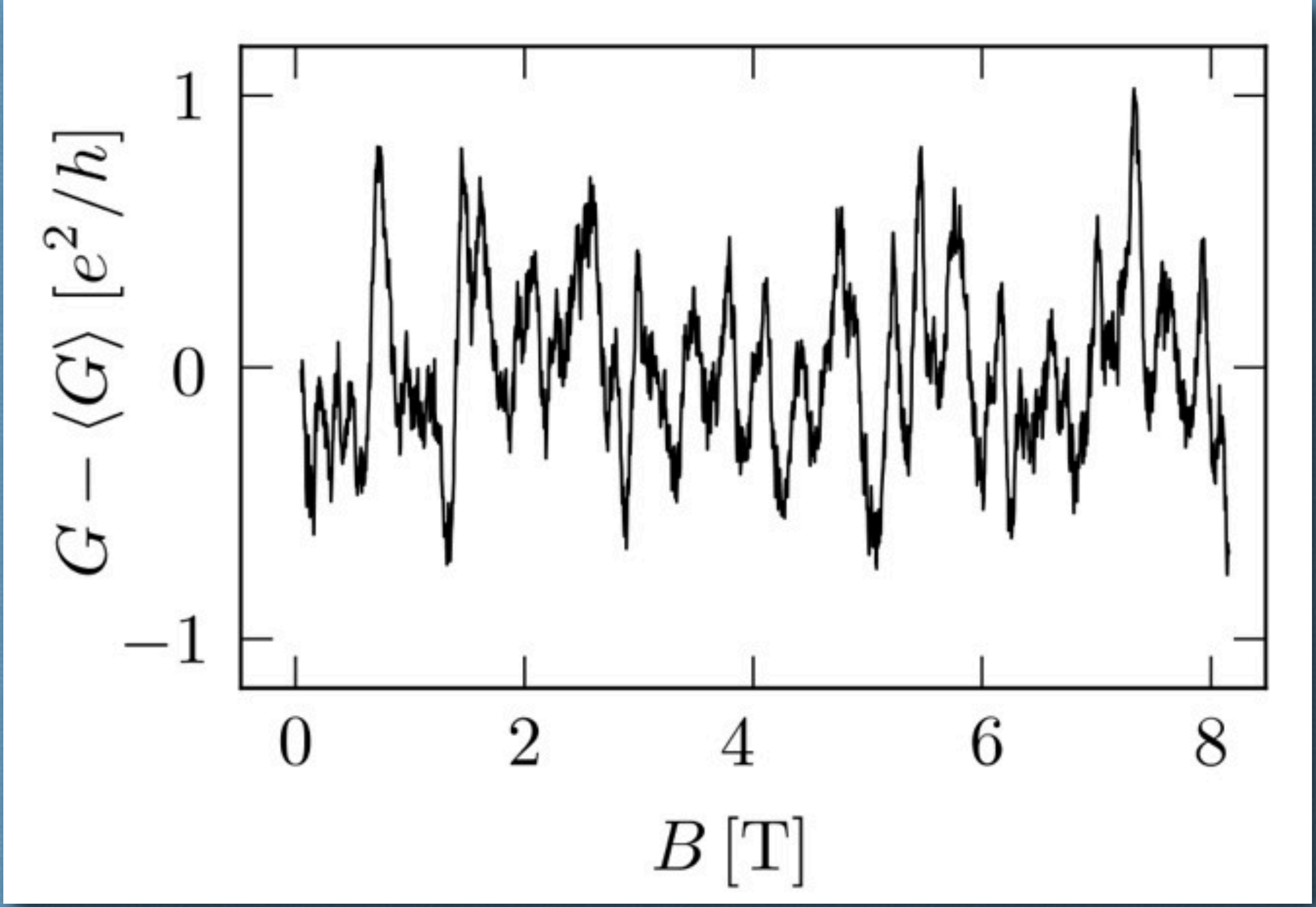
Random-matrix theory

II. random scattering matrices

RMT & mesoscopics



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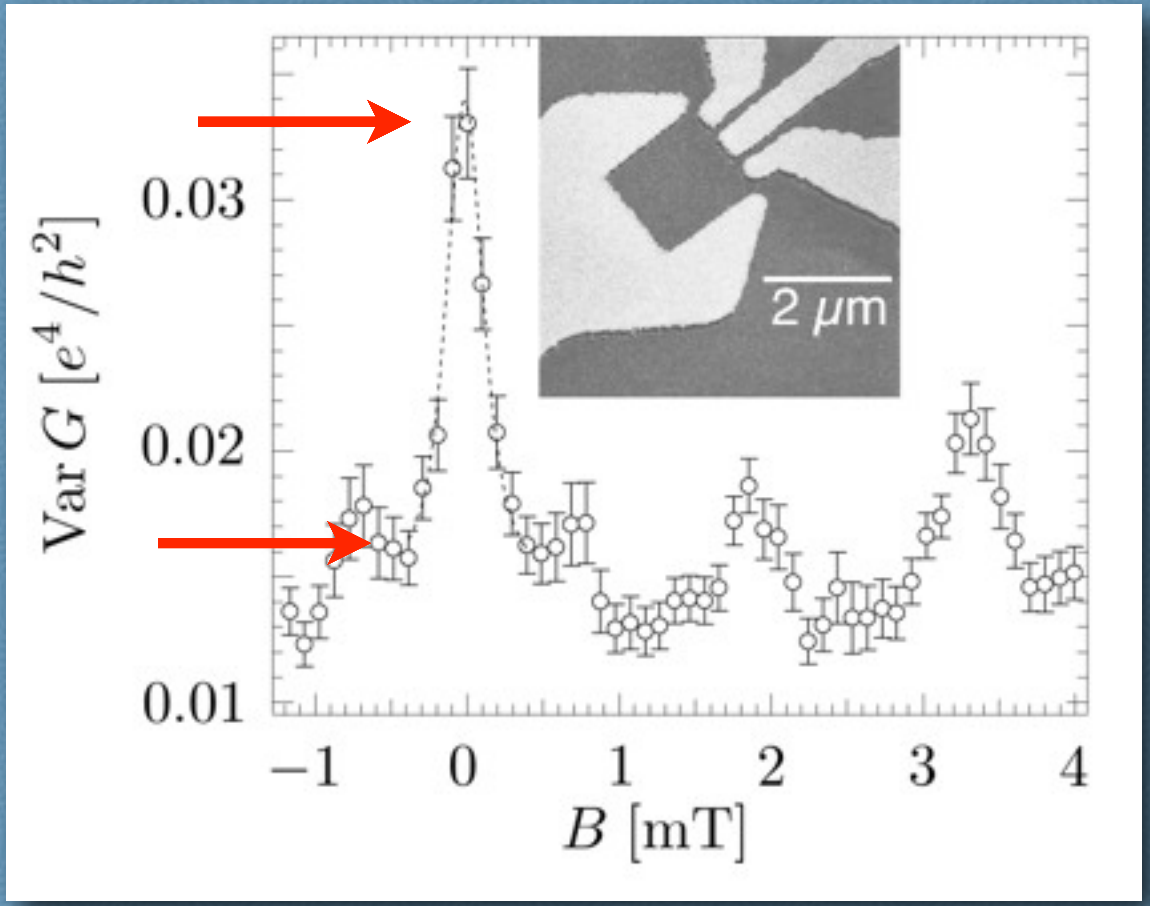
reproducible fluctuations
(magnetofingerprint)

Universal Conductance Fluctuations (UCF)

Altshuler, Lee & Stone (1985)

- $\delta G \sim e^2/h$ regardless of sample size or degree of disorder (so regardless of $\langle G \rangle$)
- sensitive to breaking of time-reversal symmetry: variance of G reduced by ½



shape fluctuations at constant *B*

spectral rigidity → UCF

Imry (1986)

$$G = \frac{e^2}{h} \mathrm{Tr}\, tt^\dagger = \frac{e^2}{h} \sum_{n=1}^{N} T_n$$ Landauer formula

- uncorrelated T_n's → Var G ~ N *too big!*
- random *t* → eigenvalue repulsion → fluctuations suppressed
- repulsion depends on symmetry → *B*-sensitivity

$$N \gg 1: \quad \mathrm{Var}\, G \propto 1/\beta$$

circular ensembles

Dyson 1962

random scattering matrix *S (CUE)*
unitary matrix $SS^{\dagger}=1$(current conservation)

symmetric matrix in the presence of time-reversal symmetry *(COE)* $S = UU^{T}$ with $U \in$ CUE

distribution of eigenvalues $e^{i\phi_n}$

$$P(S) = \text{constant} \Rightarrow P(\{\phi_n\}) \propto \prod_{n<m} |e^{i\phi_n} - e^{i\phi_m}|^{\beta}$$

