GEORGE UHLENBECK AND THE DISCOVERY OF ELECTRON SPIN

How two young Dutchmen, one with only a master's degree, the other a graduate student, made a most important finding in theoretical atomic physics.

Abraham Pais

The owl depicted on the signet ring George Uhlenbeck used to wear—"Uhlenbeck" in German means "owl's brook"—derives from his family's coat of arms. The shield reads, in the language of heraldry: Azure, on a tree trunk proper rising from water argent, an owl contourné, head affronty. In plain language, it depicts an owl with its head turned toward you, sitting on a tree trunk in natural color, which rises up out of a silvery brook. (I owe the transcription of the Dutch blazon into English heraldry to Michael Maclagan, the Richmond Herald in the College of Arms, in London.)

The Uhlenbeck ancestry can be traced to German roots. Records for the years 1634 and 1656 kept at the Staatsarchiv in Düsseldorf, at one time the capital of the duchy of Berg, mention that at those times a Jan in der Ulenbeck was the proprietor of the estate Üllenbeck, situated near the township of Velbert in the district of Angermund. The next four generations of George's ancestors were born and raised on that same estate. A great-great-grandson of Jan in der Ulenbeck, Johannes Wilhelmus Uhlenbeck, went into military service under King Frederick II—"the Great"—of Prussia. On account of a duel he had to flee the country. In 1768 he entered the military service of the Dutch East India Company on the island of Ceylon, a Dutch colony from 1658 until 1796. He is the first of the Dutch branch of the Uhlenbeck family. (I am deeply grateful to Else Uhlenbeck for information on her husband's family background.)

Eugenius Marius Uhlenbeck, a great-great-grandson of Johannes Wilhelmus, born in 1863 on the island of Sumatra in the Dutch East Indies (now Indonesia), served with the Dutch East Indian Army, eventually as lieutenant colonel. During the Atjeh wars in northern Sumatra (1873–1904), two of his brothers, also army officers, threw

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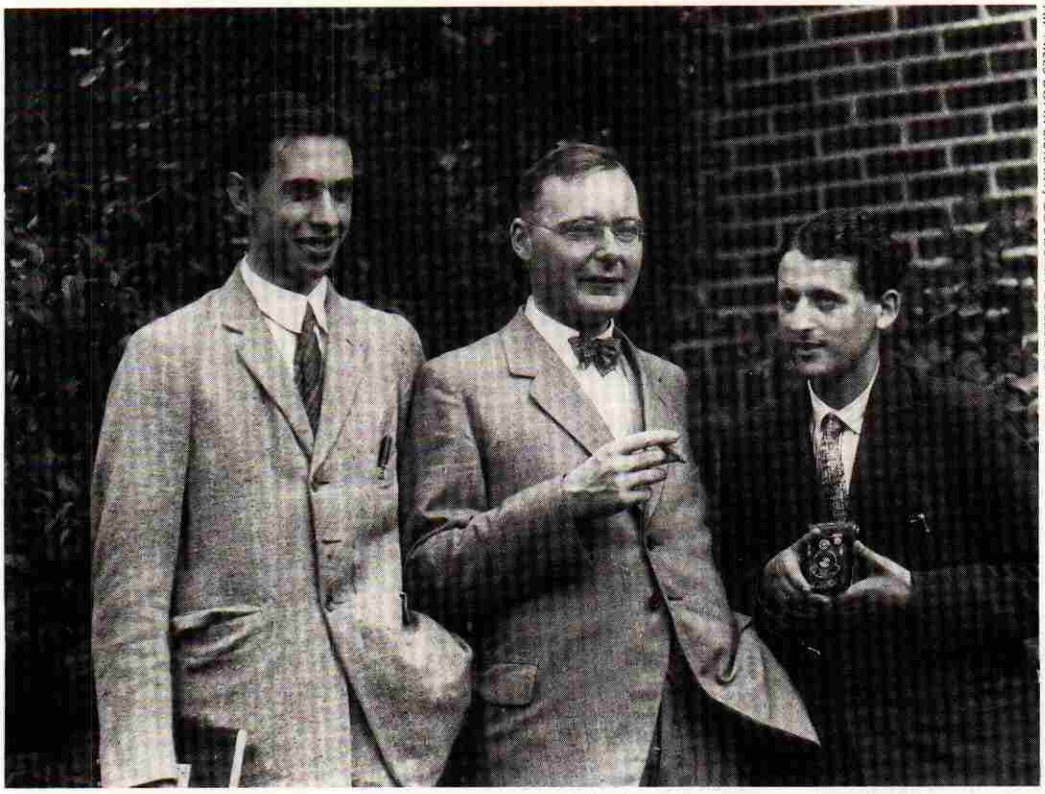

