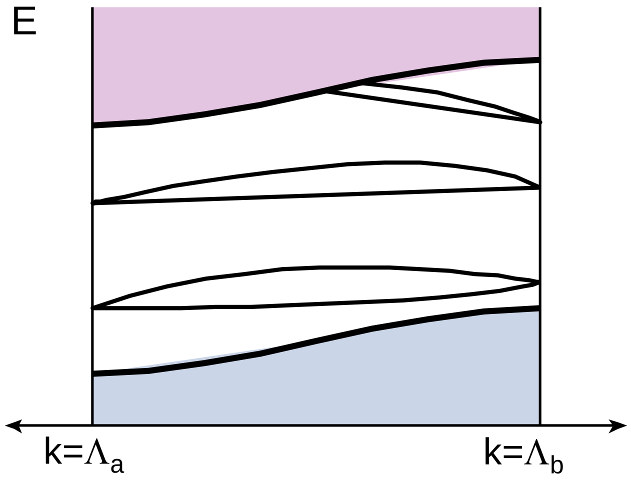
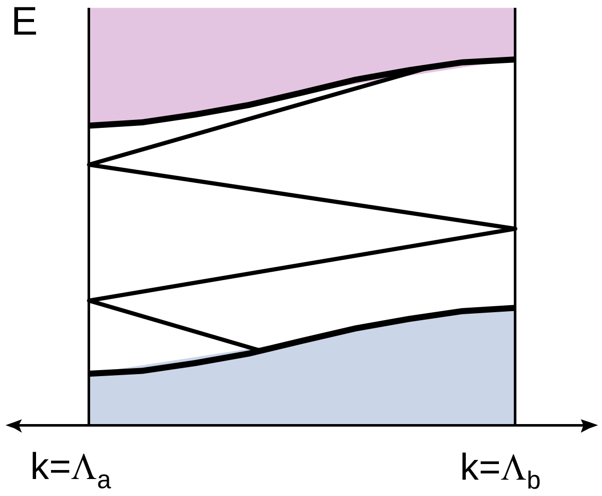
Symmetry, Topology and Electronic Phases of Matter



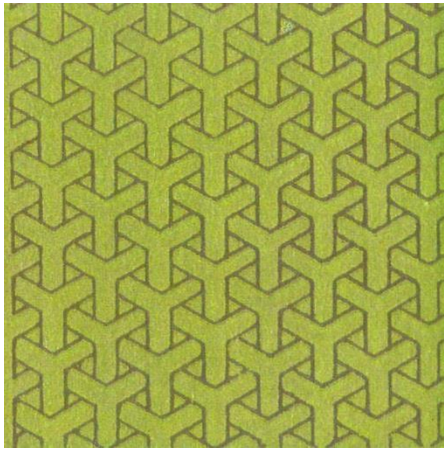


Organizing Principles for Understanding Matter

Symmetry

- What operations leave a system invariant?
- Distinguish phases of matter by symmetries


symmetry group p4


symmetry group p31m

Topology

- What stays the same when a system is deformed?
- Distinguish topological phases of matter


genus = 0


genus = 1

Symmetry, Topology and Electronic Phases of Matter

I. Introduction

- Topological band theory

II. Topological Insulators in 2 and 3 Dimension

- Time reversal symmetry & Boundary States
- Experiments: Transport, Photoemission

III. Topological Superconductivity

- Majorana fermion bound states
- A platform for topological quantum computing?

IV. The Frontier

- Many more examples of topological band phenomena
- Beyond band theory: states combining topology and strong interactions

Thanks to :

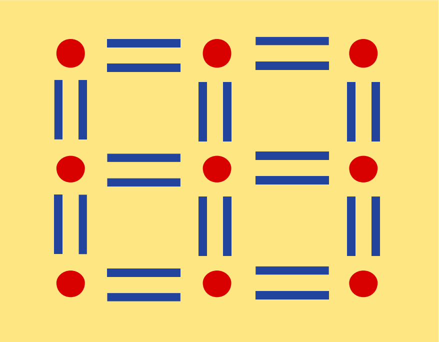
Gene Mele, Liang Fu, Jeffrey Teo, Fan Zhang,
Steve Young, Saad Zaheer, Ben Wieder,
Youngkuk Kim, Andrew Rappe (U. Penn.)

