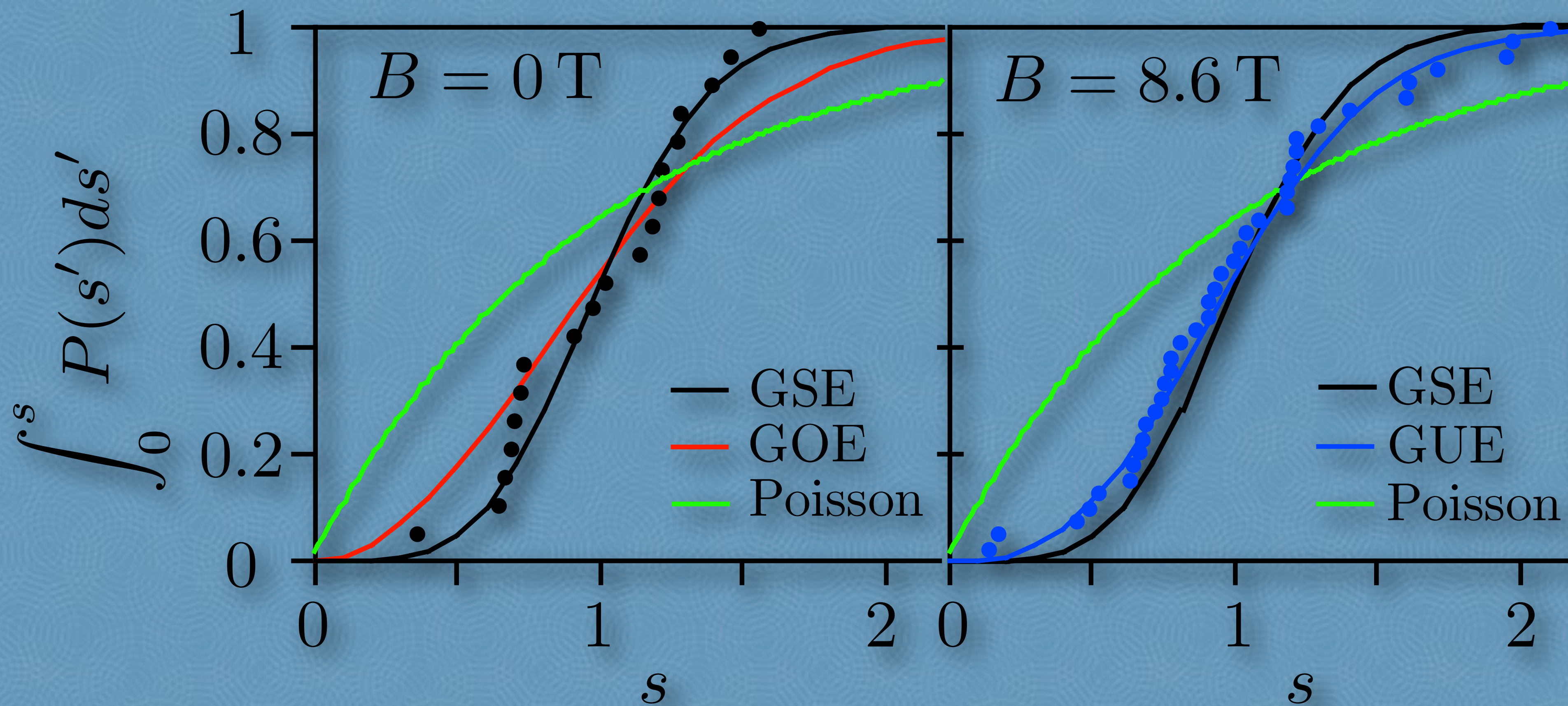
- ergodic time: $\tau_{erg} = L/v$ (ballistic) or $(L/\ell)L/v$ (diffusive)