scattering phase shifts – not observable in conduction

we need the distribution of the transmission eigenvalues

$$S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

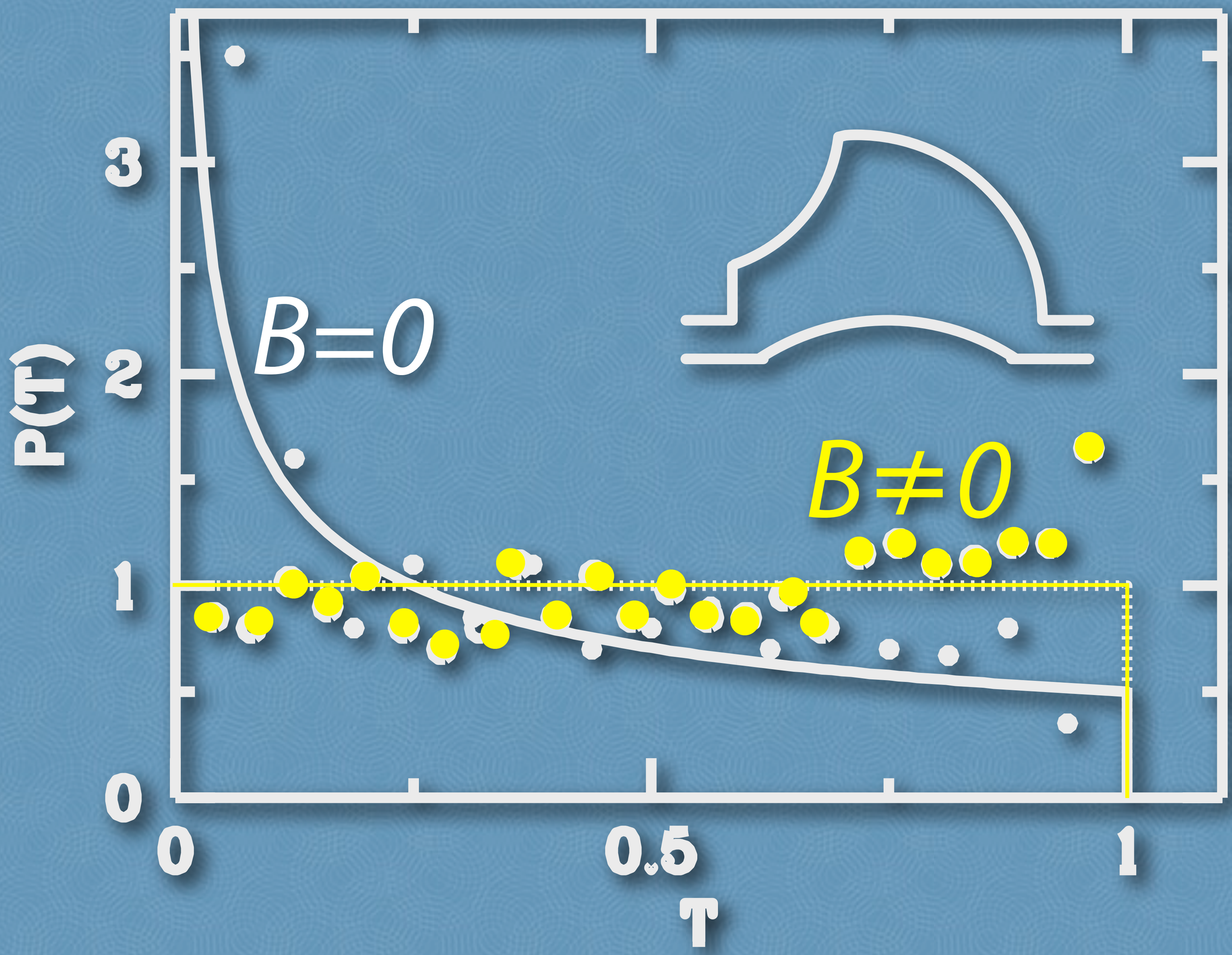
$2N \times 2N$ $\qquad$ $N \times N$

T_n is an eigenvalue of $tt^\dagger$

$$P(\{T_n\}) \propto \prod_{i<j} |T_i - T_j|^\beta \prod_n T_n^{-1+\beta/2}$$

Baranger & Mello | Jalabert, Pichard & CB 1994

quantum dot with single-channel point contacts



N=1

$$P(T) \propto T^{-1+\beta/2}$$

$$G = \frac{e^2}{h} T$$

magnetic field removes smallest T's: increases conductance

RMT of UCF

transport property $A = \sum_{n=1}^{N} a(T_n)$ *linear statistic*

$$P(\{T_n\}) \propto e^{-\beta W}$$

$$W(\{T_n\}) = \sum_{i<j} u(T_i, T_j) + \sum_i V(T_i)$$

we seek the variance in the large-N limit, to show that it is O(1) and ~1/β eigenvalue density $\rho(T) = \langle \sum_i \delta(T - T_i) \rangle$

$$\rho(T) = \frac{\int dT_1 \cdots \int dT_N \, e^{-\beta W} \sum_i \delta(T - T_i)}{\int dT_1 \cdots \int dT_N \, e^{-\beta W}}$$

$$W = \sum_{i<j} u(T_i, T_j) + \sum_i \int dT \, V(T) \delta(T - T_i)$$

two-point correlation function

$$K(T, T') = \left\langle \sum_{i,j} \delta(T - T_i) \delta(T' - T_j) \right\rangle - \rho(T)\rho(T')$$

variance $\mathrm{Var}\, A = \int dT \int dT' \, a(T) a(T') K(T, T')$