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George Uhlenbeck (left) with Hendrik Kramers (middle) and Samuel Goudsmit in Ann Arbor, Michigan. This photograph was taken around 1928, three years after Uhlenbeck and Goudsmit proposed the idea that each electron rotates with an angular momentum $\hbar/2$ and carries a magnetic moment of one Bohr magneton, $e\hbar/2mc$.

themselves on their sabres to avoid capture by cannibals. In 1893 he married Annie Beegers, who was born on Sumatra in 1874, the daughter of a Dutch major general.

George Eugene Uhlenbeck, born in Batavia (now Jakarta) on 6 December 1900, was one of the six children of that marriage. Two of the other children died very young in the Indies, of malaria. Military duties caused the family to move about a good deal. Thus it happened that George received his first schooling at a kindergarten in Padangpandjang, on Sumatra.

Early interest in physics

In 1907 the family moved permanently to Holland and settled in the Hague, where Uhlenbeck's regular education began, first at an elementary school, then at a higher burgher school (what is now called an atheneum). A three-year course in physics first drew him to the subject that was to be his life's devotion. Eager to learn more, he would bicycle to the Royal Library in the Hague to seek further information. There he absorbed Hendrik Lorentz's *Lessen over de Natuurkunde* (*Lectures on Physics*), an undergraduate university text. Uhlenbeck's particular interest in kinetic gas theory dates from those early days. His knowledge of physics, uncommonly deep for a high school student, brought him to the attention of his physics teacher, A. H. Borgesius, who discussed science with him and gave him books from which to study differential and integral calculus.

In July 1918 Uhlenbeck passed his final high school examination. He could not enter a Dutch university, however, because his school had not provided training in Greek and Latin, at that time a prerequisite by law for university study in any discipline. (Johannes van der Waals and Jacobus van't Hoff, in similar positions at earlier times, had been able to enter a university only by special governmental dispensation.) In September 1918 he therefore entered the Technische Hogeschool (Institute of Technology) in Delft, intent on studying chemical engineering. However, almost immediately thereafter a new law was enacted dispensing with the Greek and Latin requirements for university training in the sciences. In January 1919 Uhlenbeck left Delft and enrolled in the University of Leiden to study physics and mathematics.

At that time, the professors at Leiden were Paul Ehrenfest, Heike Kamerlingh Onnes and J. P. Kuenen. Every Monday, Lorentz, Ehrenfest's predecessor, would come from Haarlem to give a physics lecture.

Uhlenbeck received his undergraduate education in physics, both theoretical and experimental, from Kuenen. Every physics undergraduate had to take a laboratory course. Uhlenbeck's laboratory reports had a strongly theoretical bent. "Even for the simplest electromagnetic experiments I started from the Maxwell equations." (Throughout this article, quotations without attribution are from private conversations with Uhlenbeck.) Kuenen's laboratory assistant, who could not quite follow those reports, showed them to Kuenen, who was impressed. Largely through Kuenen's advocacy, Uhlenbeck obtained a scholarship, a quite welcome development because his father did not find it easy to raise four children on a military pension. (From September 1921 until June 1922 Uhlenbeck partially supported himself by teaching ten hours a week at a high school in Leiden; the flirtatious young girls there contributed greatly to his difficulties in keeping order in his classes.)

In addition to his physics studies, Uhlenbeck took courses in mathematics by J. Droste, Jan Cornelis Kluyver and W. van der Woude, in astronomy by J. Woltjer, in crystal structure by K. Martin, in physical chemistry by F. Schreinemaker and in inorganic chemistry by Willem Jorissen. Among his fellow students who later would make names for themselves in physics were Dirk Coster, Gerard Dieke and Samuel Goudsmit.