The Insulating State

Characterized by energy gap: absence of low energy electronic excitations

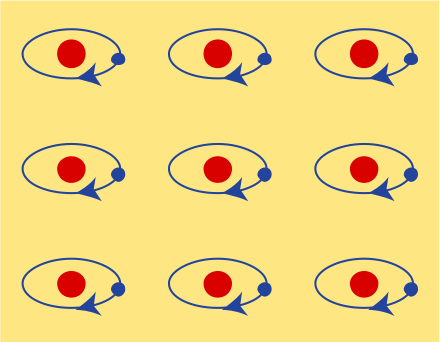
Covalent Insulator

e.g. intrinsic semiconductor

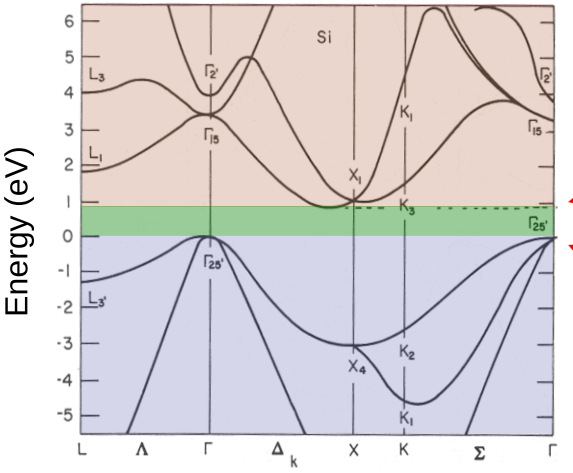


Atomic Insulator

e.g. solid Argon

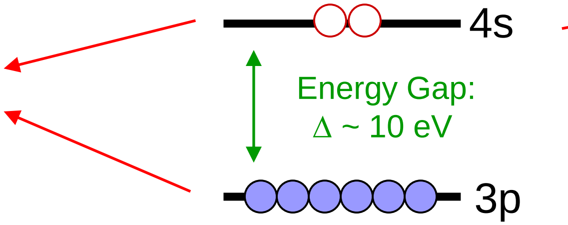


The vacuum

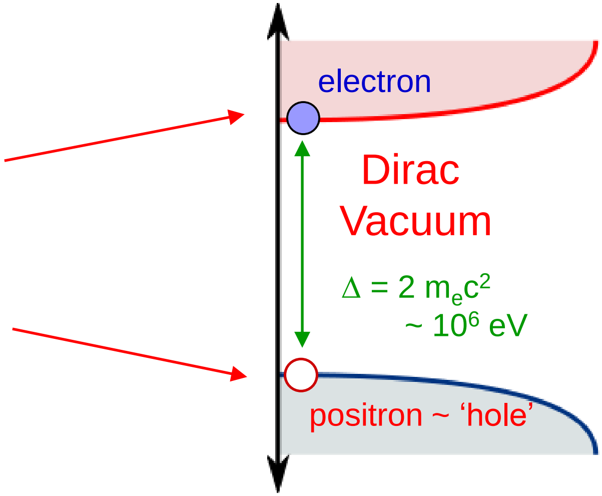


Silicon

momentum



Energy Gap:
Δ ~ 10 eV



Dirac
Vacuum

$\Delta = 2 m_e c^2$
~ 10^6 eV

Topology and Adiabatic Continuity

Insulators are topologically equivalent if they can be continuously deformed into one another without closing the energy gap

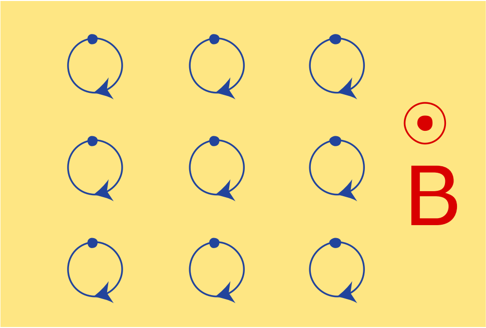
genus = 0

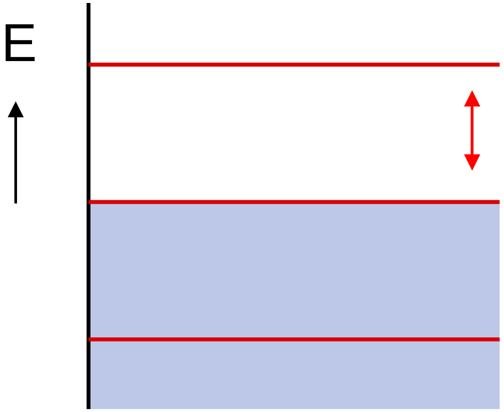
Are there “topological phases” that are not adiabatically connected to the trivial insulator (ie the vacuum) ?

