Gravitational Radiation:

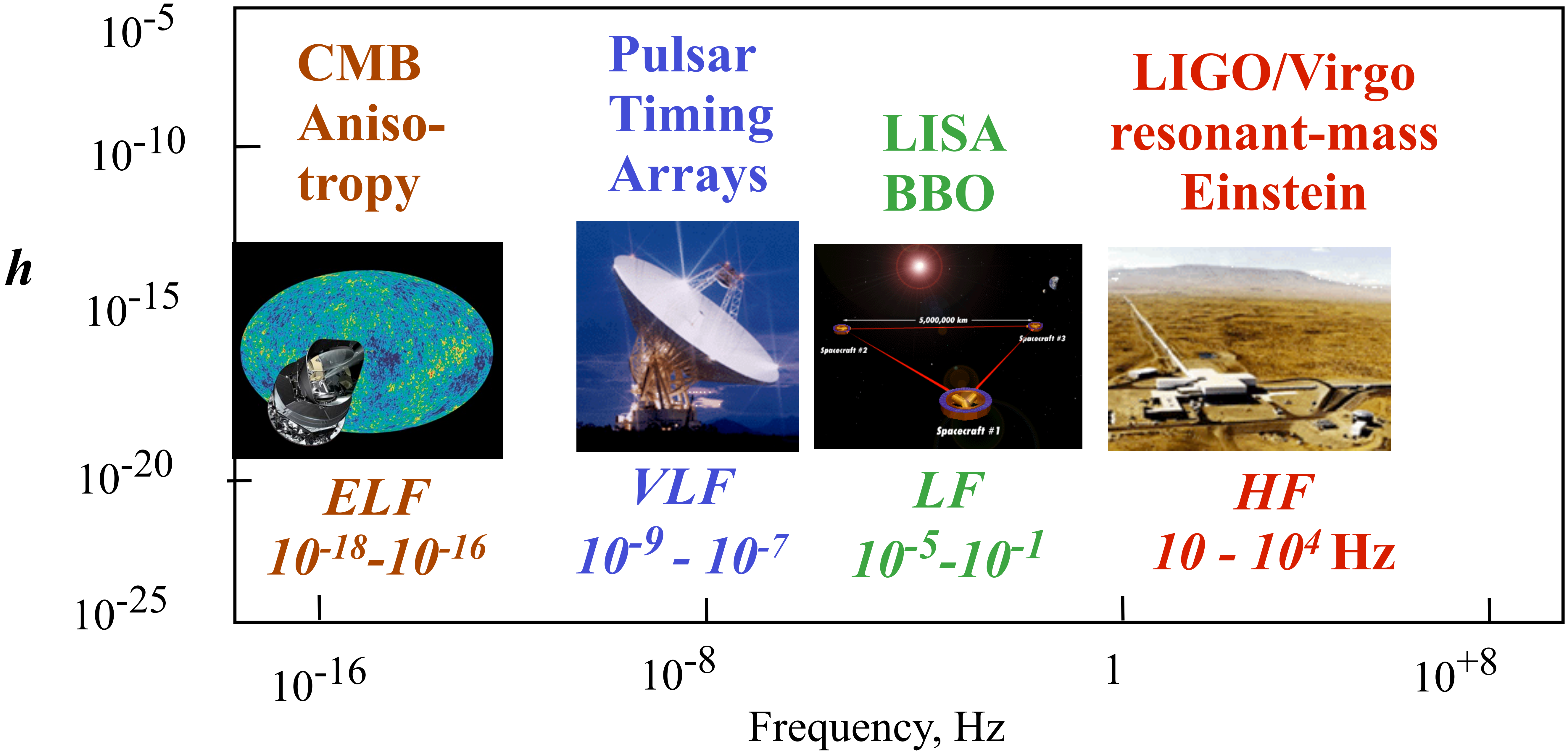
3. Gravitational-Wave Detection: Methods, Status and Plans

Kip S. Thorne

Lorentz Lectures, University of Leiden, September & October 2009

PDFs of lecture slides are available at
http://www.cco.caltech.edu/~kip/LorentzLectures/
each Thursday night before the Friday lecture

Frequency Bands and Detectors



CMB Aniso-tropy

ELF

10^{-18}-10^{-16}

Pulsar Timing Arrays

VLF

10^{-9} - 10^{-7}

LISA
BBO

LF

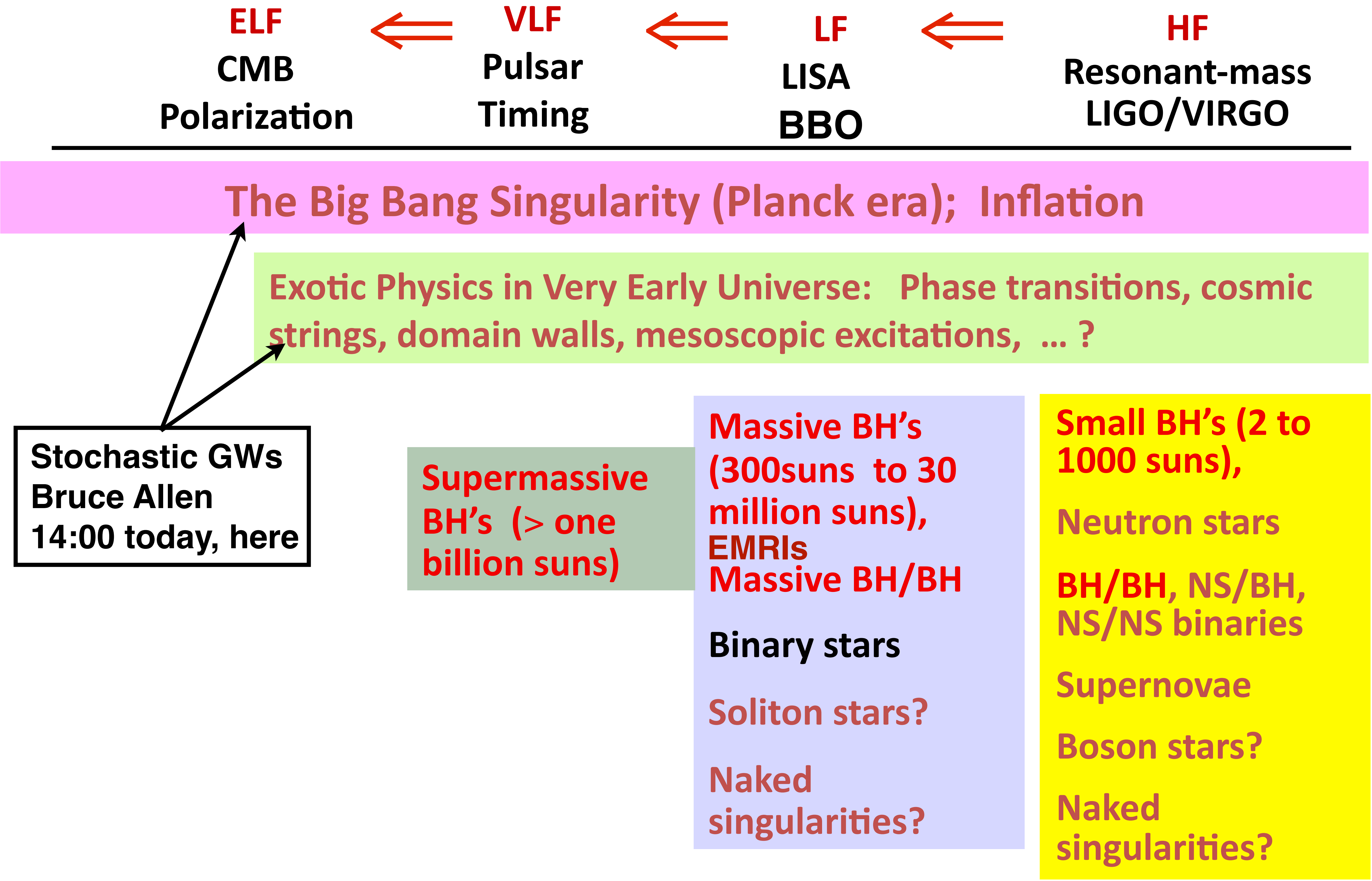
10^{-5}-10^{-1}

LIGO/Virgo
resonant-mass
Einstein

HF

10 - 10^{4} Hz

Some Sources in Our Four Bands



ELF ⇐ **VLF** ⇐ **LF** ⇐ **HF**

ELF
CMB
Polarization

VLF
Pulsar
Timing

LF
LISA
BBO

HF
Resonant-mass
LIGO/VIRGO

The Big Bang Singularity (Planck era); Inflation

Exotic Physics in Very Early Universe: Phase transitions, cosmic strings, domain walls, mesoscopic excitations, … ?

Stochastic GWs
Bruce Allen
14:00 today, here

Supermassive BH's (> one billion suns)

Massive BH's (300suns to 30 million suns),
EMRIs
Massive BH/BH

Binary stars

Soliton stars?

Naked singularities?

Small BH's (2 to 1000 suns),

Neutron stars

BH/BH, NS/BH, NS/NS binaries

Supernovae

Boson stars?

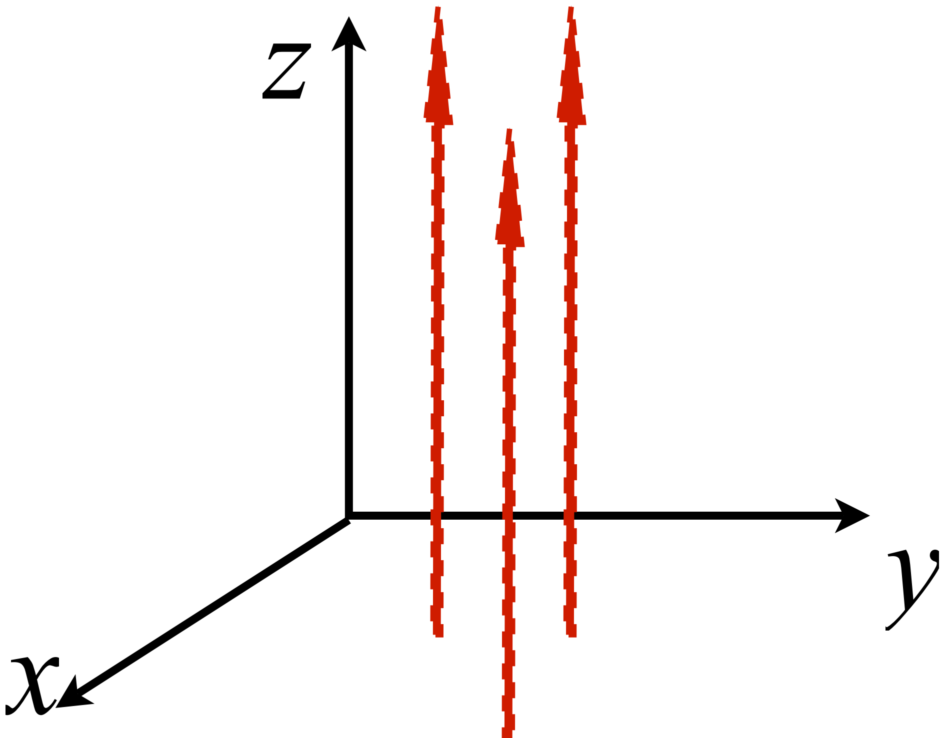
Naked singularities?