All through his student years, Uhlenbeck commuted by train between Leiden and his family's home on the Lübeckstraat in the Hague. His mother would pack his lunch and give him a *kwartje* (25 cents) for coffee. He saved the money until one day he spent it on a secondhand copy of Boltzmann's *Vorlesungen über Gastheorie* (*Lectures on Gas Theory*), lecture notes that he found hard to grasp. Not long thereafter his brother-in-law, a chemist, introduced him to Paul and Tatiana Ehrenfest's encyclopedia article on statistical mechanics.[1] "That was a revelation. I began to see what Boltzmann was up to."

Ehrenfest

After graduating in December 1920, Uhlenbeck attended courses by Ehrenfest and Lorentz and also the celebrated Wednesday evening "Ehrenfest colloquium," which one could attend by invitation only, but to which one had to go once admitted. Ehrenfest even took attendance.

Ehrenfest was by far the most important scientific figure in Uhlenbeck's life. In all the years I knew Uhlenbeck, in Utrecht, in Ann Arbor and in New York, a single picture always stood on his office desk: a small photograph of a warmly smiling Ehrenfest. In 1956, upon receiving the Oersted Medal from the American Association of Physics Teachers, Uhlenbeck publicly expressed[2] his veneration for his respected and beloved teacher, whose life had long since come to a tragic end. In his acceptance lecture he recalled some characteristic Ehrenfest sayings:

> *Was ist der Witz*...? [Do you say that to make a point, or only because it happens to be true?] *Weshalb habe ich solche gute Studenten? Weil ich so dumm, bin.* [Why do I have such good students? Because I am so stupid.]

Uhlenbeck also described some typical traits of Ehrenfest's style of lecturing and conducting seminars:

> First the assertion, then the proof.... His famous clarity, not to be confused with rigor.... He never gave or made problems; he did not believe in them; in his opinion the only problems worth considering were those you proposed yourself.... He worked essentially with only one student at a time, and that practically every afternoon during the week.... In the beginning, at the end of the afternoon one was dead tired.

Uhlenbeck also added a personal touch:

> One of the compliments I treasure most is when some of [my own students] told me of the identical experience they had working with me, especially the fact of the extreme exhaustion in the beginning.

It is one of the good fortunes of my own life to have been in a position to pay Uhlenbeck this compliment myself, with feeling.

Let us return to the early 1920s. Ehrenfest's graduate lectures consisted of a two-year course: Maxwell theory, ending with the theory of electrons and some relativity, one year; and statistical mechanics, ending with atomic structure and quantum theory, the other. Uhlenbeck attended these lectures and took additional instruction in mathematics. One day toward the end of his second graduate year, Ehrenfest asked in class whether anyone might be interested in a teaching position in Rome. Uhlenbeck raised his hand. So it came to pass that from September 1922 until June 1925 he became the private tutor in mathematics, physics, chemistry, Dutch, German and Dutch history of the younger son of the Dutch ambassador J. H. van Royen. The summers were spent in Holland, however, and in September 1923 Uhlenbeck obtained the degree of "doctorandus," the equivalent of a master's degree.

Fermi

For about a year right after his arrival in Rome, Uhlenbeck took Italian language lessons at the Berlitz school. Thereafter he continued his Italian studies by taking private tuition two hours a week, eventually reading Dante's *Divina Commedia* with his teacher. (He reread Dante in later years and was fond of occasionally reciting passages of this work.) By the fall of 1923 he had mastered the language sufficiently to attend mathematics courses taught at the University of Rome by Federigo Enriques, Tullio Levi-Civita and Vito Volterra. He also made contact with Roman physicists. When Uhlenbeck was in Holland during the summer of 1923, Ehrenfest told him of a young Italian physicist by the name of Enrico Fermi who had written a paper on the ergodic theorem. Ehrenfest had not understood Fermi's reasoning and asked Uhlenbeck to carry a letter to Rome with questions for Fermi. Thus it came about that Uhlenbeck and Fermi, who was nearly one year younger, met for the first time in the autumn of 1923. Their acquaintance grew into a friendship that lasted throughout Fermi's life. Together with a few other young Italian physicists they organized a small colloquium. "Fermi was the born leader and did most of the talking."