genus = 1

The Integer Quantum Hall State

2D Cyclotron Motion, Landau Levels

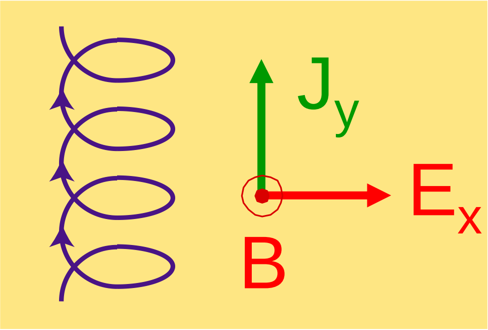




$$E_{gap} = \hbar\omega_c$$

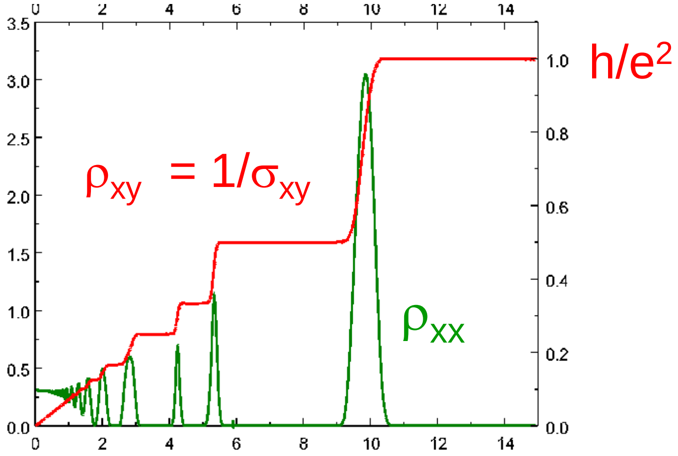
Energy gap, but NOT an insulator

Quantized Hall conductivity : $J_y = \sigma_{xy} E_x$

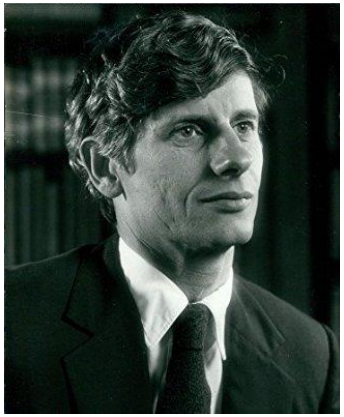


$$\sigma_{xy} = n\frac{e^2}{h}$$

Integer accurate to 10^{-9}



Topological Band Theory



David Thouless

Thouless et al., 1982: The distinction between a conventional insulator and the quantum Hall state is a *topological* property of the band structure.

- When there is an energy gap, the occupied electronic states (valence bands) vary smoothly as a function of momentum **k** and can be classified by an integer topological invariant .
- Integer Chern (or TKNN) number:

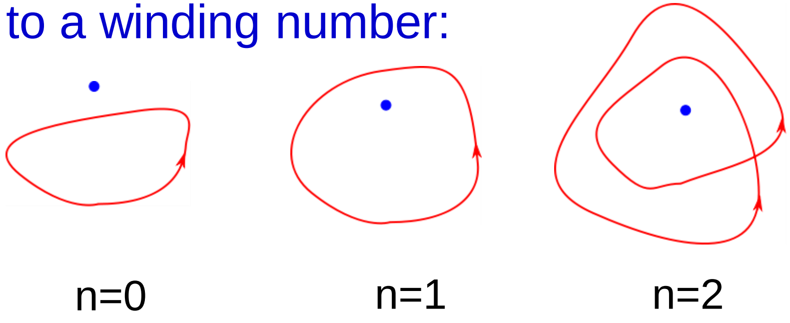
$$n = \frac{1}{2\pi i}\int_{BZ} d^2\mathbf{k}\cdot\langle\nabla_{\mathbf{k}}u(\mathbf{k})|\times|\nabla_{\mathbf{k}}u(\mathbf{k})\rangle \in \mathbb{Z}$$

u(**k**) = Bloch wavefunction

- n characterizes the quantized Hall conductivity:

Insulator: n = 0 ; IQHE state: $\sigma_{xy} = n\,e^2/h$

- Similar to a winding number:

