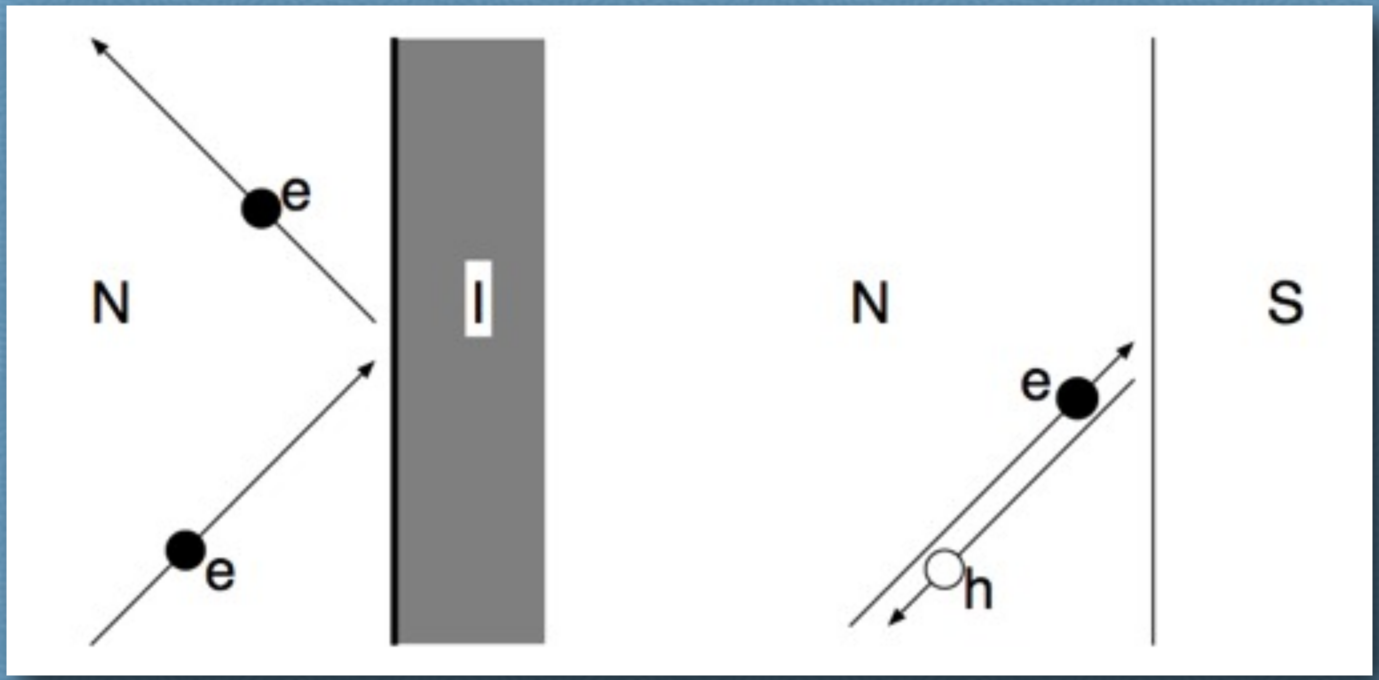
Random-matrix theory

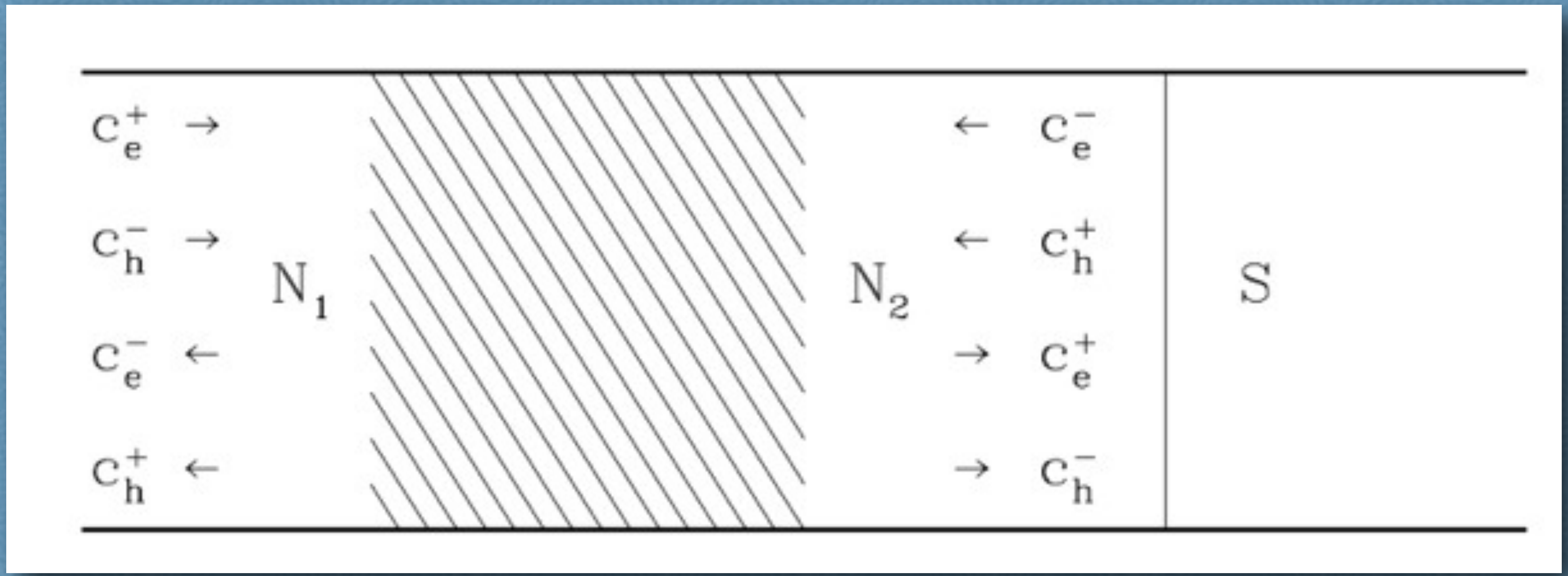
IV. Andreev reflection & topological superconductors

specular reflection | Andreev reflection



- *energy conservation:* Yes | Yes
- *momentum conservation:* No | Yes
- *charge conservation:* Yes | No
- *phase shift:* π | –π/2
- *spin band switched:* No | Yes

scattering matrices



normal scattering: $S_N(\varepsilon) = \begin{pmatrix} s(\varepsilon) & 0 \\ 0 & s^*(-\varepsilon) \end{pmatrix}$

Andreev reflection: $S_A = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$

combined: $S = \begin{pmatrix} s_{ee} & s_{eh} \\ s_{he} & s_{hh} \end{pmatrix}$

$s_{hh}(\varepsilon) = s_{ee}^*(-\varepsilon)$

$s_{he}(\varepsilon) = -s_{eh}^*(-\varepsilon)$

$$s = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

$$r_{he}(\varepsilon) = -it^*(-\varepsilon)[1 + r'(\varepsilon)r'^*(-\varepsilon)]^{-1}t'(\varepsilon)$$

multiple Andreev reflections

$$1 + r'r'^\dagger = 2 - t't'^\dagger$$

at the Fermi level (ε=0), zero magnetic field

$$\mathrm{Tr}\, r_{he}r_{he}^\dagger = \sum_n T_n^2(2 - T_n)^{-2}$$

Andreev conductance

$$G_{NS} = \frac{2e^2}{h}\mathrm{Tr}\left(1 - r_{ee}r_{ee}^{\dagger} + r_{he}r_{he}^{\dagger}\right)$$