Ground-Based GW Detectors:

High-Frequency Band (HF)

1Hz - 10,000 Hz

GWs: Review



- The gravitational-wave field, h_{jk}^{GW}

Symmetric, transverse, traceless (TT);
two polarizations: +, x

- + Polarization

$$h_{xx}^{\mathrm{GW}} = h_+(t - z/c) = h_+(t - z)$$

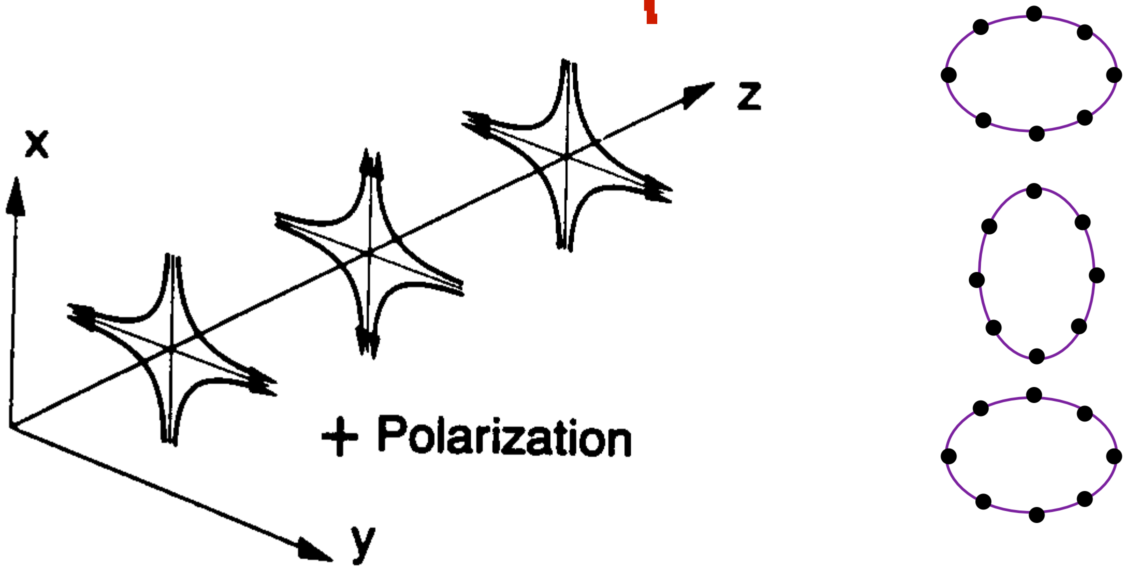
$$h_{yy}^{\mathrm{GW}} = -h_+(t - z)$$

Lines of force

$$\ddot{x}_j = \frac{1}{2}\ddot{h}_{jk}^{\mathrm{GW}} x_k$$

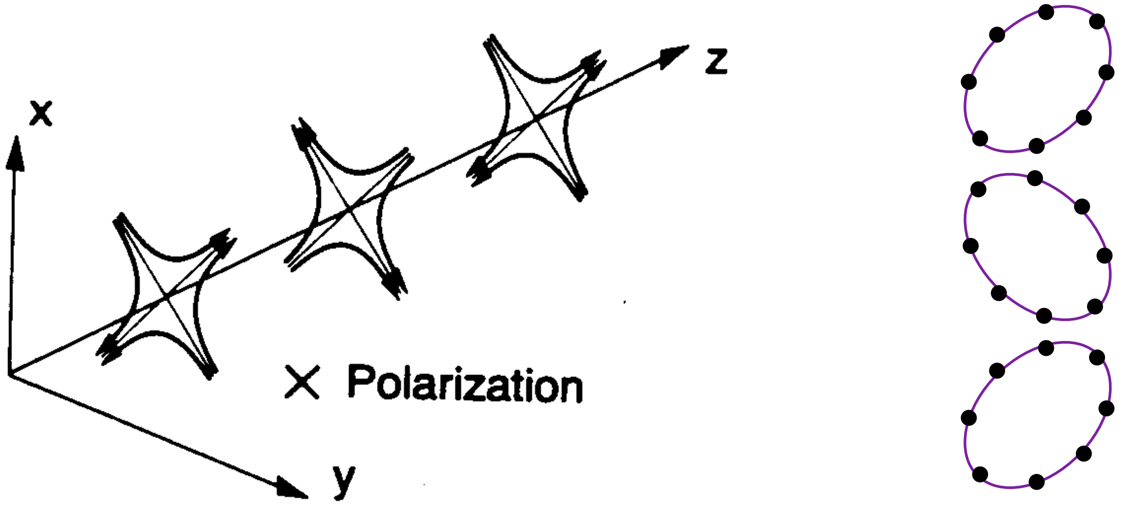
$$\ddot{x} = \ddot{h}_+ x$$

$$\ddot{y} = -\ddot{h}_+ y$$

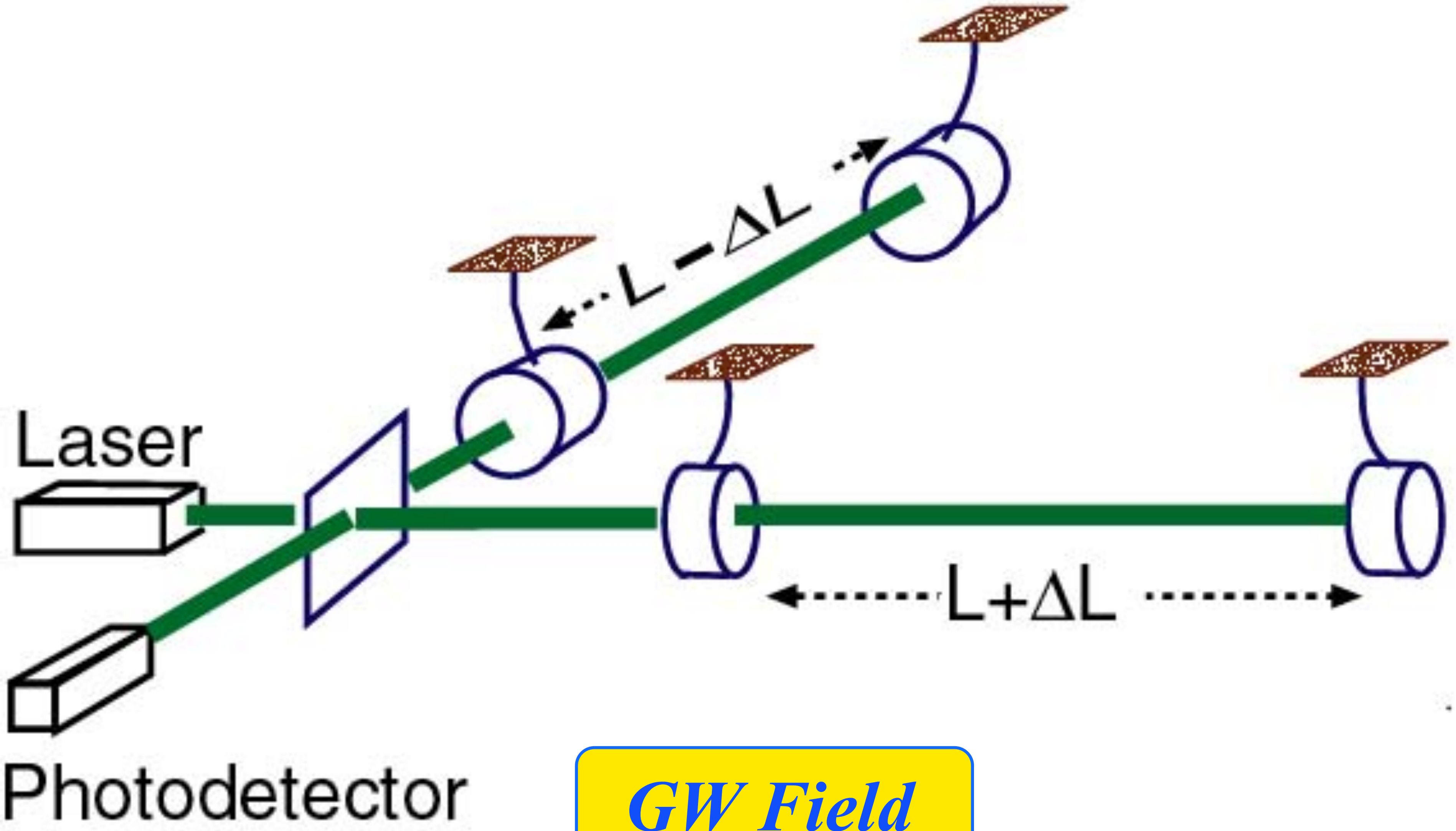


- x Polarization

$$h_{xy}^{\mathrm{GW}} = h_{yx}^{\mathrm{GW}} = h_\times(t - z)$$

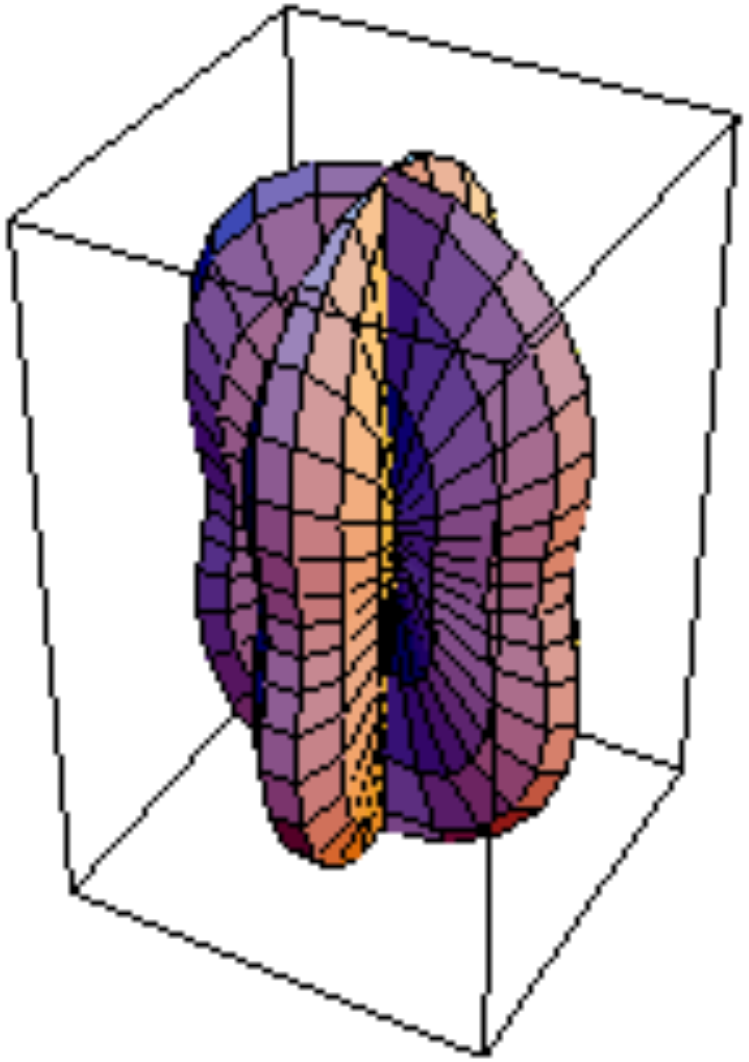


Gravitational-Wave Interferometer