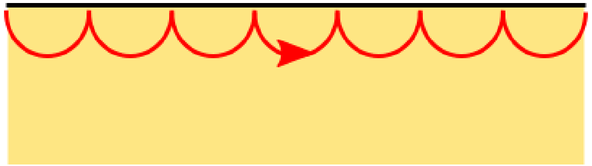


Edge States

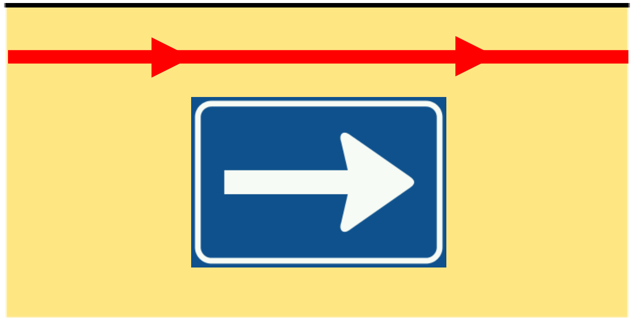
Halperin '82

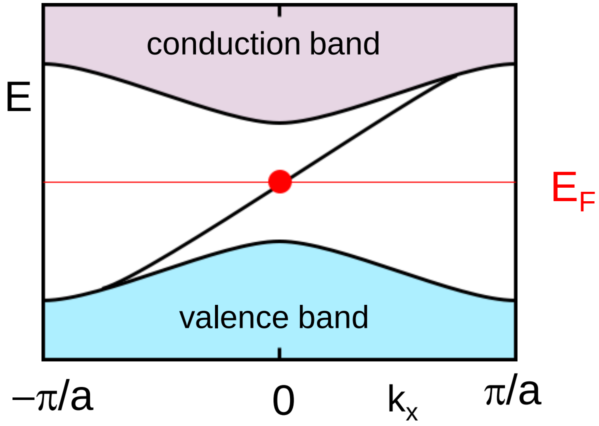
Bert Halperin

Classical : Skipping Orbits



Quantum : 1D Chiral Dirac Fermions $E = v p$





Chiral edge states are topologically protected

- Electric current flows without dissipation
- Precisely quantized conductance
- Insensitive to disorder

Bulk-Boundary Correspondence : (related to index theorems in mathematics)

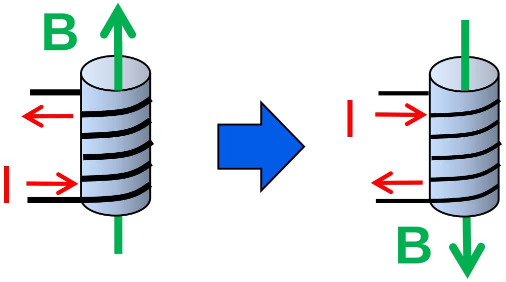
- Bulk Invariant = Boundary Invariant
- Chern number n = # chiral edge modes

Time Reversal Symmetry

Under the reversal of the direction of time :

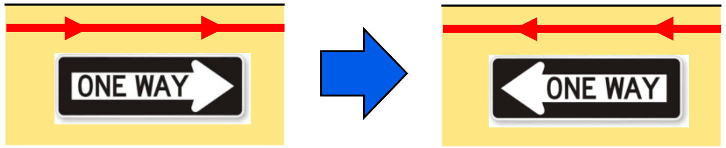
- Magnetic Field :

  B → –**B**



- Chiral Edge state :

  Right mover → Left mover



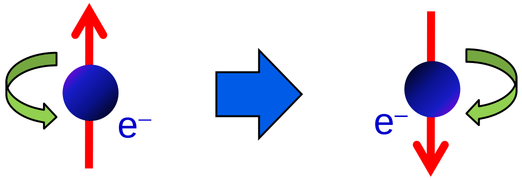
- Spin Angular Momentum :

  S → –**S**

  $\phi_\uparrow(\mathbf{r}) \to \phi_\downarrow(\mathbf{r})^*$

  $\phi_\downarrow(\mathbf{r}) \to -\phi_\uparrow(\mathbf{r})^*$

spin ½ electron



"spin up" "spin down"

- Kramers' Theorem : $T^2 = -1$:

  For spin ½ particles with T symmetry

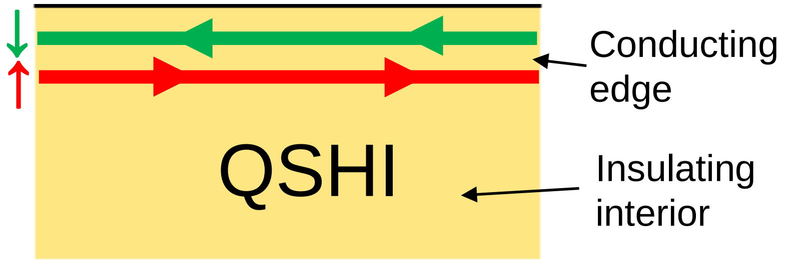
  all states are at least two fold degenerate.