$$K(T, T') = -\frac{1}{\beta} \frac{\delta \rho(T)}{\delta V(T')}$$

$$V(T) + \int dT'\, u(T,T')\rho(T') = \text{constant}$$

large-N limit (mechanical equilibrium)

$$\Rightarrow K(T,T') = -\frac{1}{\beta}\frac{\delta\rho(T)}{\delta V(T')} = \frac{1}{\beta}u^{\text{inv}}(T,T')$$

$$\text{Var}\,A = \frac{1}{\beta}\int dT \int dT'\, a(T)a(T')u^{\text{inv}}(T,T')$$

CB 1993

- independent of V ⇒ “universal”
- $\sim 1/\beta$

quantum dot: $u(T,T') = -\ln|T - T'|$

$$0 < T, T' < 1$$

$$-\int_0^1 dT'' \ln|T - T''|\, u^{\mathrm{inv}}(T'', T') = \delta(T - T')$$

$$u^{\mathrm{inv}}(T,T') = -\frac{1}{\pi^2}\frac{\partial}{\partial T}\frac{\partial}{\partial T'}\ln\left|\frac{\sqrt{\lambda}-\sqrt{\lambda'}}{\sqrt{\lambda}+\sqrt{\lambda'}}\right| \qquad \lambda = (1-T)/T$$

$$\mathrm{Var}\,A = -\frac{1}{\beta\pi^2}\int_0^1 dT \int_0^1 dT' \frac{da(T)}{dT}\frac{da(T')}{dT'}\ln\left|\frac{\sqrt{\lambda}-\sqrt{\lambda'}}{\sqrt{\lambda}+\sqrt{\lambda'}}\right|$$

UCF: $G/G_0 = \sum_n T_n \Rightarrow a(T) = T$

$G_0 = e^2/h$

$$\mathrm{Var}\,G/G_0 = \frac{1}{8\beta}$$

*from UCF to U*F: any linear statistic has variance O(1) and ~1/β*

- conductance $a(T) = T \Rightarrow \text{variance} = 1/8\beta$
- shot noise $a(T) = T(1-T) \Rightarrow \text{variance} = 1/64\beta$
- …+ many other transport properties