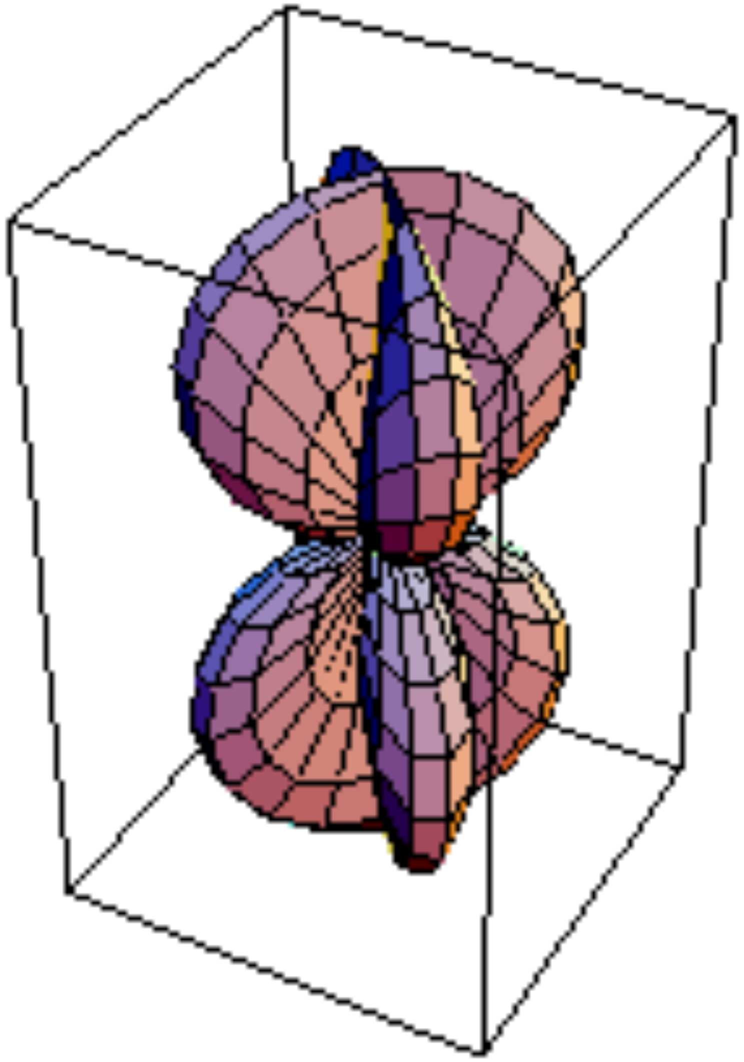


$$\Delta L = \underset{\lesssim 10^{-21}}{\overset{\text{GW Field}}{h}}\ \underset{4\ \text{km}}{L} \lesssim 4\times10^{-16}\ \text{cm}$$

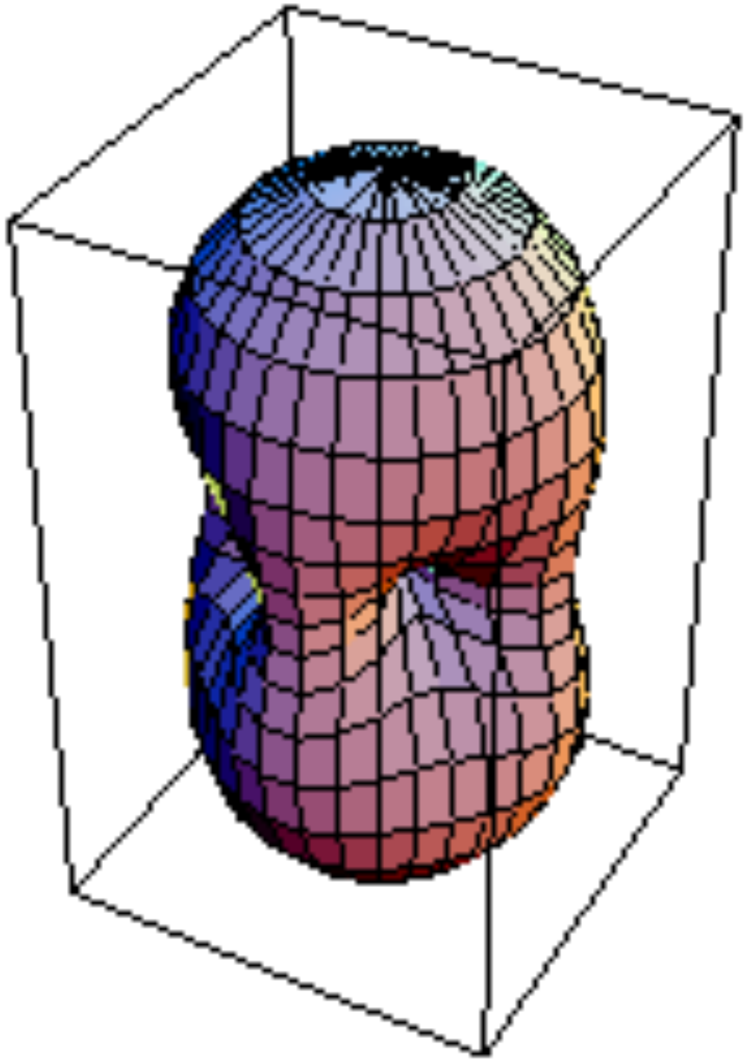
Beam Patterns



+ Polarization



x Polarization

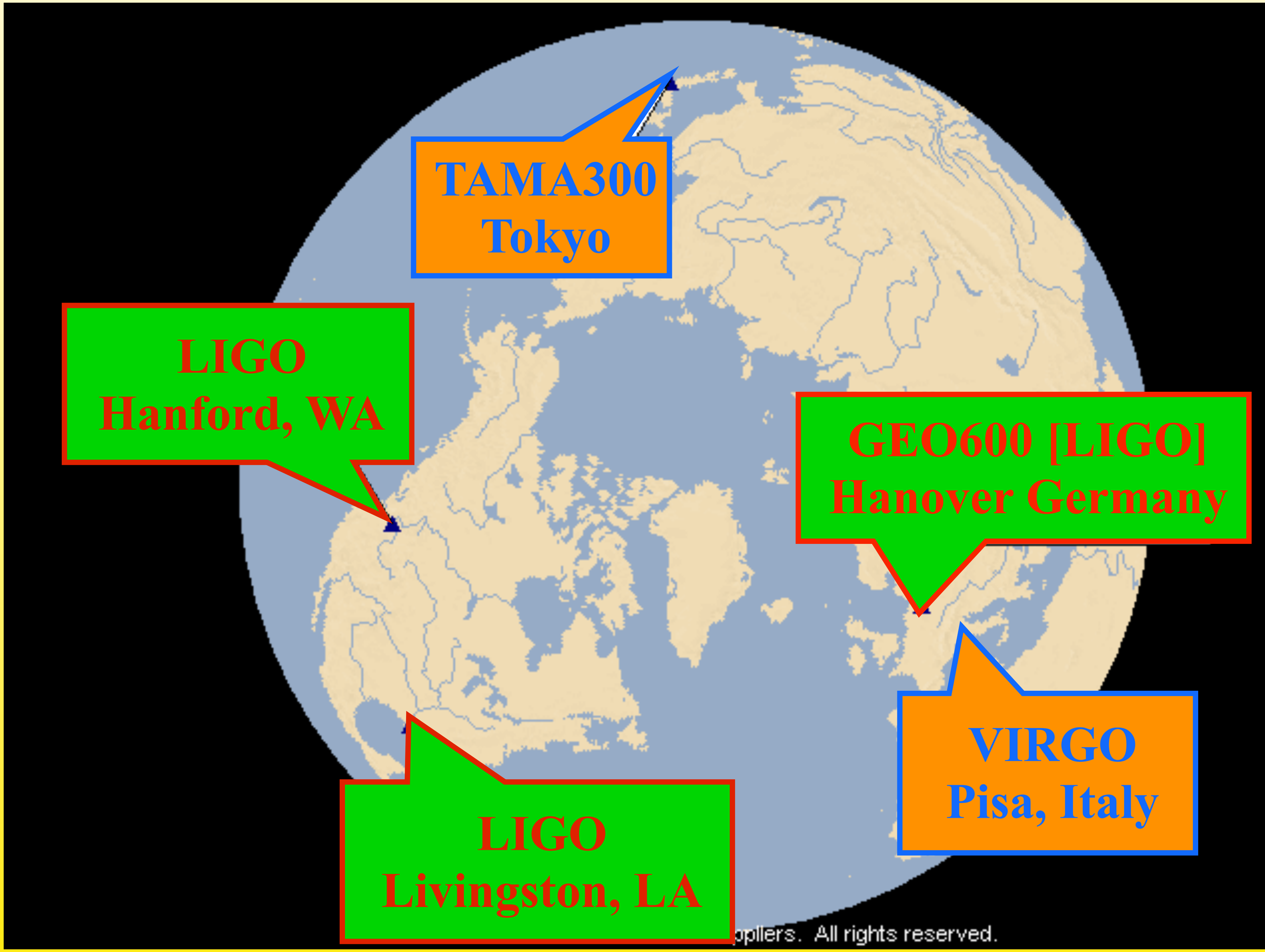


Unpolarized

International Network

Network Required for:

- Detection Confidence
- Waveform Extraction
- Direction by Triangulation



LIGO: Laser Interferometer Gravitational Wave Observatory

Collaboration of ~500 scientists at ~50 institutions in 8 nations [J. Marx, Director; D. Rietze, Spokesman]

LIGO

USA, UK, Germany, Australia, India, Japan, Russia, Spain

**Livingston,
Louisiana**

GEO600 [Germany/UK]