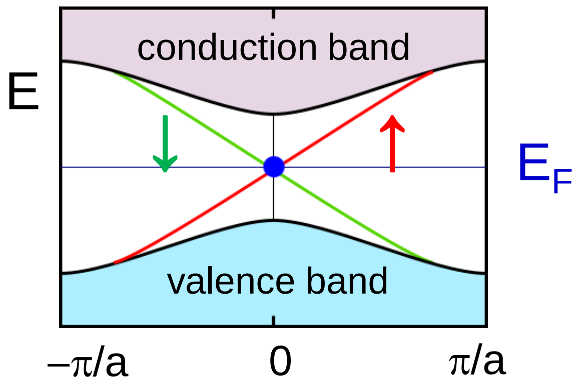
The integer quantum Hall state requires broken time reversal symmetry.
Are there topological phases with unbroken time reversal symmetry?

Quantum Spin Hall Insulator

Simplest version : 2 copies of quantum Hall effect

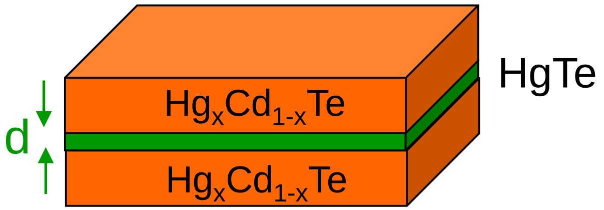
Kane and Mele ’05
Bernevig and Zhang ‘06

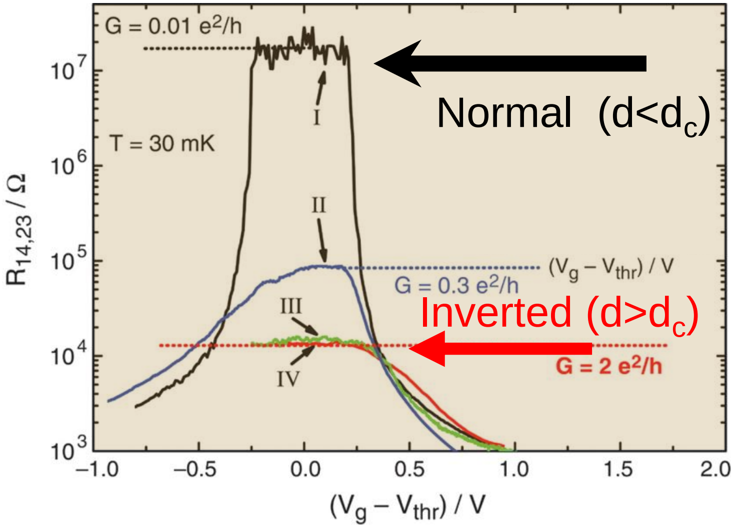




“Helical” edge states protected by time reversal

HgCdTe quantum wells





- Theory: Bernevig, Hughes and Zhang, Science ’06

  Predict inversion of conductance and valence band for d > d_c = 6.3 nm ⇒ QSHI

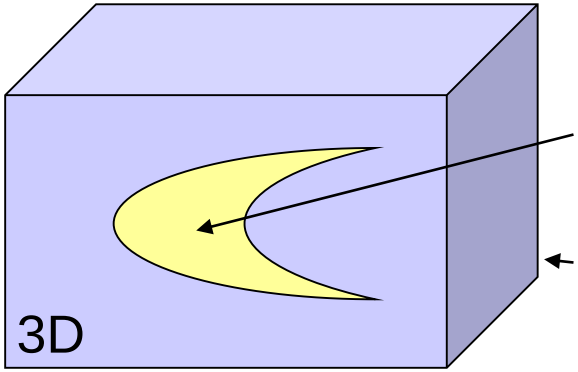
- Experiment: Konig et al. Science ’07

  Measure electrical conductance due to edge states

3D Topological Insulator

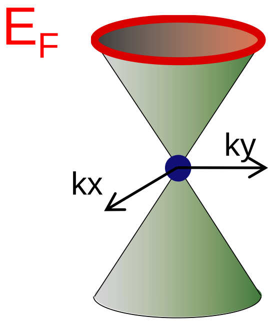
Moore & Balents '06;
Roy '06;
Fu & Kane '06