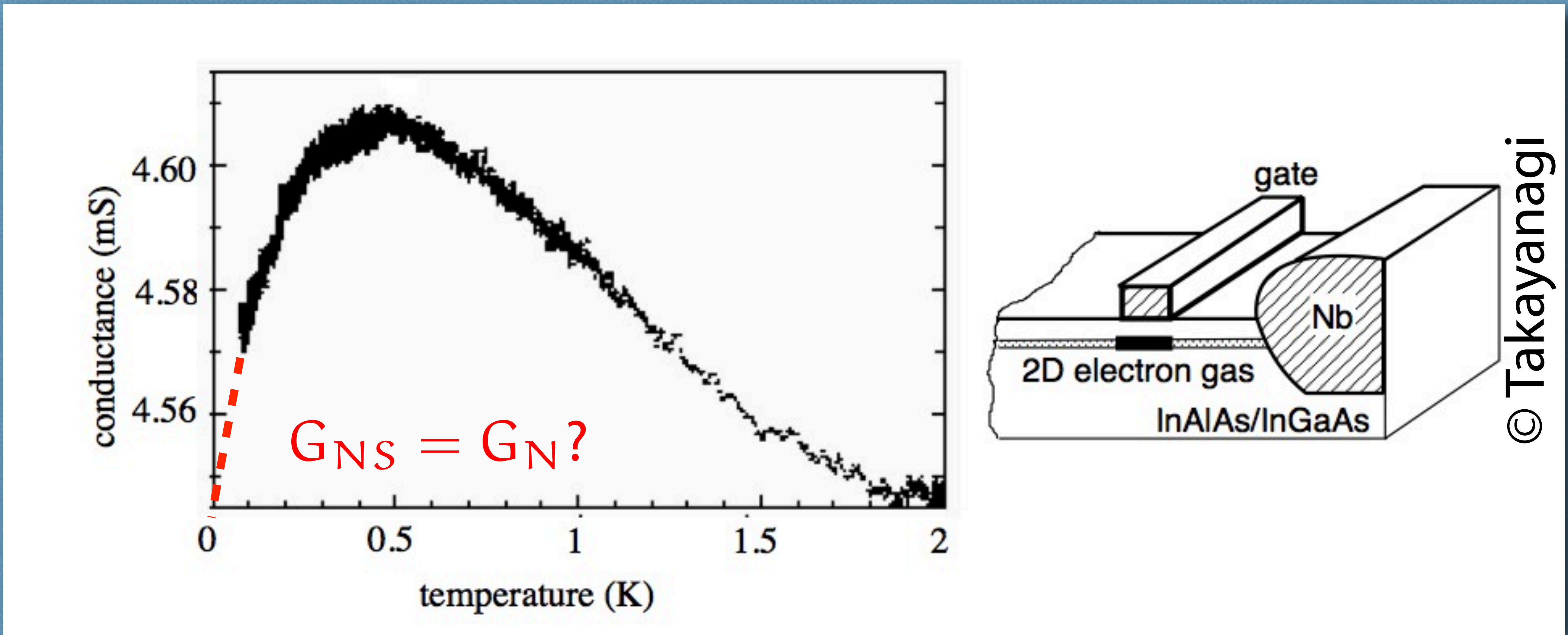
$$= \frac{4e^2}{h}\mathrm{Tr}\, r_{he}r_{he}^{\dagger}$$

$$= \frac{4e^2}{h}\sum_{n}\frac{T_n^2}{(2-T_n)^2}$$

$$G_N = \frac{2e^2}{h}\sum_{n} T_n$$

$$G_{NS} \leqslant 2G_N$$

what is the conductance of a disordered NS junction?