Hannover, Germany

Next-Generation Technology

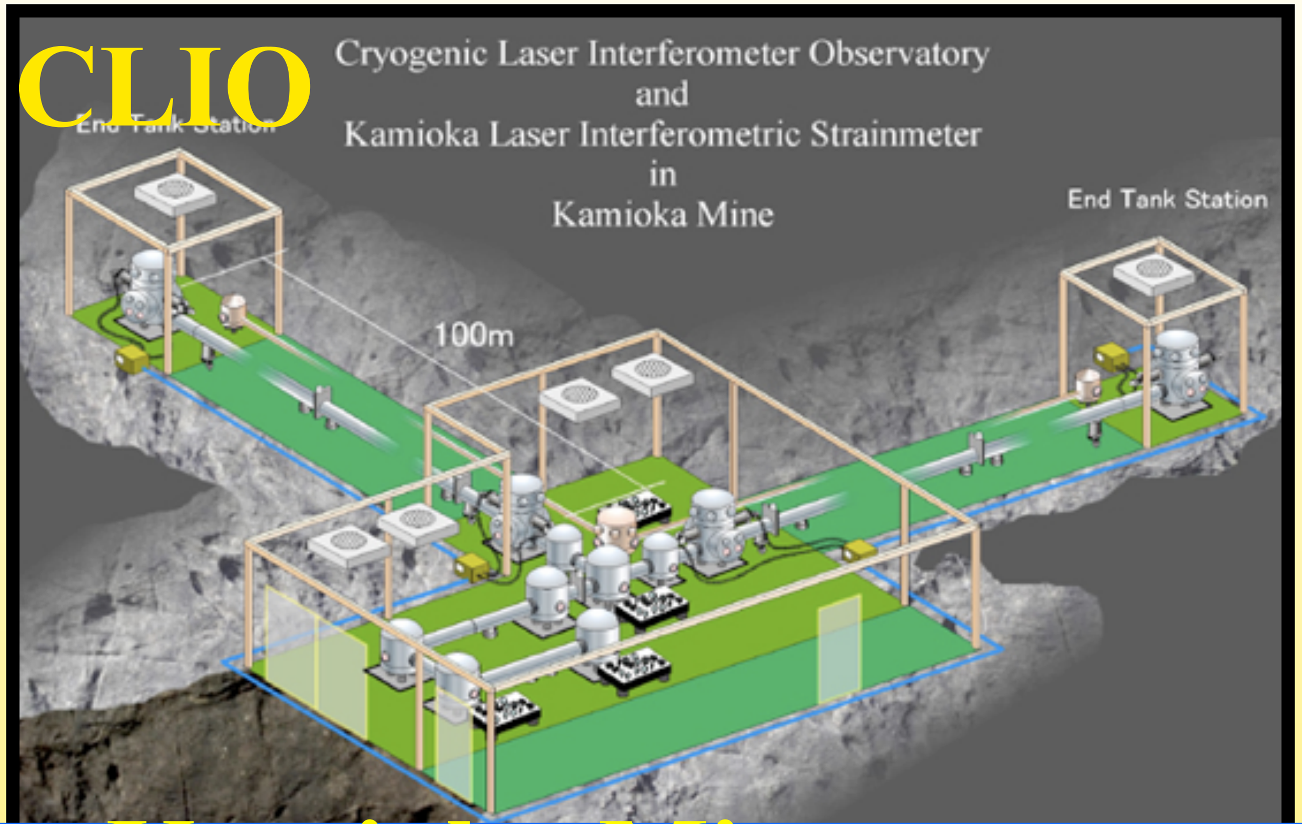
Directors: J Hough, Glasgow, K. Danzmann, Hannover

VIRGO [France, Italy; ... NIKHEF]

Pisa, Italy

JAPAN:


TAMA: in Tokyo


CLIO


Kamioka Mine

Precursors to LCGT: Large Cryogenic Gravitational Telescope

AIGO [Australia] - 5 km Arms

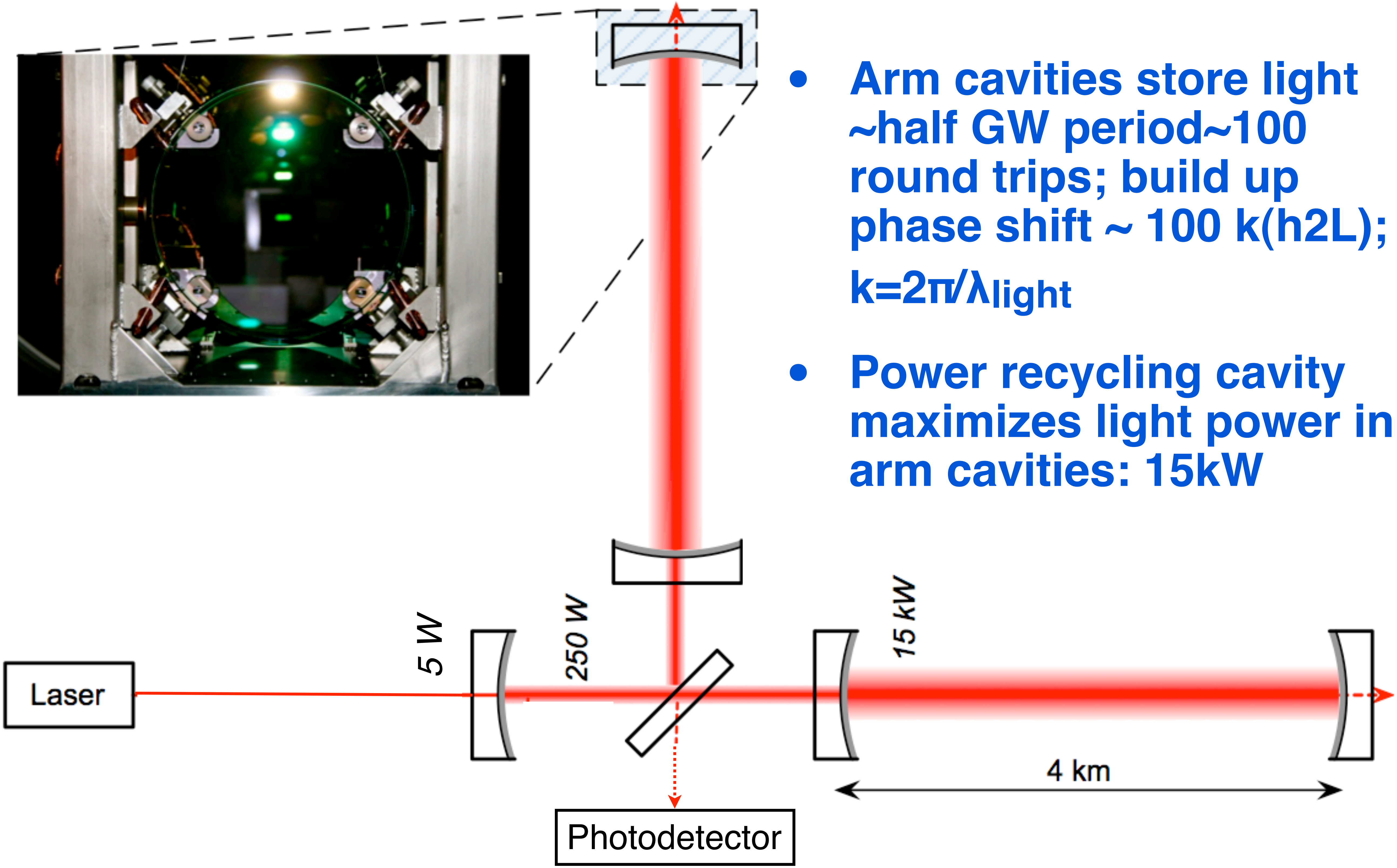
Australian International Gravitational Observatory

Director: D. McClelland

Gin-Gin, West Australia

80 meter test facility

Initial LIGO's Optical System



- Arm cavities store light ~half GW period~100 round trips; build up phase shift ~ 100 k(h2L); $k=2\pi/\lambda_{light}$
- Power recycling cavity maximizes light power in arm cavities: 15kW

How characterize noise: *Spectral Density, $S_h(f)$*

- $h(t) \equiv \dfrac{\Delta L(t)}{L} = \text{(interferometer's strain)}$

- $S_h(f) = \text{(spectral density of } h\text{, at frequency } f\text{)}$

  $= \text{(``noise power'' in } h(t) \text{ fluctuations, per unit bandwidth)}$

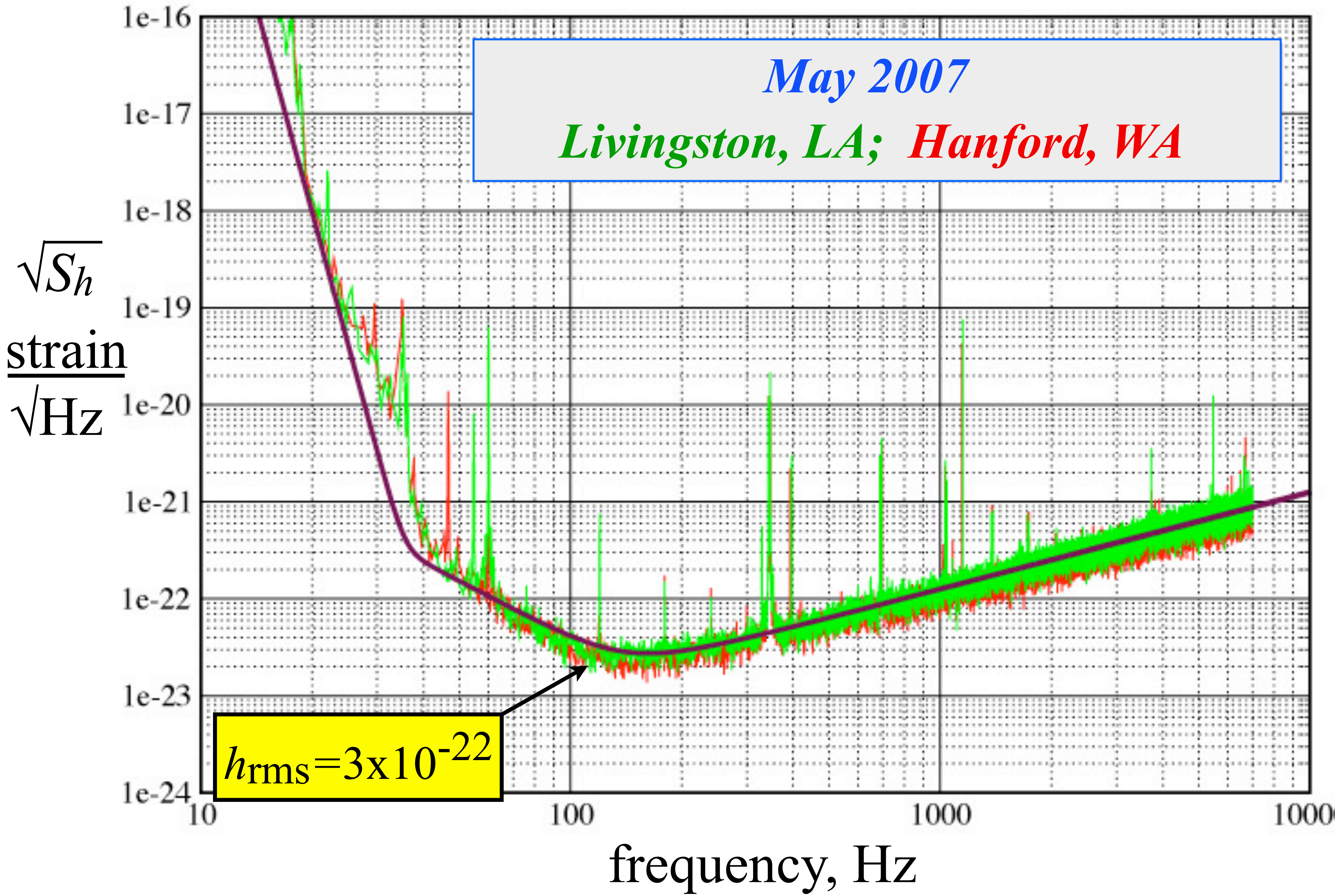
  Units: $1/\text{Hz}$

- $\sqrt{S_h(f)\Delta f} = \text{rms fluctuations of } h(t) \text{ in bandwidth} \Delta f$