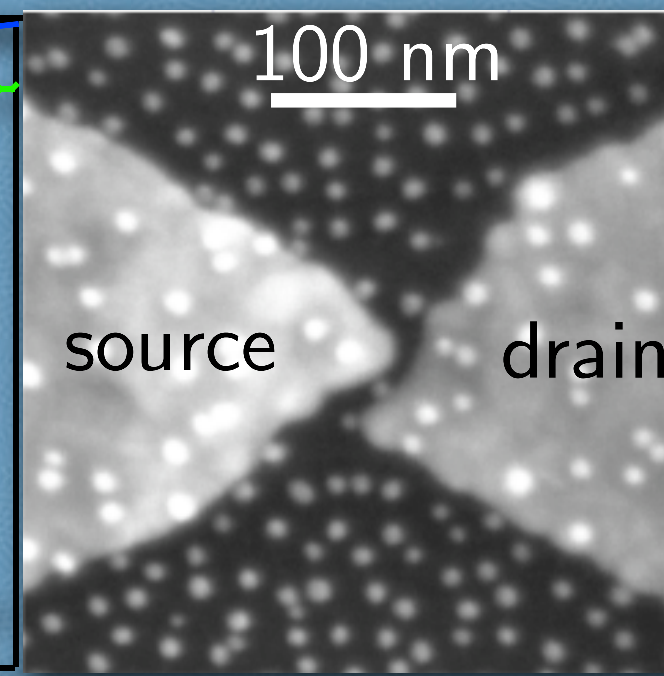
- correlation energy $E_c = \hbar/\tau_{erg}$
- levels further than E_c are uncorrelated

- conductance $G \sim (e^2/h)E_c/\delta$ (Thouless), so Wigner-to-Poisson crossover at $G \sim e^2/h$



time-reversal-symmetry breaking





gold nanoparticle: strong spin-orbit coupling
→ symplectic-to-unitary crossover

$$P(s) \simeq s^{\beta}, \;\; \beta = 4 \to 2$$

how big should the magnetic field be?

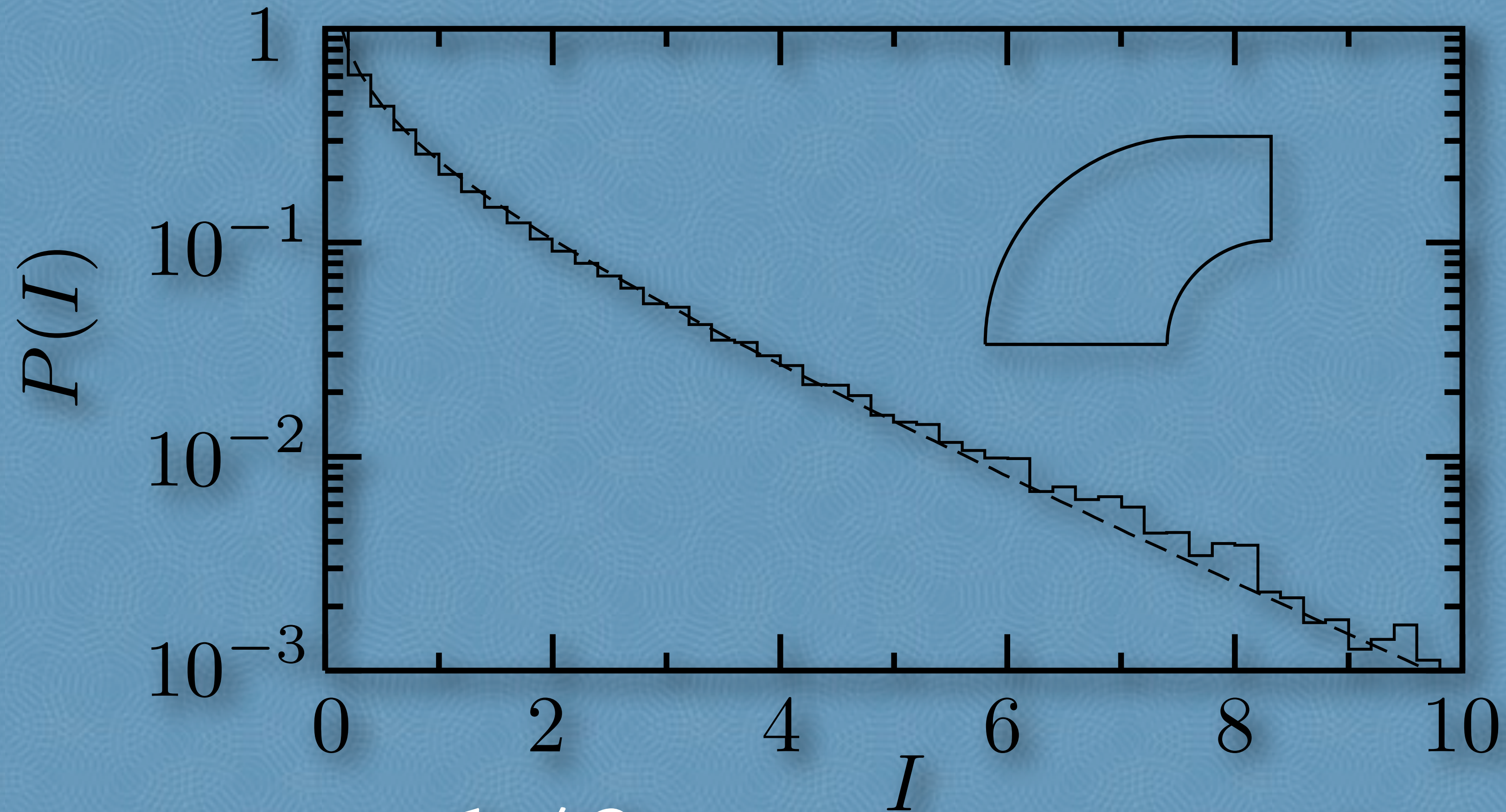
flux Φ through system in units of flux quantum h/e