without phase coherence

$$G_{NS}(L) \approx 2G_N(2L)$$
$$= G_N(L)$$

$$G_{NS} = G_0 \sum_n \frac{2T_n^2}{(2 - T_n)^2} \qquad T_n = 1/\cosh^2 x_n$$

$$= G_0 \sum_n \frac{2}{\cosh^2 2x_n}$$

$$G_N = G_0 \sum_n T_n = G_0 \sum_n \frac{1}{\cosh^2 x_n}$$

uniform density of the x_n's $\Rightarrow$

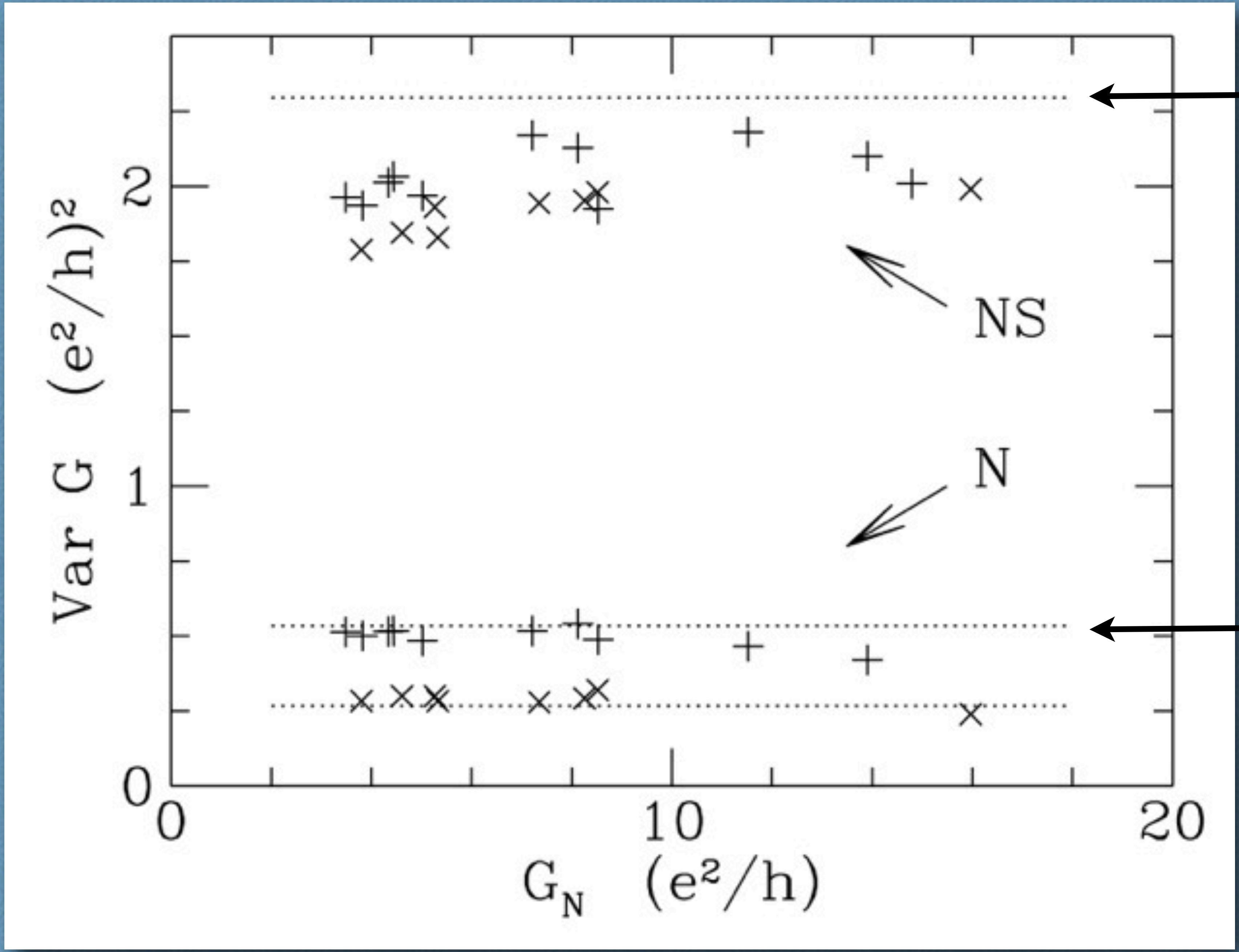
$$\int_0^\infty f(x)\,dx = \int_0^\infty 2f(2x)\,dx \qquad \langle G_{NS} \rangle = \langle G_N \rangle$$

fully phase coherent!

CB 1992

UCF in an NS junction

Marmorkos, Jalabert & CB (1993)



$\frac{64}{15} - \frac{192}{\pi^4} \approx 2.29$

$8/15$

+ β=1

x β=2

Var $G_{NS} \approx 4$ Var G_N for $\beta = 1$

Var $G_{NS}(\beta = 2) \approx$ Var $G_{NS}(\beta = 1)$???

Andreev conductance is not a linear statistic for β=2

$$G_{NS} = \frac{4e^2}{h} \mathrm{Tr}\,(1 + rr^*)^{-1} tt^\dagger (1 + r^T r^\dagger)^{-1} t^T t^*$$