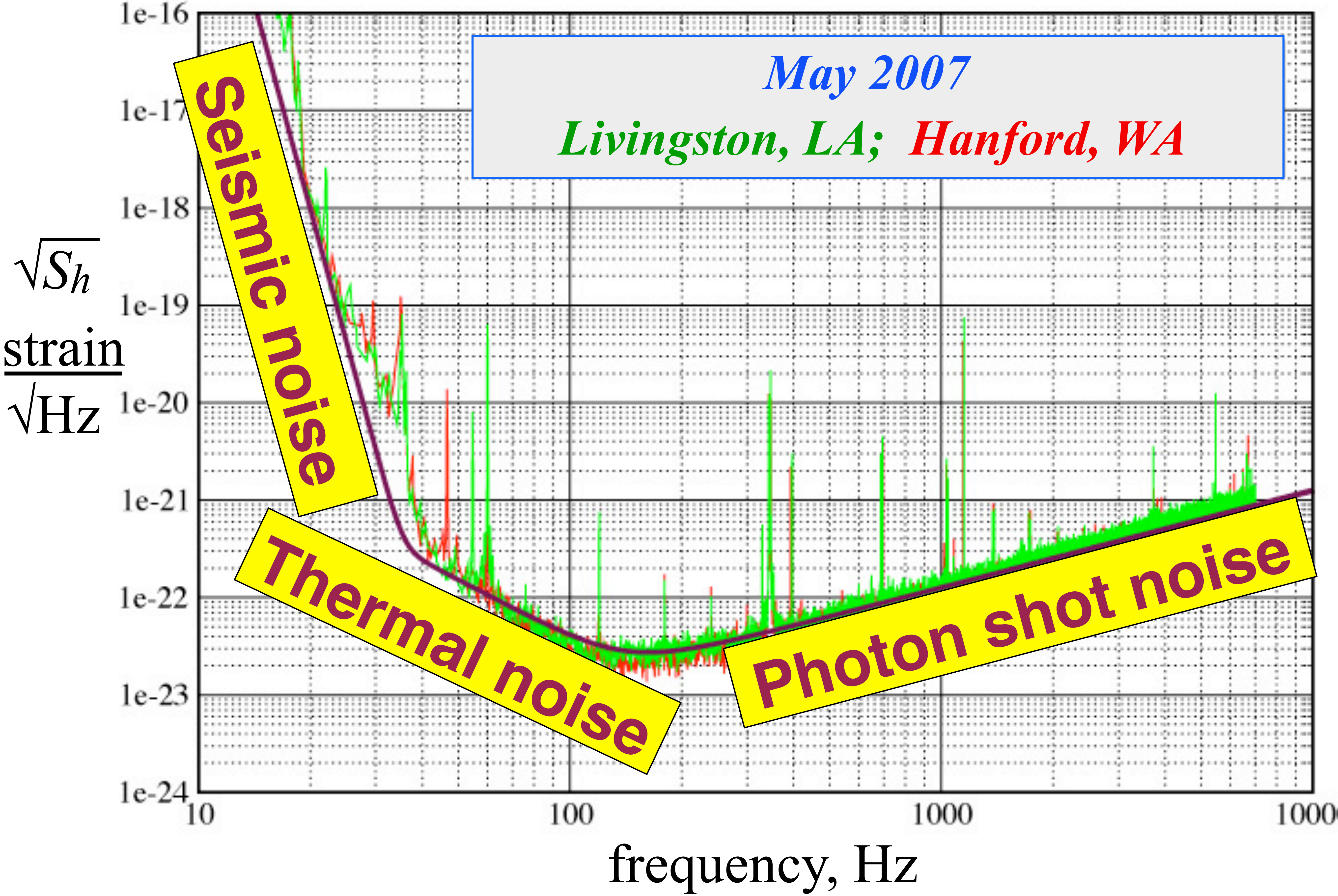
- $\sqrt{f\, S_h(f)} = \text{rms fluctuations of } h(t) \text{ in bandwidth equal to frequency}$

  $\equiv h_{\text{rms}}(f)$

Initial-LIGO Noise Curves



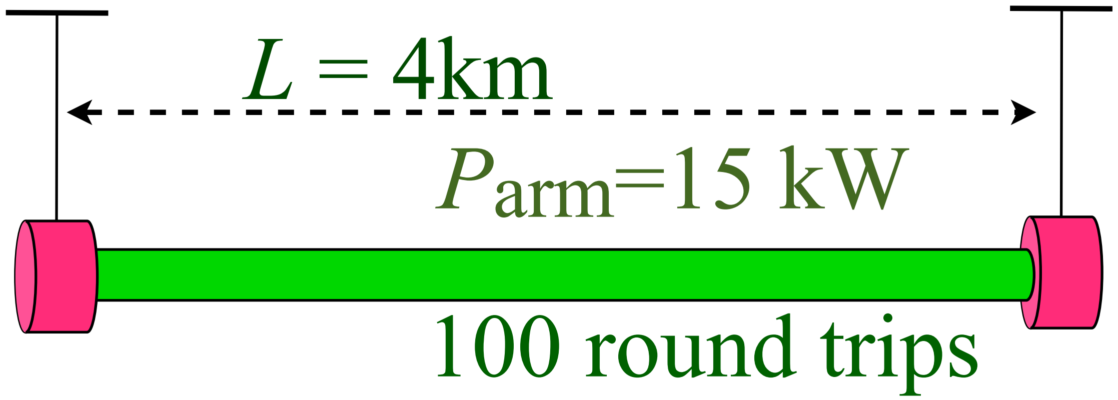
Fundamental Noise Sources



May 2007

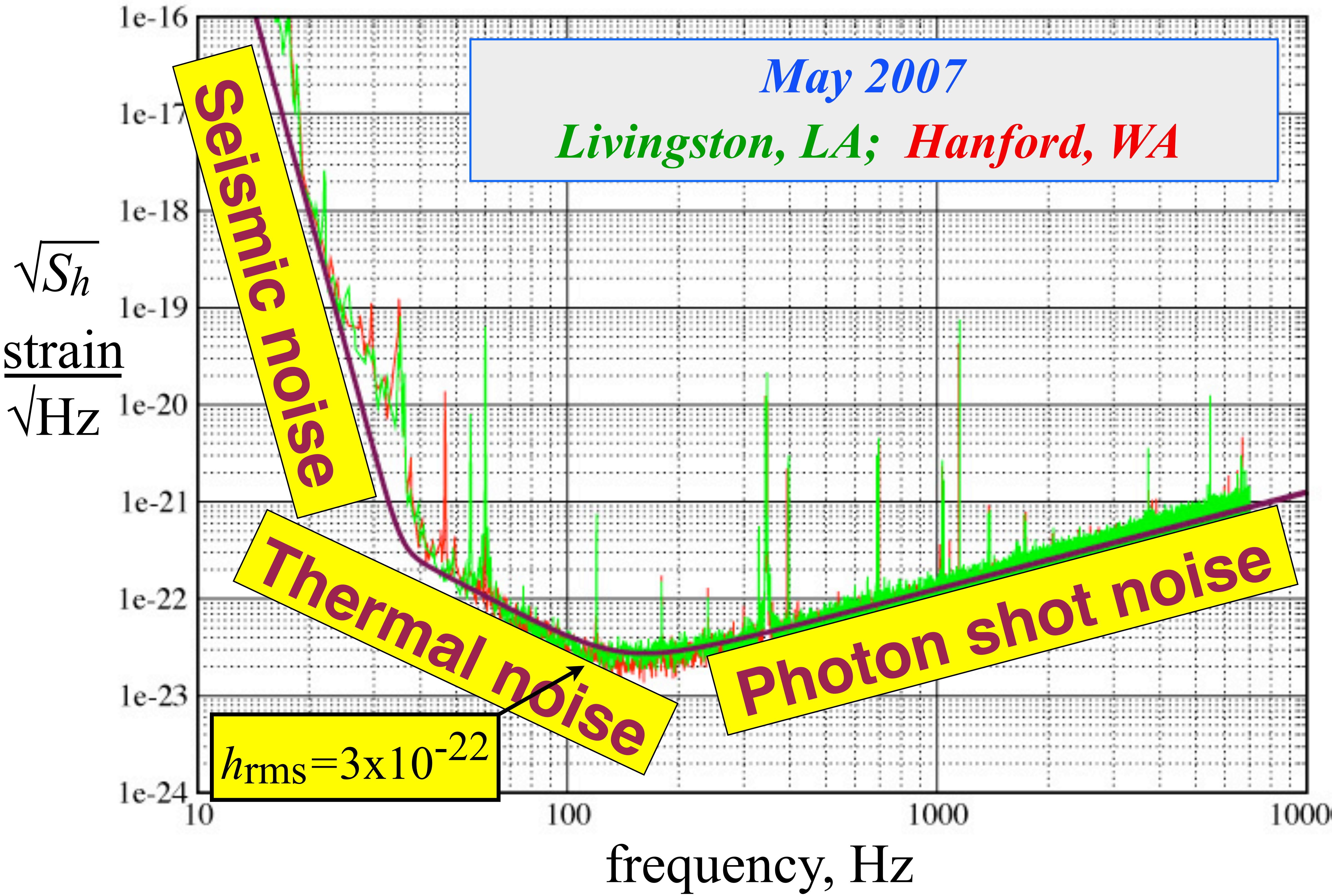
Livingston, LA; Hanford, WA

Photon Shot Noise



- **At GW frequency $f = 100$ Hz (optimal sensitivity)**
  - $P_{\rm arm}=15$ kW ⇒ Number of photons in arm cavity: $N = 2\text{x}10^{18}$
  - Light in coherent state ⇒ variance is $\Delta N=\sqrt{N} \simeq 10^9$
  - Uncertainty Principle: $\Delta\Phi\ \Delta N \geq 1$ ⇒ rms phase fluctuations in arm cavity light: $\Delta\Phi=10^{-9}$
  - GW moves mirrors, produces phase shift on the arm cavity light $\Phi = 100\, k\, 2hL \simeq 10^{-9}$ for $h = 3\times10^{-22}$; so light's phase fluctuations correspond to $h_{\rm rms} = 3\times10^{-22}$ at $f = 100$ Hz; $\sqrt{S_h} = 3\times10^{-23}$
- **At GW frequency $f > 100$ Hz**
  - GW has less time to put phase shift onto light, so: $\sqrt{S_h} \sim 1/f$