Fermi wrote his paper on the ergodic theorem in 1923 during his stay in Göttingen, Germany, a visit that adversely affected his self-confidence. The learned Göttingen style did not agree with him. At Uhlenbeck's urgings, Fermi went to Leiden for three months in 1924; he even published a paper in Dutch. One of Uhlenbeck's contributions to physics lies in initiating the personal contact between Ehrenfest and Fermi, which helped greatly to restore Fermi's self-confidence.

Thus Uhlenbeck stayed in touch with the sciences during his Italian period. Yet they receded from the center of his attention. He became deeply involved in history, especially cultural history. He became a regular visitor of the Nederlandsch Historisch Instituut te Rome; befriended a Dutch contemporary, Johan Quiryn van Regteren Altena (who later became a professor of art history in Amsterdam); and studied the works of Johan Huizinga, a professor in Leiden, and other cultural historians. The first article Uhlenbeck ever published is historical and is written in Dutch.[3] It deals with the Dutchman Johannes Heckius, one of the four cofounders of the Acedemia dei Lincei in Rome, in 1603. That it was the Dutchman Uhlenbeck who introduced Fermi, born and raised a Roman, to Michelangelo's Moses in the church of San Pietro in Vincoli says something about the personalities of the two physicists.

When Uhlenbeck left Rome for good to return to Holland, in mid-June 1925, he was seriously considering giving up physics to become a historian. He called on Huizinga in Leiden, who gave him a friendly reception; and he discussed the matter with his uncle, the distin-

Paul Ehrenfest (front, center) in Leiden, 1924. In January 1919 Uhlenbeck enrolled at the University of Leiden, where Ehrenfest was a professor. Ehrenfest became by far the most important scientific figure in Uhlenbeck's life. In the photograph with Ehrenfest are, from left to right, Gerard Dieke, Goudsmit, Jan Tinbergen, Ralph Kronig and Enrico Fermi. (Brookhaven National Laboratory photograph, courtesy AIP Niels Bohr Library.)