not restricted to logarithmic eigenvalue repulsion

disordered wire: Rejaei & CB 1993

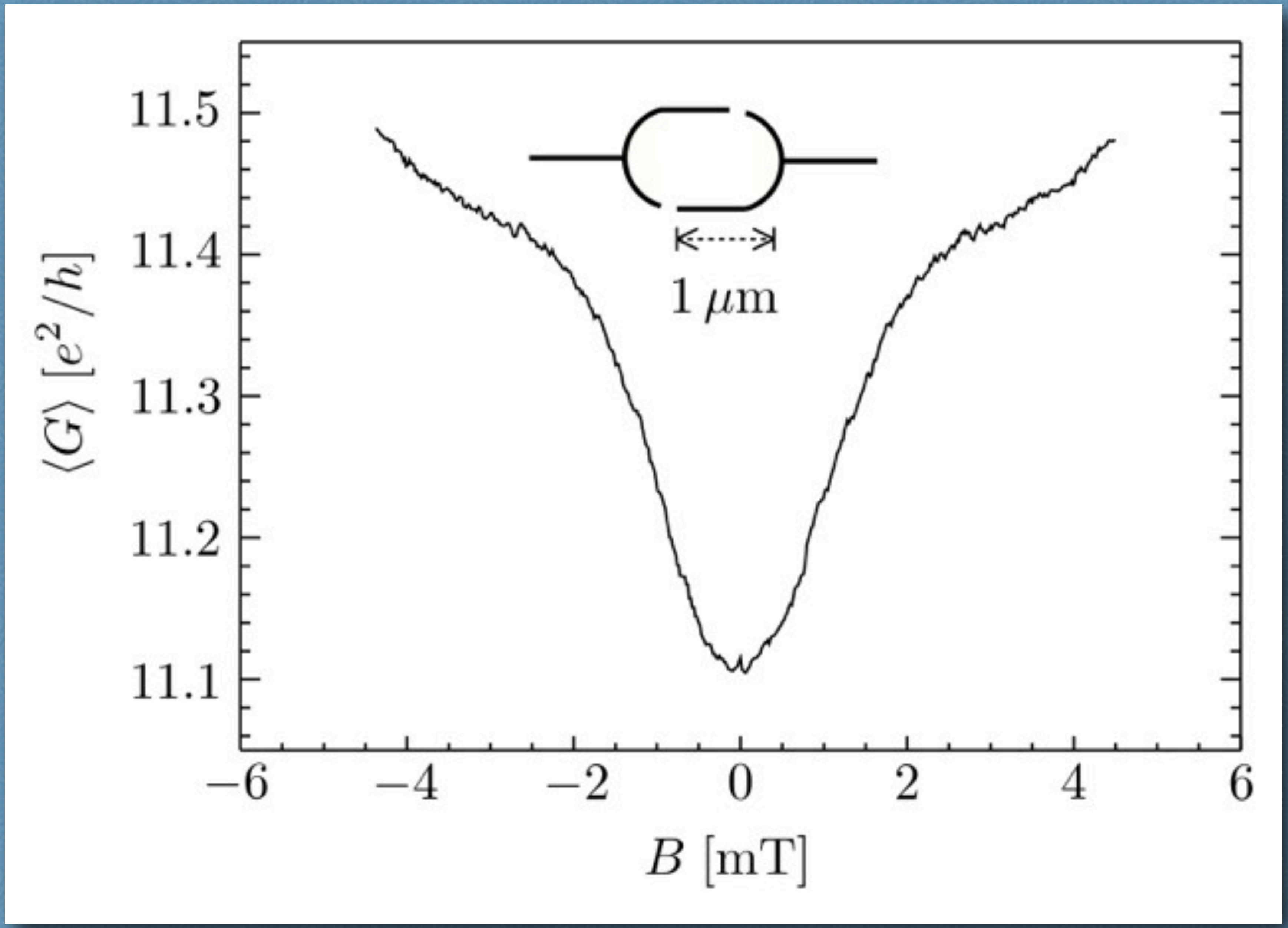
$$u(T,T') = -\frac{1}{2}\ln|T-T'| - \frac{1}{2}\ln|\mu-\mu'|$$

$$\mu = \text{arsinh}^2\sqrt{1/T-1}$$

conductance variance $2/15\beta$

weaker repulsion gives larger variance (2/15>1/8)

weak localization



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$$N \gg 1: \quad \langle G(B) \rangle - \langle G(0) \rangle = \frac{1}{4} \times \frac{2e^2}{h}$$

finite *N averages*

polynomial averages over *U(N)* [β=2]

only nonzero if

$\{\alpha_i\} = \{\beta_i\}, \{a_i\}=\{b_i\}$

$$\langle U_{\alpha a} U^*_{\beta b} \rangle = \frac{1}{N} \delta_{\alpha\beta} \delta_{ab}$$

$$\langle U_{\alpha a} U_{\alpha' a'} U^*_{\beta b} U^*_{\beta' b'} \rangle = \frac{1}{N^2-1} \left(\delta_{\alpha\beta} \delta_{ab} \delta_{\alpha'\beta'} \delta_{a'b'} + \delta_{\alpha\beta'} \delta_{ab'} \delta_{\alpha'\beta} \delta_{a'b} \right)$$