3D insulators are characterized by four Z_2 topological invariants



Insulating interior

Conducting surface

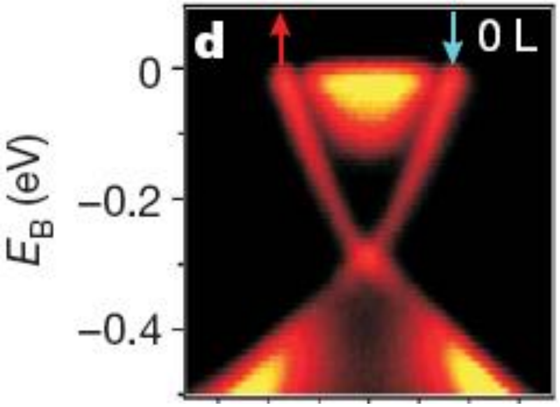


"Surface Dirac Cone"

$Bi_2\ Se_3$

ARPES Experiment : Y. Xia et al., '09
Band Theory : H. Zhang et. al, '09

- Energy gap: $\Delta \sim .3$ eV :
  A room temperature topological insulator
- Simple surface state structure :
  A textbook Dirac cone, with a spin texture



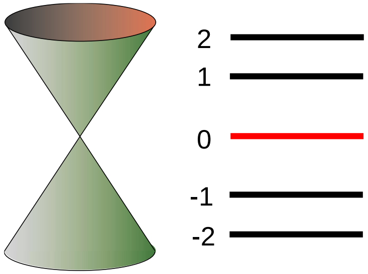
Angle resolved photoemission spectroscopy (Xia et al '09)

Surface of a topological insulator: A route to new gapped topological states

I. Break time reversal symmetry : "Half integer" Quantum Hall Effect

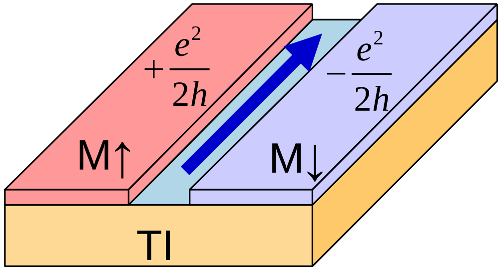
1. Orbital QHE:

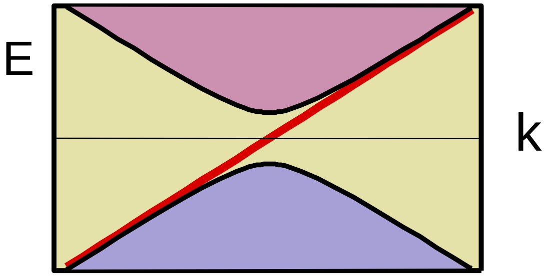
Landau levels



$$\sigma_{xy} = \frac{e^2}{h}\left(n+\frac{1}{2}\right)$$

2. Magnetic insulator on surface : Chiral Dirac fermion at domain wall

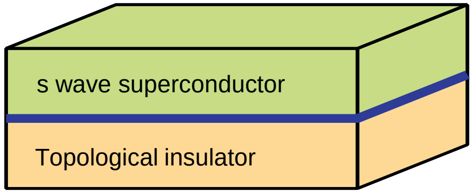




3. Quantum anomalous Hall effect recently observed in thin film magnetic topological insulators C-Z Chang, … Q-K Xue, et al. Science '13

II. Break gauge symmetry : Superconducting Proximity Effect

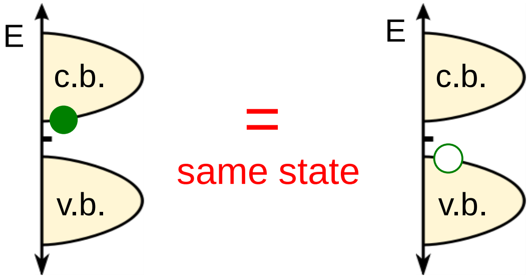
A route to topological superconductivity using ordinary superconductors



Topological Superconductivity

Key ingredients of BCS model of superconductivity :

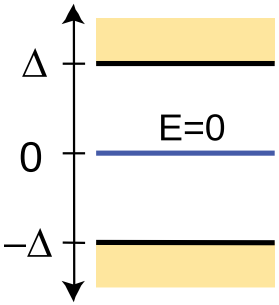
- Similar to insulator: energy gap for quasparticle excitations
- Intrinsic Particle – Hole symmetry