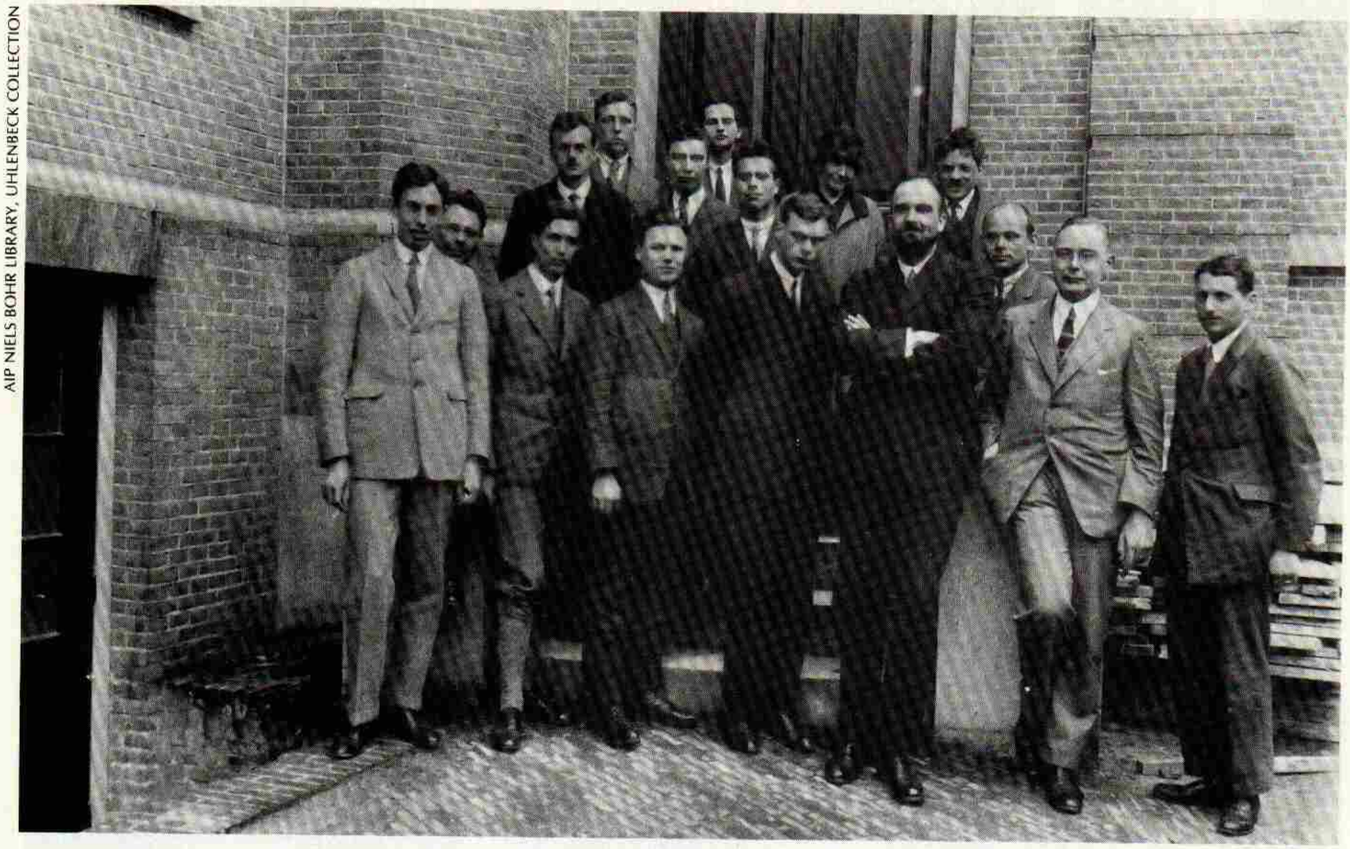
guished linguist Christianus Cornelis Uhlenbeck, an expert on American Indian languages and a professor at Leiden. His uncle was sympathetic to the idea, but suggested that it might be best to obtain first a PhD in physics, because he had already progressed quite far. Ehrenfest also responded benevolently to the new project but suggested that Uhlenbeck first find out what was currently happening in physics. He proposed that Uhlenbeck work with him for a while, and also that he learn from Goudsmit what was going on in *Spektralzoologie*, as Pauli used to call the study of spectra. Uhlenbeck accepted both suggestions, at the same time arranging for lessons in Latin from a friend in the Hague. His work with Ehrenfest on wave equations in multidimensional spaces (with special emphasis on the differences between odd and even numbers of spatial dimensions) led to a mathematical paper,[4] followed by a joint paper with Ehrenfest.[5] Uhlenbeck enjoyed this collaboration. So did Ehrenfest, who in the fall of 1925 appointed Uhlenbeck to succeed the mathematician Dirk Struik as his assistant.

Throughout the summer of 1925 Goudsmit came to the Lübeckstraat to educate Uhlenbeck in spectra. In his later years Uhlenbeck would refer to this period as the "Goudsmit summer."

Then, in mid-September 1925, doctorandus Uhlenbeck and graduate student Goudsmit discovered spin. Gone were Uhlenbeck's aspirations of becoming a historian.

Work with Goudsmit

Samuel Abraham Goudsmit—"Sem" to his friends—was born in 1902 in the Hague, the son of a prosperous wholesale dealer in bathroom fixtures. His mother owned a fashionable hat shop. He got his first taste of physics at the age of 11 when browsing through an elementary physics text; he was particularly struck by a passage explaining how spectroscopy shows that stars are composed of the same elements as the Earth. As Goudsmit recalled, "Hydrogen in the sun and iron in the Big Dipper made Heaven seem cozy and attainable."[6] After finishing high school in one year less than the usual time, he became a physics student in Leiden, where Ehrenfest turned his interest into devotion. It soon became evident that he had a bent for intuitive, rather than analytical, thinking, starting from empirical hunches. Uhlenbeck later said of Goudsmit: "Sem was never a conspicuously reflective man, but he had an amazing talent for taking random data and giving them direction. He's a wizard at cryptograms." I. I. Rabi said: "He thinks like a detective. He is a detective."[6] In fact, Goudsmit once took an eight-month course in detective work, in which he learned to identify fingerprints, forgeries and bloodstains. A two-year uni-



At the Kamerlingh Onnes Laboratory in Leiden, 1926. Uhlenbeck is at the far left. Goudsmit is at the far right, next to Kramers. Ehrenfest is at the right, rear, next to his wife. Paul Dirac is the one in the dark coat at the left.