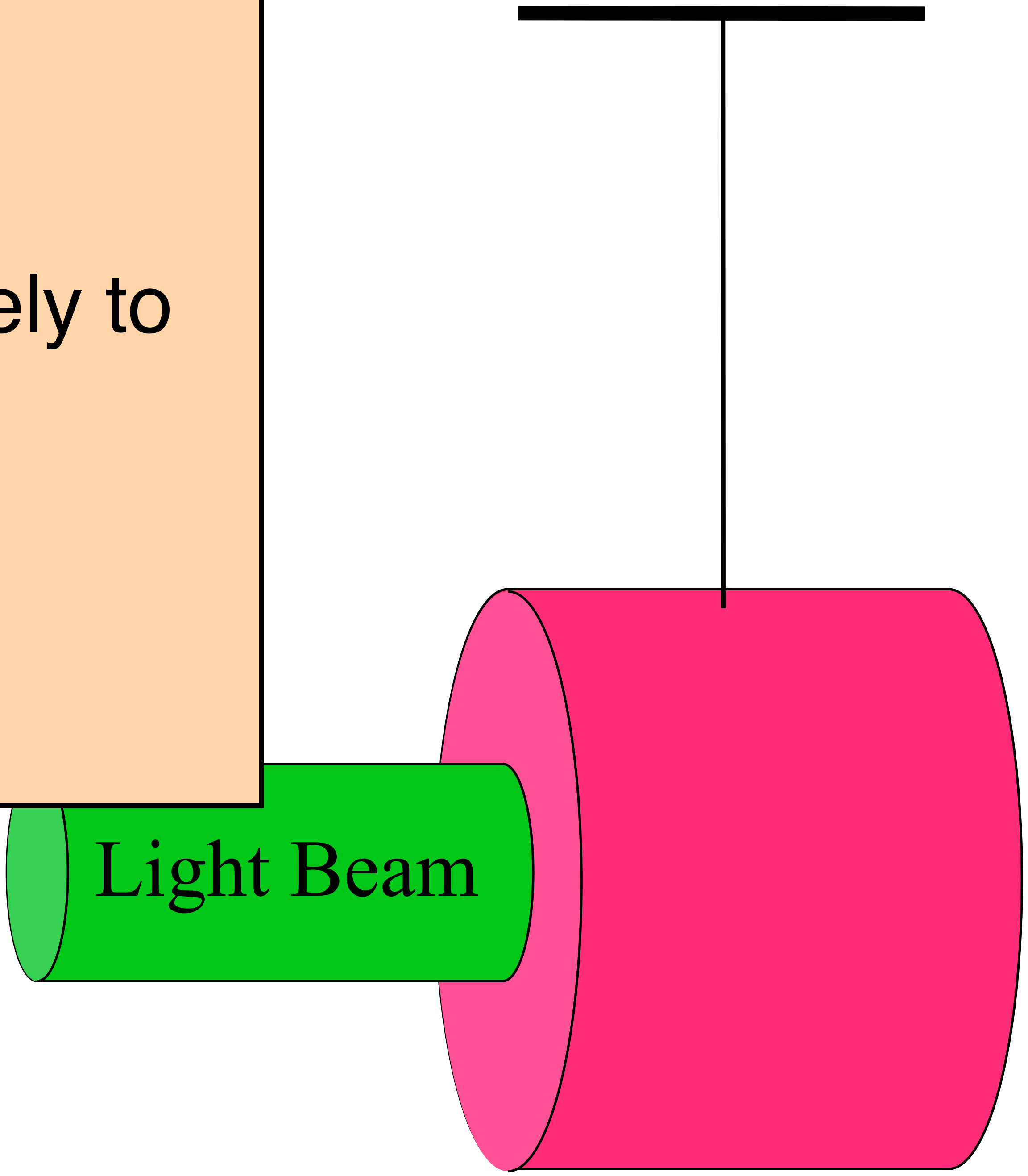
Fundamental Noise Sources



Thermal Noise

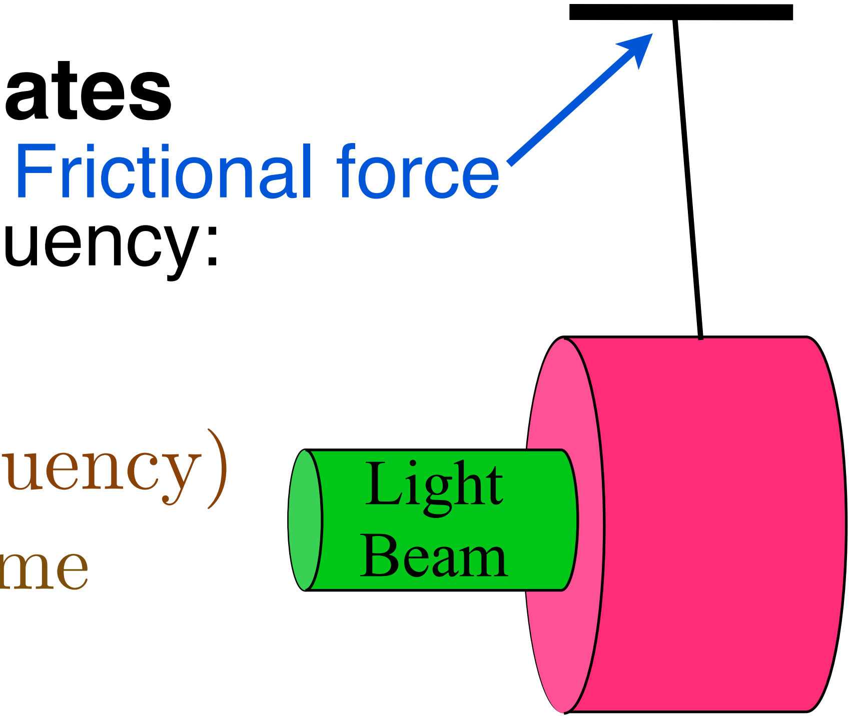
- rms thermal motions of atoms in mirror surface:
  - ◊ amplitude ~ $(kT/\mu\omega_o^2)^{1/2} \sim 10^{-11}$ meters ~ $10^7\,\Delta L$; at $\omega_o \sim 10^{13}$ Hz

- Light beam averages over:
  - ◊ ~50 cm^2 (~10^9 surface atoms)
  - ◊ ~0.01 sec (~10^{11} atomic vibrations)
- Result: Light is sensitive almost solely to center-of-mass motion
- **So mirror behaves like a 40 kg "particle"**
- **Residual motions: thermal noise**



Thermal Noise in Initial LIGO Interferometers

- **Thermal noise in suspension wires dominates**



  - mirror's center of mass motion near its eigenfrequency:

$$M\ddot{x} + M\frac{2}{\tau}\dot{x} + M\omega_o^2 = F(t)$$

$2\pi \times$ (1 Hz pendulum frequency)

$\sim 10^5$ sec pendulum damping time

  - Fluctuating force $F(t)$, together with damping, can produce $x_{\rm rms} = \sqrt{\frac{kT}{M\omega_o^2}}$ only if $S_F = 8MkT/\tau$ at $f = \omega_o/2\pi f$ [fluctuation-dissipation theorem]

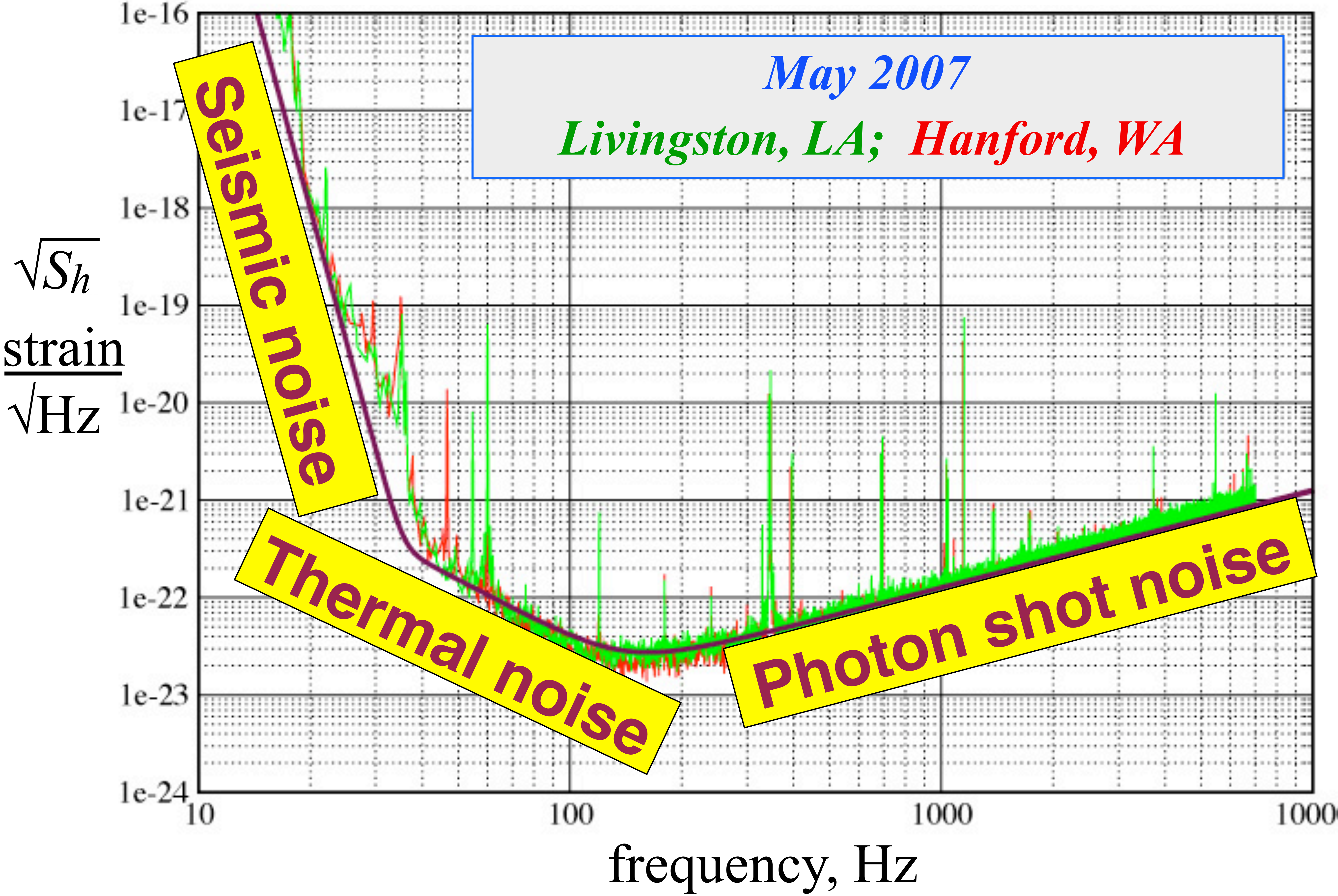
  - We are interested in the noise at f=10 to 100 Hz ; experiment shows that S_F scales $\sim \omega_o / 2\pi f$ so $S_F = 8MkT/\tau\ (\omega_o/2\pi f)$

  - Since $2\pi f \gg \omega_o \gg 1/\tau$, amplitudes at frequency f are $-(2\pi f)^2 Mx = F$, so spectral densities are $S_x = S_F / [(2\pi f)^2 M]^2$

  - Combining:

$$\boxed{\sqrt{S_h} = \left[\frac{32kT}{ML^2(2\pi f)^4\tau}\left(\frac{\omega_o}{2\pi f}\right)\right]^{1/2} = \frac{2\times 10^{-23}}{\sqrt{\rm Hz}}\left(\frac{100{\rm Hz}}{f}\right)^{5/2}}$$