1D Topological Superconductor

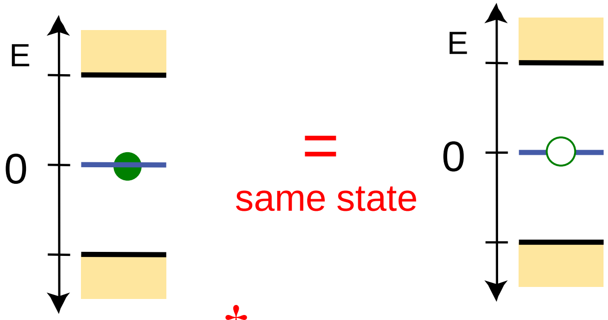
(Kitaev 2000)

- Two topological classes
- Protected zero energy end state



Particle = Anti particle

- Defines a **Majorana Fermion** bound state
- “Half” an ordinary particle



$$\gamma^\dagger_0 = \gamma_0$$

In search of Majorana

1937 : Majorana publishes his modification of the Dirac equation that allows spin ½ particles to be their own antiparticle.

1938 : Majorana mysteriously disappears at sea

2013 : Italian police concludes Majorana was alive in Venezuela until the 1950's. *

Ettore Majorana
1906–1938?

Observation of a Majorana fermion is among the great challenges of physics today

Particle physics :

Fundamental particles (eg neutrino) might be Majorana fermions

Condensed matter physics:

Kitaev '03: Zero energy Majorana bound states provide a new method for storing and manipulating quantum information

- 2 Majorana bound states store 1 qubit of quantum information nonlocally
- Immune from local sources of decoherence
- "Braiding" can perform quantum operations

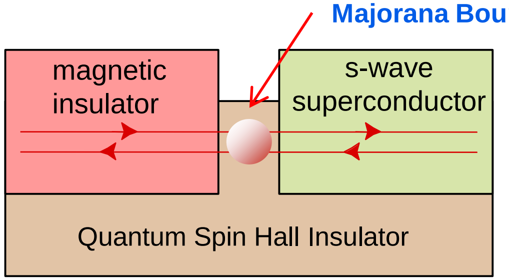
Alexei Kitaev

Quest for Majorana in Condensed Matter

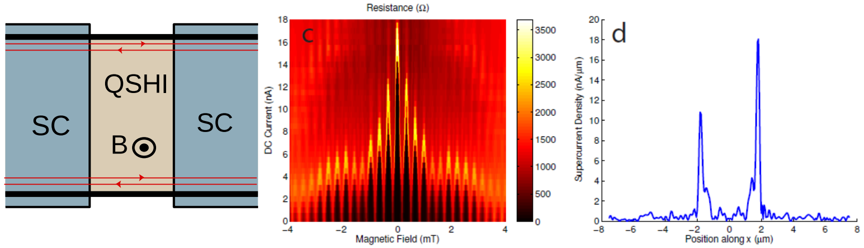
Superconducting Proximity Effect: Use ordinary superconductors and topological materials
to engineer topological superconductivity

Superconductor - Topological Insulator Devices

Theory: Fu, Kane '07, '08



Expt: Hart, … Yacoby '14 (HgTe);
Pribiag, … Kouwenhoven '14 (InAs/GaSb)

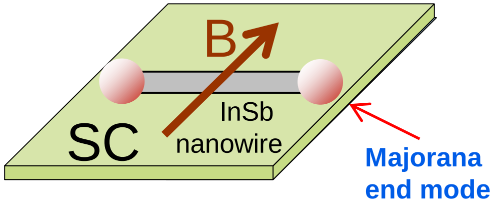


"Two slit" interference pattern in a S-TI-S Josephson Junction
Demonstrates edge superconductivity

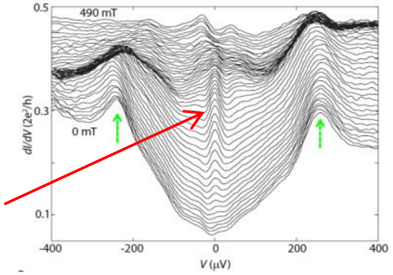
Superconductor - Semiconductor Nanowire Devices