versity course taught him to decipher hieroglyphics. In physics the decoding of spectra became his passion. At age 18 he completed his first paper, on alkali doublets.[7] Uhlenbeck called it "a most presumptuous display of self-confidence but . . . highly creditable."[6]

In August 1925 the two men started their regular meetings in the Hague. George was the more analytic one, better versed in theoretical physics, a greenhorn in physics research and an aspiring historian with a paper on Heckius to his credit. Sem was the detective, thoroughly at home with spectra (on which he had already published several papers), known in the physics community and a part-time assistant to Pieter Zeeman in Amsterdam. In almost no time Sem's tutelage of George turned into joint research and publication, and their relationship into a close and lasting friendship. I know, more from my own later personal friendships with both than from their writings,[8,9,10] how each remained forever beholden to the other for his share of the work during those months. (See Goudsmit's article in PHYSICS TODAY, June 1976, page 40, and Uhlenbeck's article in PHYSICS TODAY, June 1976, page 43). There was no politesse, but deep appreciation.

Among the topics that Sem taught George that summer was Alfred Landé's theory of the anomalous Zeeman effect, those splittings of spectral lines that do not follow the patterns predicted much earlier by Lorentz on the basis of classical theory. In 1921 Landé had found it possible to explain those anomalies by the new and quite daring assumption that angular momentum quantum numbers can take on half-integer values. Sem went on to tell the story: How Werner Heisenberg, in his first published paper, had gone further by proposing that in alkalis the valence electron and the residual atomic *Rumpf*, the core, each have angular momentum $\frac{1}{2}$ (in units of $h/2\pi$). How then Landé deduced from this that g, the gyromagnetic ratio, should have the value 2 for the core instead of 1, the classical prediction. How next Pauli had shown that the core had to have zero angular momentum. How he, Sem, had written that Landé's $g = 2$ is "completely incomprehensible" but that, using this assumption, one nevertheless "masters completely the extensive and complicated material of the anomalous Zeeman effect."[11] How Pauli thereupon—we are now in January 1925—had proposed to assign a new, a fourth, half-integer-valued quantum number, not to the core but to the electron itself. And how Pauli was thereby led to the discovery of the exclusion principle.

Another subject Sem taught George was Arnold Sommerfeld's formula for the fine structure of the hydrogen spectrum: how it worked very well, how there was no problem with the Zeeman effect that experimentally appeared to be (but of course was not) normal at that time.

George was unhappy. "He knew nothing; he asked all those questions which I never asked," Goudsmit would later recall.[8] Why two distinct models if the alkalis and hydrogen were so much alike? Why not try the half-integer quantum numbers on hydrogen as well? In August 1925 this led to their first joint paper, a little-known but quite good piece of work, written in Dutch, in which they modified the quantum number assignments Sommerfeld had given to the atomic levels and reported an improved treatment of He^+ fine structure.[12]

Goudsmit wrote about what happened next: "Our luck was that the idea [of spin] arose just at the moment when we were saturated with a thorough knowledge of the