Weingarten coefficients

$$- \frac{1}{N(N^2-1)} \left(\delta_{\alpha\beta} \delta_{ab'} \delta_{\alpha'\beta'} \delta_{a'b} + \delta_{\alpha\beta'} \delta_{ab} \delta_{\alpha'\beta} \delta_{a'b'} \right)$$

$$G = G_0 \sum_{n,m=M+1}^{2M} |U_{nm}|^2, \quad N = 2M$$

$$\langle G \rangle = \frac{M}{2} G_0, \quad \mathrm{Var}\, G = \frac{M^2}{16M^2 - 4} G_0^2$$

analogously, for β=1

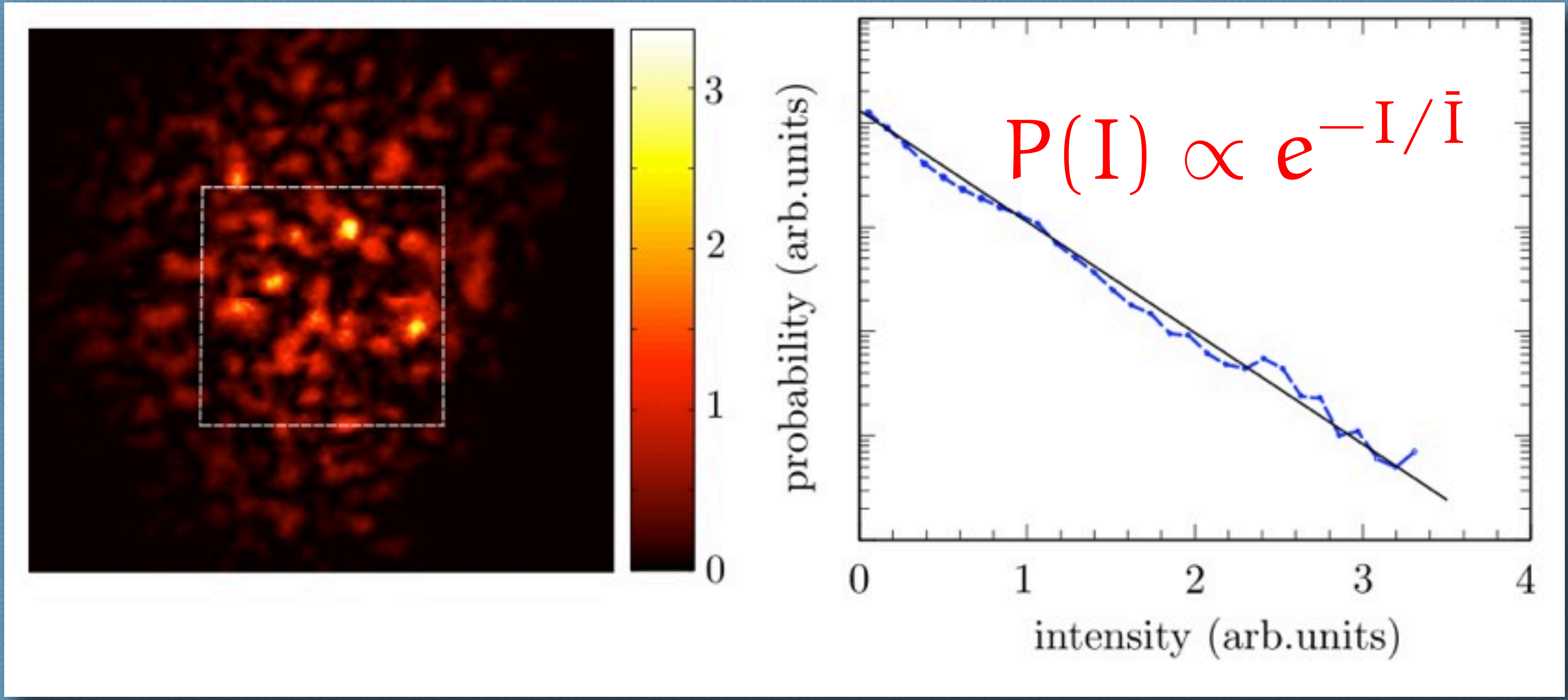
$S = UU^t, \quad U$ uniform in $U(N)$

$$\langle G \rangle = \frac{M^2}{2M+1} G_0$$

$< M/2$ "weak localization"