$$r^* \neq r^\dagger \quad \text{for} \quad \beta = 2$$

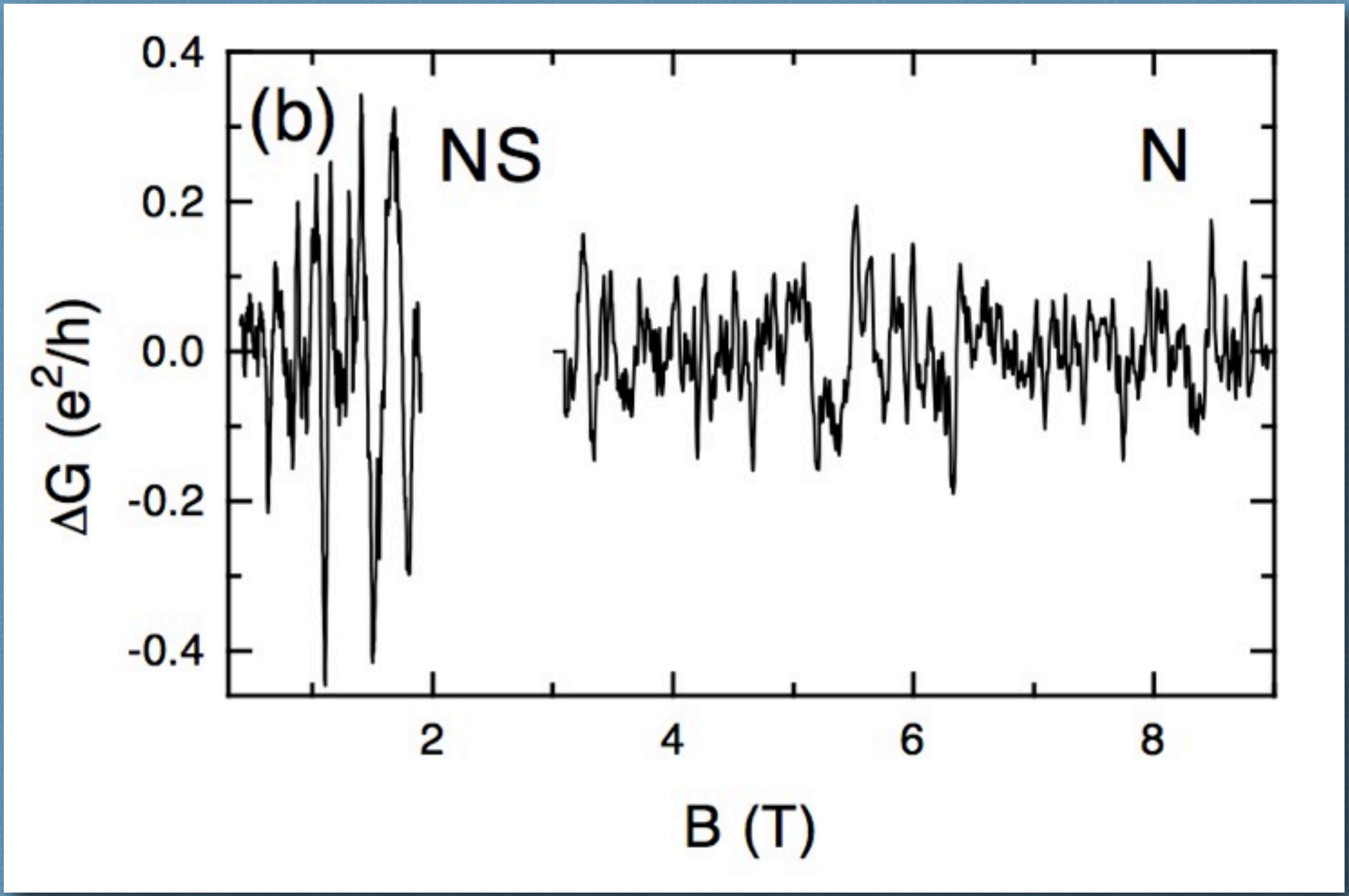
G_{NS} depends on eigenvectors as well as on eigenvalues

result: $\mathrm{Var}\, G_{NS} = \frac{32}{15}(e^2/h)^2 = 8\, \mathrm{Var}\, G_N$

≈ 2.13

Brouwer & CB (1995)

gold wire + niobium contact



$$\mathrm{Var}\, G_{NS} \approx 7.8\, \mathrm{Var}\, G_N$$

Hecker, Hegger, Altland & Fiegle (1997)

Topological superconductors

spin-singlet *(s-wave)* superconductor:

$$s_{\mathrm{hh}}(\varepsilon) = s^*_{\mathrm{ee}}(-\varepsilon) \qquad s_{\mathrm{he}}(\varepsilon) = -s^*_{\mathrm{eh}}(-\varepsilon)$$

spin-triplet *(p-wave)* superconductor:

$$s_{\mathrm{hh}}(\varepsilon) = s^*_{\mathrm{ee}}(-\varepsilon) \qquad s_{\mathrm{he}}(\varepsilon) = s^*_{\mathrm{eh}}(-\varepsilon)$$

no minus sign because
Andreev reflection without
switch of spin band

circular ensemble

$$S \mapsto uSu^\dagger \qquad u = \sqrt{\frac{1}{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

change of basis (e ± h)