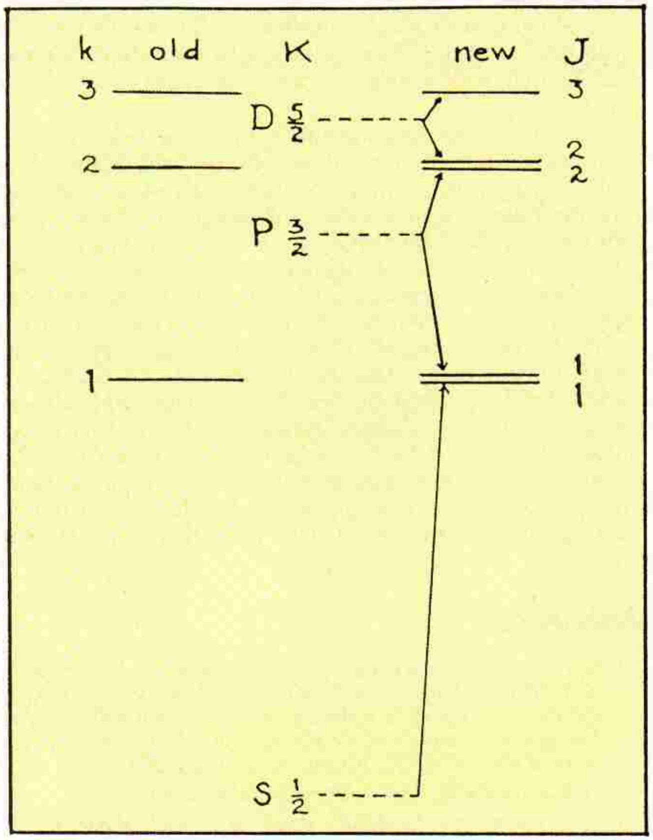


Diagram from L. H. Thomas's February 1926 *Nature* paper explaining the extra factor of 2 in the formula for fine-structure splitting in hydrogen-like spectra as derived from a semiclassical treatment of spin precession. Thomas's relativistically correct calculation reduced the angular velocity of the electron (as seen by the nucleus) by the needed factor of 2. The dotted lines represent the levels calculated without spin. (From *Nature* **117**, 514, 1926.)

structure of atomic spectra, had grasped the meaning of relativistic doublets, and just after we had arrived at the correct interpretation of the hydrogen atom."[9] Uhlenbeck recalled: "It was then that it occurred to me that, since (as I had learned) each quantum number corresponds to a degree of freedom of the electron, the fourth quantum number must mean that the electron had an additional degree of freedom—in other words the electron must be rotating!"[10]

Everything fell into place. The electron had spin $\frac{1}{2}$. Landé's $g = 2$ does not apply to the core but to the electron itself!

Sem asked whether this g value could be given a physical meaning.[10] Following a hint by Ehrenfest, George found in an old article by Max Abraham[13] that an electron considered as a rigid sphere with only surface charge does have $g = 2$. All this was written up in a short note[14] that includes the Abraham model, but with a caveat: If that model were the explanation of $g = 2$, then the peripheral rotational velocity should be much larger than the velocity of light, assuming the electron to be an extended object with "classical radius" e^2/mc^2.

That last comment is quite important. It makes clear that the discovery of spin, made after Heisenberg had already published the first paper on quantum mechanics, is an advance in the spirit of the old quantum theory, that wonderfully bizarre mixture of classical reasoning supplemented by *ad hoc* quantum rules.

The discovery note was published with Uhlenbeck as first author and Goudsmit second because (George told me) Ehrenfest suggested that this order would avoid the impression that George was only Sem's student, while Sem himself preferred to come second because it was George who had first thought of spin.

The discovery note is dated 17 October 1925. One day earlier Ehrenfest had written to Lorentz asking him for an opportunity to have "his judgment and advice on a *very* witty idea of Uhlenbeck about spectra."[15] Lorentz listened attentively when George went out to see him soon thereafter, and then raised an objection. The spinning electron should have a magnetic energy on the order of μ^2/r^3, where μ is its magnetic moment and r its radius. Equate this energy to mc^2. Then r would be on the order of 10^{-12} cm, too big to make sense. (The weak point in this argument was to be revealed years later by the positron theory.) George, upset, went to Ehrenfest to suggest that the paper be withdrawn. Ehrenfest replied that he had already sent off their note, and he added that its authors were young enough to be able to afford a stupidity.[10] Some time later Lorentz handed Uhlenbeck a sheaf of papers with calculations of spinning electrons orbiting a nucleus. This work was to become the last paper[16] by the grand master of the classical electron theory. It was presented to the Como conference in September 1927.

No sooner had George and Sem's note appeared when Goudsmit received a letter from Heisenberg congratulating him on his "*mutige Note* [brave note]" and inquiring "wie Sie den Faktor 2 losgeworden sind [how you have got rid of the factor 2]" in the formula for the fine-structure splitting in hydrogen as derived from a semiclassical treatment of spin precession.[17] The young Leideners had not even thought of calculating this splitting. After some struggle they found that Heisenberg was right: The fine structure came out too large by a factor of 2. That puzzle was still unresolved when, in December 1925, Niels Bohr arrived in Leiden to attend the festivities for the golden jubilee of Lorentz's doctorate. Late one evening in 1946, Bohr told me in his home in Gamle Carlsberg what happened to him on that trip.