$\delta G \to -G_0/4$ for $M \to \infty$

optical speckle



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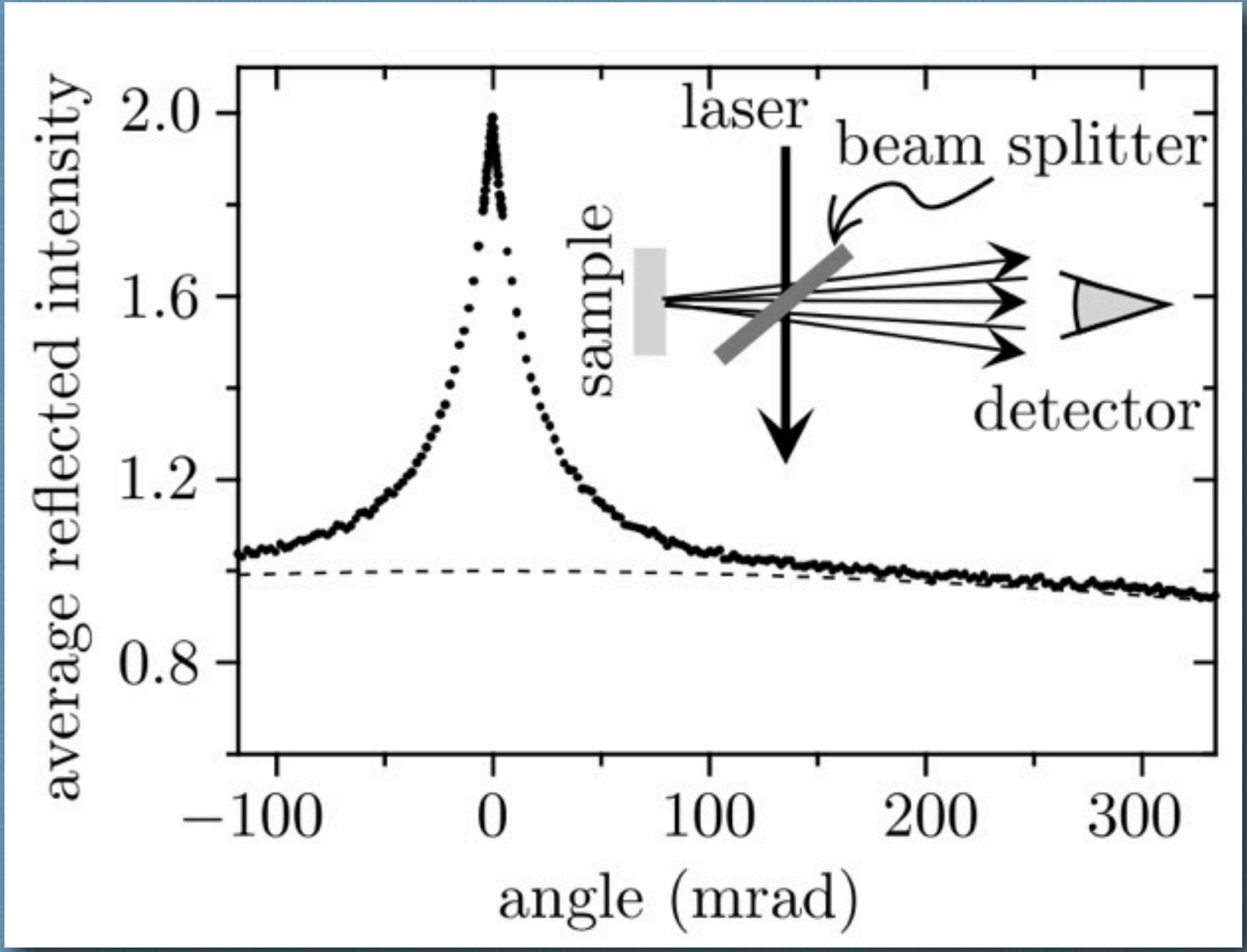
$$I_n = |S_{nm}|^2$$ *S* random unitary matrix

for incident radiation in mode *m*, scattered into mode *n*

(mode ~ speckle in the far field)