$$S^*(-\varepsilon) = S(\varepsilon)$$

S is a *real orthogonal* matrix at $\varepsilon=0$

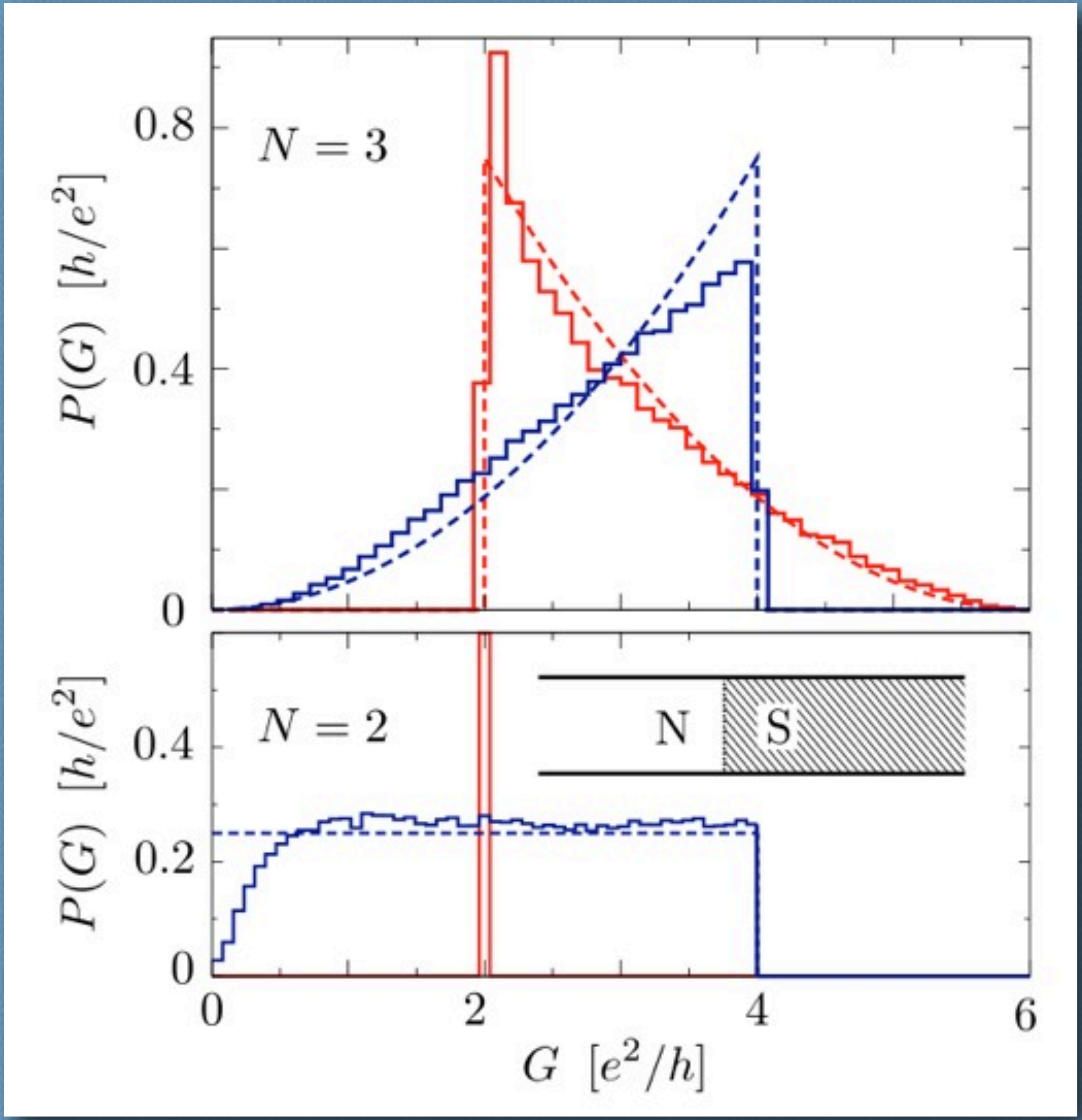
circular real ensemble (CRE)

topological quantum number

real orthogonal matrix has
determinant ± 1

Det $S = 1$: topologically trivial
(connected to unit matrix)

Det $S = -1$: topologically nontrivial
(disconnected from unit matrix)



p-th cumulant independent of *Det S* if *p*<*N*

2N×2N scattering matrix in CRE, with Det S=1 or Det S =−1