Theory:
Lutchyn, Sau , Das Sarma '10
Oreg, Refael, von Oppen '10

Expt: Mourik, …Kouwenhoven '12



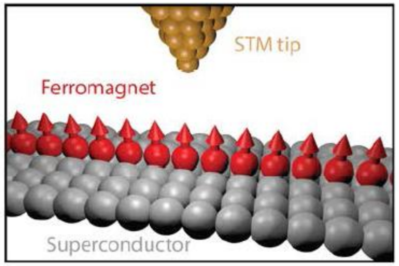
tunneling conductance

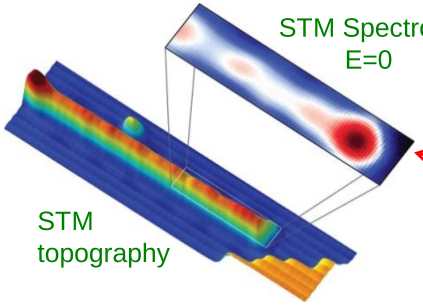


Leo Kouwenhoven

Ferromagnetic Atomic Chains on Superconductors

Nadj-Perg, …, Yazdani '14 (Fe on Pb)



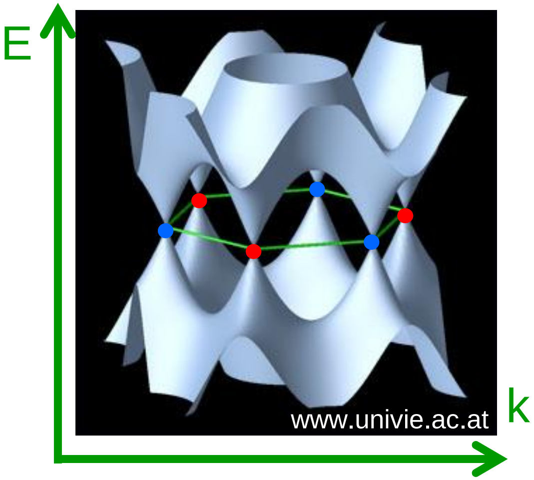


A Vast Frontier I: Many more examples of topological Band Phenomena

Example: Symmetry protected topological semimetals

1. Graphene

2D Dirac points protected by
inversion symmetry,
time reversal symmetry,
spin rotation symmetry (no spin orbit)



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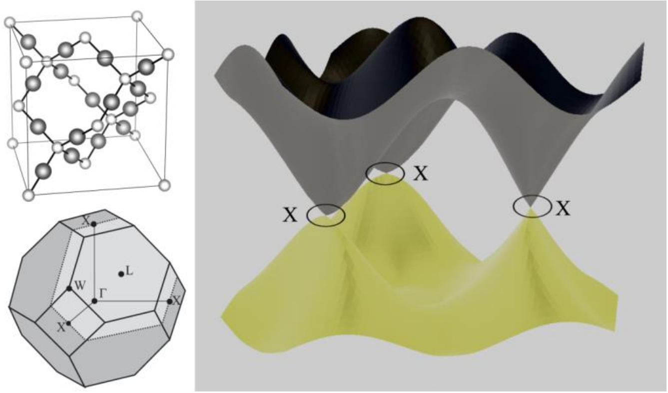
2D Dirac point

$$H = \mathrm{v}\vec{\sigma}\cdot\mathbf{p}$$

2. 3D Dirac Semimetal

3D Dirac points with strong spin-orbit protected by
time reversal symmetry
space group symmetries
Observed in many real materials



3D Dirac point

$$H = \mathrm{v}\vec{\gamma}\cdot\mathbf{p}$$

Current status :

- Strong interaction between theory, computation and experiment.
- Many real materials have been shown to exhibit topological band phenomena.

A Vast Frontier II : states that combine band topology and strong interactions

Strongly interacting systems can exhibit intrinsic topological order, which is distinct from band topology in insulators.

- Excitations with fractional quantum numbers
- Long ranged quantum entanglement in ground state
- Ground state degeneracy depends on topology of space

Example: Laughlin state of fractional quantum Hall effect

Can we engineer topologically ordered states in materials or devices?

- Fractional Chern Insulators ?
- Fractional Topological Insulators ?
- Fractional Majorana Fermions (aka Z_n parafermions) ?
- … and beyond

Current status :

- There has been much recent progress in models for such states.
- More work needs to be done to achieve them in the real world.

Conclusion

- Symmetry and Topology provide a powerful framework for the discovery of novel electronic phases with protected low energy states.
  - topological insulators in 2D and 3D
  - topological superconductivity
  - topological semimetals
- Experimental Challenges
  - Perfect known topological materials and discover new ones
  - Superconducting, Magnetic structures
  - Create heterostructure devices
- Theoretical Challenges
  - Materials physics : predicting and optimizing materials for topological phases
  - Many body physics : What phases are possible and how can you make them?