Fundamental Noise Sources



May 2007

Livingston, LA; Hanford, WA

Seismic noise

Thermal noise

Photon shot noise

Seismic Noise

- Spring between masses 1 and 2; rigidity $m_2\,\omega_o^2$ so

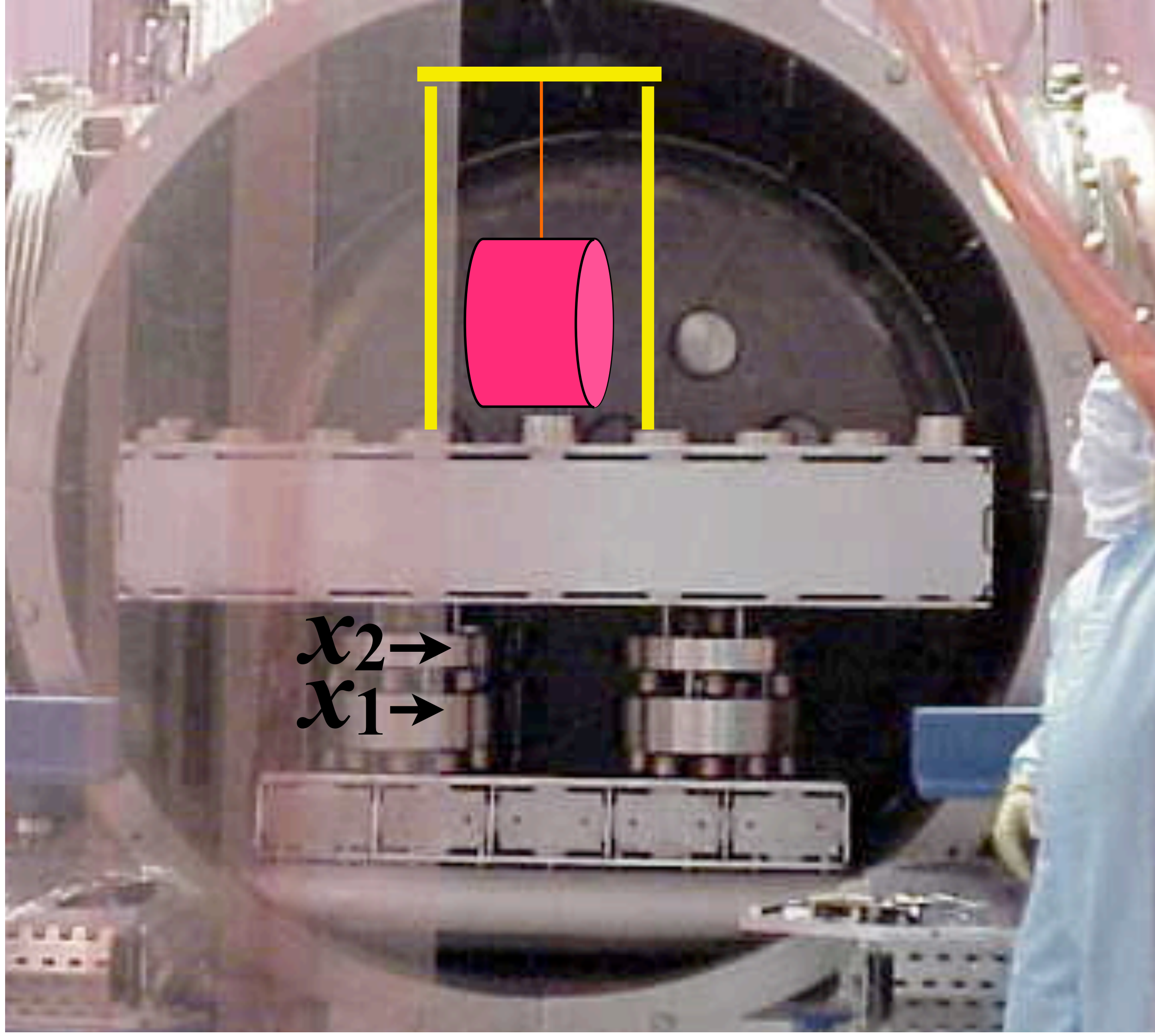
$$m_2\ddot{x}_2 = m_2\omega_o^2(x_1 - x_2)$$

- If seismic noise is driving x_1 at frequency $f = \omega/2\pi$, then

$$-\omega^2 x_2 = \omega_o^2(x_1 - x_2)$$

- $\omega_o = 2\pi$ (10 Hz); GW frequencies are $\omega >> \omega_o$ so $x_2 = (\omega_o/\omega)^2 x_1$ and

$$\frac{S_{x_2}}{S_{x_1}} = \left(\frac{\omega_o}{\omega}\right)^4$$



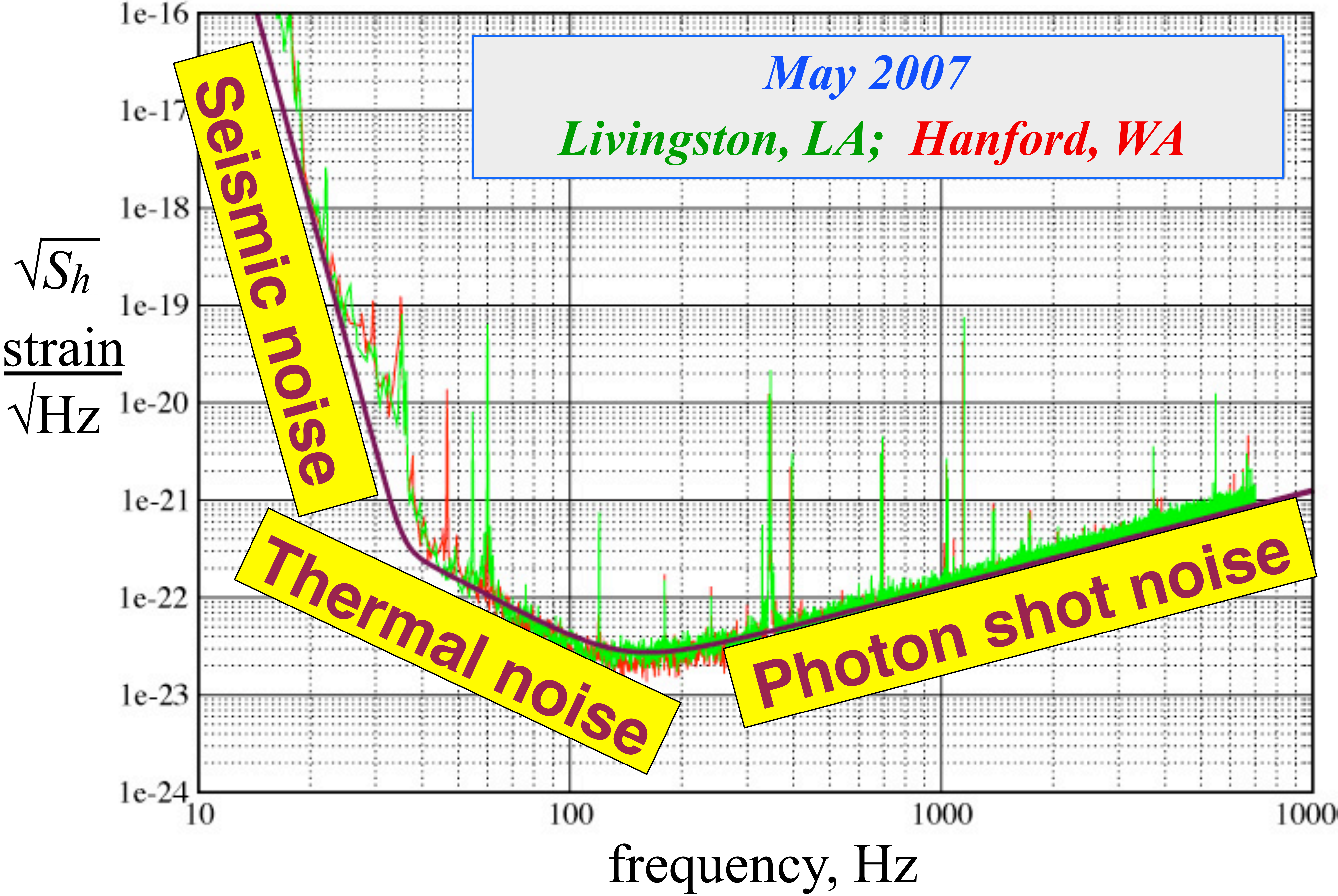
- Four mass-spring sets with $\omega_o \simeq 10$ Hz , plus pendulum with $\omega_o \simeq 1$ Hz

- Ground motion at LIGO sites: $S_{x_g} \simeq 10^{-18}\,\frac{\mathrm{m}^2}{\mathrm{Hz}}\left(\frac{10\mathrm{Hz}}{f}\right)^4$

- So: LIGO seismic strain noise is $S_h = \frac{4S_x}{L^2} = \frac{4S_{x_g}}{L^2}\left(\frac{10\ \mathrm{Hz}}{f}\right)^{16}\left(\frac{1\ \mathrm{Hz}}{f}\right)^4$

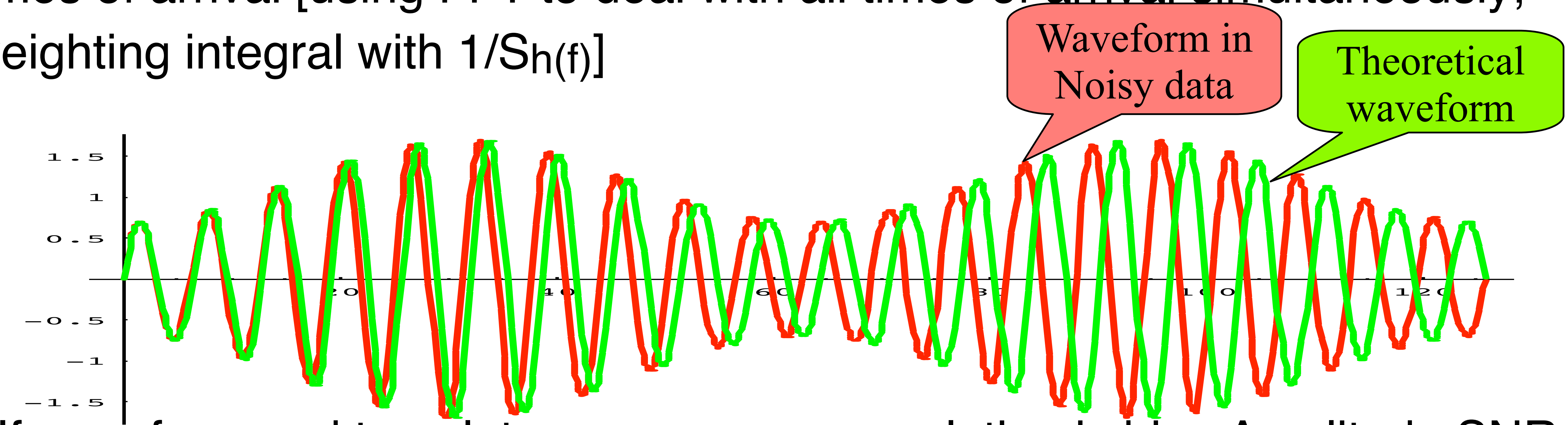
$$\boxed{\sqrt{S_h} = \frac{1\times 10^{-22}}{\sqrt{\mathrm{Hz}}}\left(\frac{40\ \mathrm{Hz}}{f}\right)^{12}}$$

Fundamental Noise Sources



GW Searches: Data Analysis

- **If waveforms are known** [e.g. 7 parameter BH/BH inspiral, merger, ringdown] and have many cycles: Matched Filter Method
  - Build discrete family of templates covering the parameter space (up to ~ 10,000 template shapes with unknown arrival times)
  - Cross correlate each template with interferometer output, for various times of arrival [using FFT to deal with all times of arrival simultaneously; weighting integral with $1/S_{h(f)}$]