$$E(\Phi)=E(0)+E_c\Phi^2$$

$$E(\Phi_c)=\delta \to \Phi_c=\sqrt{\delta/E_c}=1/\sqrt{g}\ll 1$$

much less than one flux quantum is sufficient to break time-reversal symmetry

wave function statistics



$$P(\mathrm{I}) \propto \mathrm{I}^{-1/2}\exp(-\mathrm{I}/2\bar{\mathrm{I}})$$

Porter-Thomas distribution (β=1)

follows from P(H) which depends only on the eigenvalues, so eigenfunctions are *uniformly* distributed in orthogonal or unitary group

β=2: eigenfunctions are rows of a matrix which is uniformly distributed in U(N)

$$P(U_{11}, U_{12}, \ldots U_{1N}) \propto \delta\left(1 - \sum_{n=1}^{N} |U_{1n}|^2\right)$$

integrate out N-1 elements

$$P(U_{11}) \propto \left(1 - |U_{11}|^2\right)^{N-2} \theta\left(1 - |U_{11}|^2\right)$$

$$\Rightarrow P(U_{11}) \propto \exp(-N|U_{11}|^2) \quad \text{for} \quad N \to \infty$$

$$I \propto |U_{11}|^2 \Rightarrow P(I) \propto \exp(-I/\bar{I})$$

$$\beta = 1: \quad P(O_{11}) \propto \exp(-NO_{11}^2)$$

$$I \propto O_{11}^2 \Rightarrow P(I) \propto I^{-1/2} \exp(-I/2\bar{I})$$

All of us theoreticians should feel a little embarassed. We know the theoretical interpretation of the reduced width: it is the value of a wave function at the boundary, and we should have been able to guess what the distribution of such a quantity is. However, none of us were courageous enough to do that...Perhaps I am now too courageous when I try to guess the distribution of the distances between successive levels.

Eugene Wigner at the conference "Neutron Physics by Time-of-Flight", held at the Oak Ridge National Laboratory in 1957.