Bohr's train trip

Bohr's train to Leiden made a stop in Hamburg, where he was met by Wolfgang Pauli and Otto Stern, who had come to the station to ask him what he thought about spin. Bohr must have said that it was very interesting (his favorite way of expressing his belief that something was wrong) but he could not see how an electron moving in the electric field of the nucleus could experience the magnetic field necessary for producing fine structure. (As Uhlenbeck admitted later, "I must say in retrospect that Sem and I in our euphoria had not really appreciated [this] basic difficulty."[10]) On his arrival in Leiden, Bohr was met at the train by Ehrenfest and Albert Einstein, who asked him what he thought about spin. Bohr must have said that it was very, very interesting but what about the magnetic field? Ehrenfest replied that Einstein had resolved that. The electron in its rest frame sees a rotating electric field; hence by elementary relativity it also sees a magnetic field. The net result is an effective spin–orbit coupling. Bohr was at once convinced. When told of the factor of 2 he expressed confidence that this problem would find a natural resolution. He urged Sem and George to write a more detailed note on their work. They did; Bohr added an approving comment.[18]

After Leiden, Bohr traveled to Göttingen. There he was met at the station by Heisenberg and Pascual Jordan, who asked what he thought about spin. Bohr replied that it was a great advance and explained about the spin–orbit coupling. Heisenberg remarked that he had heard this remark before but that he could not remember who made it and when. (I will return to this point shortly.) On Bohr's way home the train stopped at Berlin, where he was met at the station by Pauli, who had made the trip from Hamburg for the sole purpose of asking Bohr what he now thought about spin. Bohr said it was a great advance, to which Pauli replied, "Eine neue Kopenhagener Irrlehre" (a new Copenhagen heresy). After his return home Bohr wrote to Ehrenfest that he had become "a prophet of the electron magnet gospel."[19]

Two additional comments:

▷ The mysterious factor of 2 was supplied in February 1926 by L. H. Thomas and has since been known as the Thomas factor.[20] Thomas noted that earlier calculations of the precession of the electron's spin had been performed in the rest frame of the electron, without taking into account the precession of the electron's orbit around its normal. Inclusion of this relativistic effect reduced the angular velocity of the electron (as seen by the nucleus) by the needed factor of 2.

▷ In March 1926 Hendrik Kramers received a letter from America written by Ralph Kronig,[21] a young Columbia University PhD who had spent two years studying in Europe, including a stay in Copenhagen from January to November 1925. Kronig reminded Kramers that prior to Goudsmit and Uhlenbeck he, Kronig, had already had the idea of spin, though he too had an extra factor of 2 in the fine structure, and that he and Kramers had discussed those matters in Copenhagen. Heisenberg's hazy recollection, mentioned a few lines earlier, of having heard part of the spin story before must refer to a discussion with Kronig. In his letter, Kronig told Kramers that he had not published because "Pauli ridiculed the idea, saying 'that is indeed very clever but of course has nothing to do with reality,' " and added, "In the future I shall trust my own judgment more and that of others less."

After Kramers told this story to Bohr, the latter wrote to Kronig, expressing his "consternation and deep regret."[22] Kronig replied, "I should not have mentioned the matter at all [to Kramers] if it were not to take a fling at the physicists of the preaching variety who are always so damned sure of, and inflated with, the correctness of their own opinion."[23] He asked Bohr to refrain from public reference to the affair since "Goudsmit and Uhlenbeck would hardly be very happy about it." Kronig is an eminent physicist and a gentleman. So was Uhlenbeck, who has written, "There is no doubt that Ralph Kronig anticipated what certainly was the main part of our ideas."[10]

I should like to conclude with a few remarks of a personal nature. In my undergraduate years in my native Amsterdam I began taking courses in physics, chemistry and mathematics, in a rather unfocused way. Then in the winter of 1938 Uhlenbeck, at that time a professor in Utrecht, came for a visit and gave two lectures on beta decay. I did not understand much. I had not yet heard about neutrinos. Nevertheless, listening to those talks given in a calm yet ever so compelling way I knew, I just knew: That is what I want to do. After graduation I moved to Utrecht, where I did my warm-up research exercises with Uhlenbeck and took his courses. Since then I have met many other physicists of distinction but never a better lecturer than George. We became personal friends in later years. We published two joint papers.[24] He had a great and lasting influence on me for which I shall remain forever grateful.

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