coherent backscattering



$$S_{nm} = S_{mn}$$

time-reversal symmetry

$$\langle |S_{nm}|^2 \rangle = \frac{1 + \delta_{nm}}{N + 1}$$

open transmission channels

Dorokhov, 1984

$$\beta = 2: \quad V(T) = 0, \quad u(T - T') = -\ln|T - T'|$$

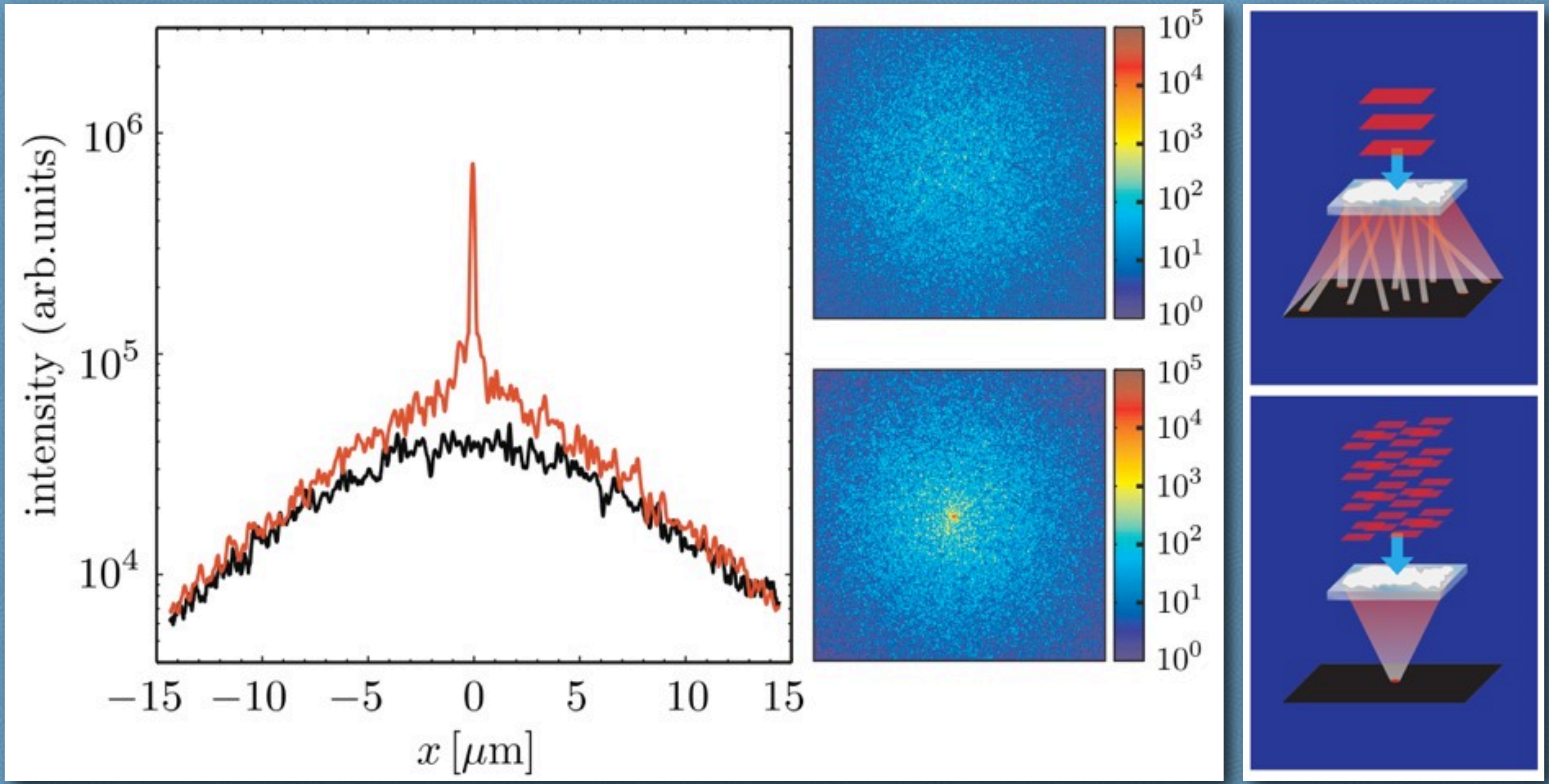
$$N \gg 1: \quad \int_0^1 dT' \, \rho(T') \ln|T - T'| = \text{constant for } 0 < T < 1,$$

$$\rho(T) = \frac{N}{\pi} \frac{1}{\sqrt{T}\sqrt{1-T}} + \mathcal{O}(1)$$

mean of T equals ½, but ρ(T) is minimal at T=½

bimodal transmission eigenvalue distribution

detection of open transmission channels



Vellekoop & Mosk (2008)