  - If waveform and template agree, cross correlation is big. Amplitude SNR

$$\boxed{\mathrm{SNR} = \int \frac{|\tilde{h}(f)|^2}{S_h(f)}\, df}$$

So a good measure of sensitivity is

$$\boxed{\int \frac{df}{S_h(f)}}$$

- **If waveforms are not known,** a variety of other data analysis methods are used. [e.g.: for stochastic background - Bruce Allen's lecture, 14:00 this afternoon]

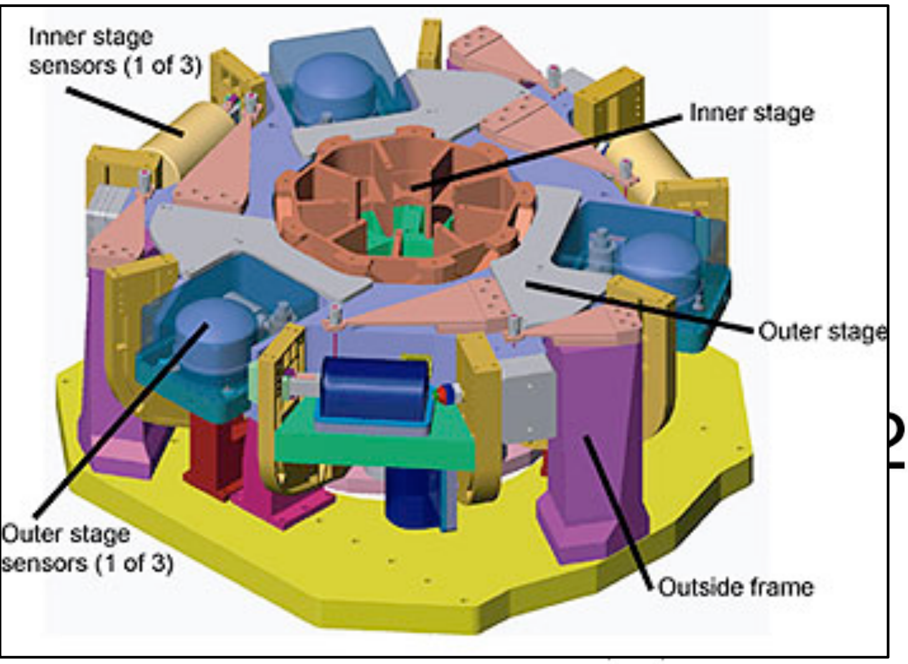
2 Year Long “S5” Search: Examples of Results

- **BH/BH Binaries with $M_{tot} < 35M_{sun}$:** <1/860 yrs in MWEG
- **GRB070201** (coincident with Andromeda) is not a NS/NS or NS/BH in Andromeda
- **Targeted Pulsar Search**
  - **Crab pulsar:** < 7% of spindown energy goes to GWs
- **Stochastic Background:** $\Omega < 7 \times 10^{-6}$ in 41-178 Hz band (Bayesian 90% confidence)

From Initial Interferometers to Advanced

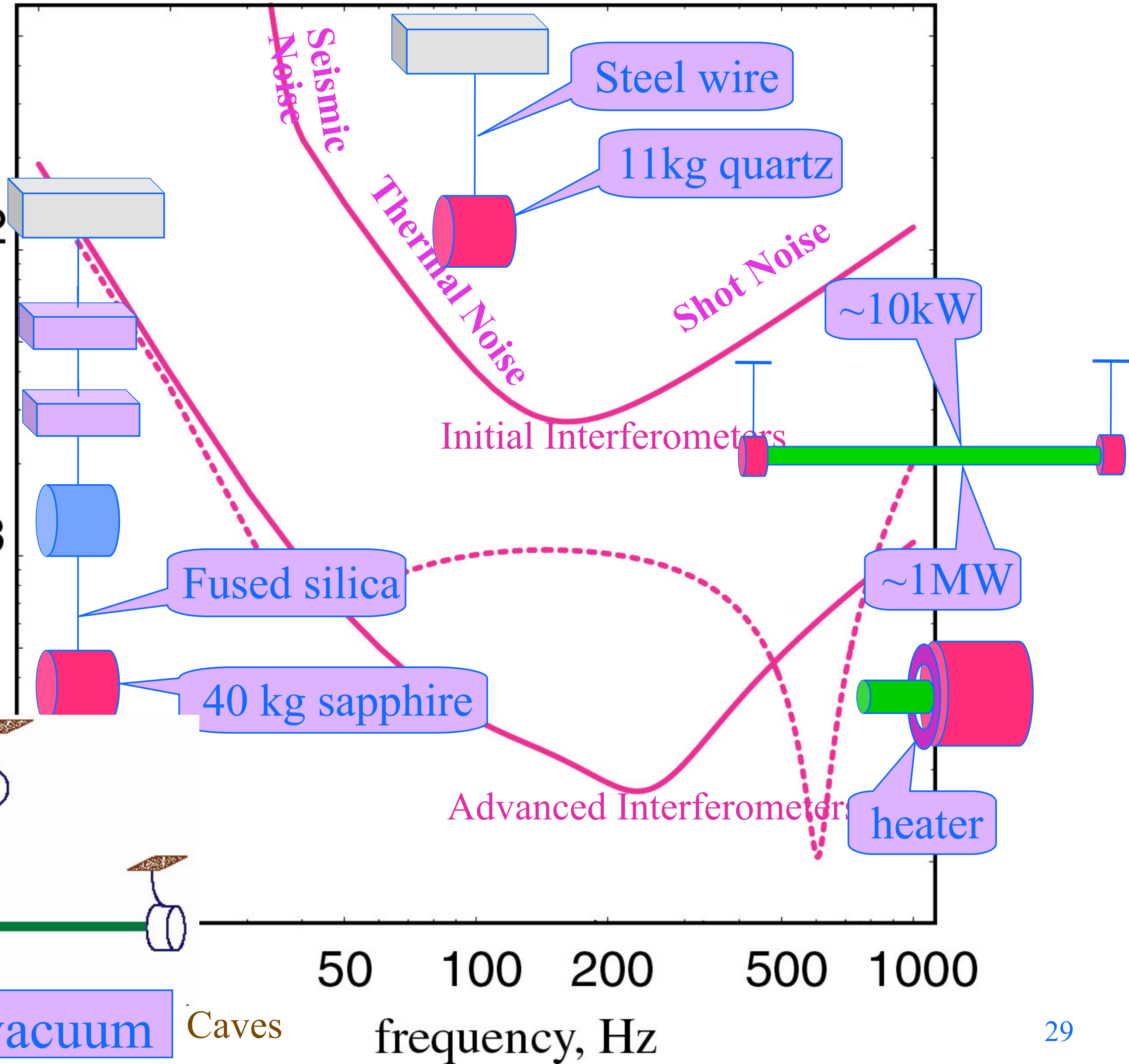
- **1989:** LIGO Proposed
- **1995-2000:** Construction; installation of initial interferometers
- **2000-2005:** Initial interferomters ommissioned
- **2005-2007:** “S5” gravitational-wave search
- **2007-2010:** Advanced interferometers procurement & preparation for installation. Initial interferometers enhanced; “S6” search now underway
- **2011-2012:** Advanced interferometers installation
- **2012 - :** Advanced interferometers commissioning

From Initial Interferometers to Advanced

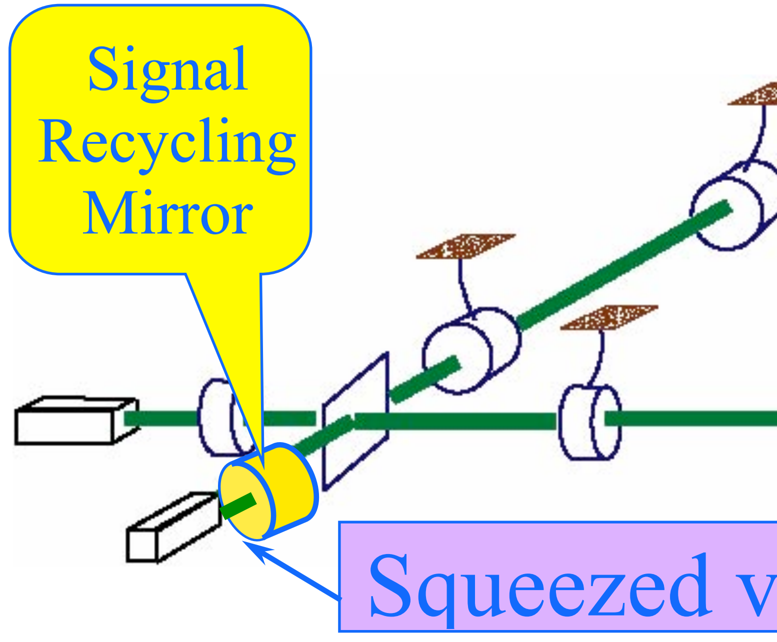


ACTIVE VIBRATION ISOLATION

Seismic wall: 40 Hz→10 Hz



Drever, Meers, Strain

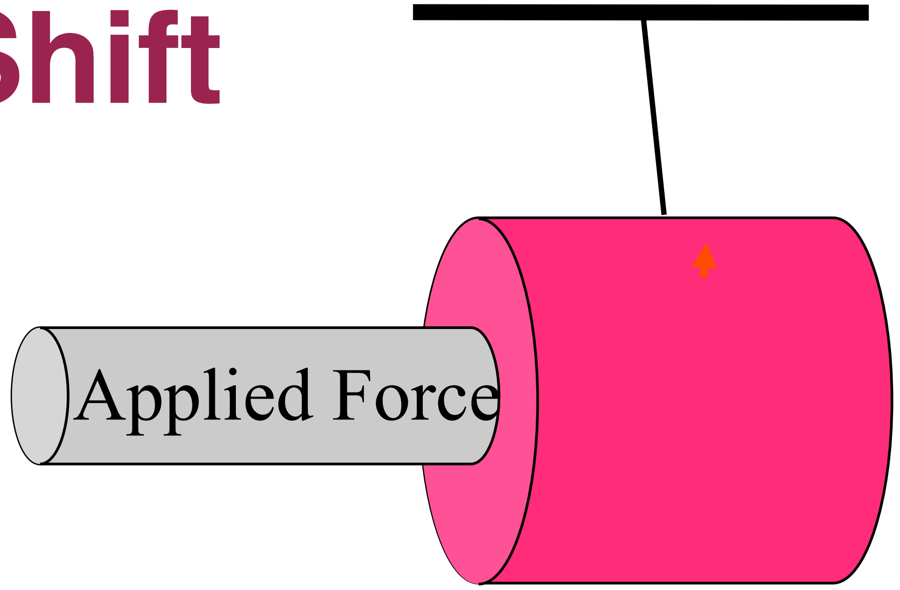


Caves

Two Paradigm Shifts on Fundamental Noises
(largely from theory students in my group)

- Thermal Noise [Yuri Levin]
- Optical Noise & Quantum Noise [Carlton Caves, ..., Alessandra Buonanno and Yanbei Chen]

Thermal Noise Paradigm Shift

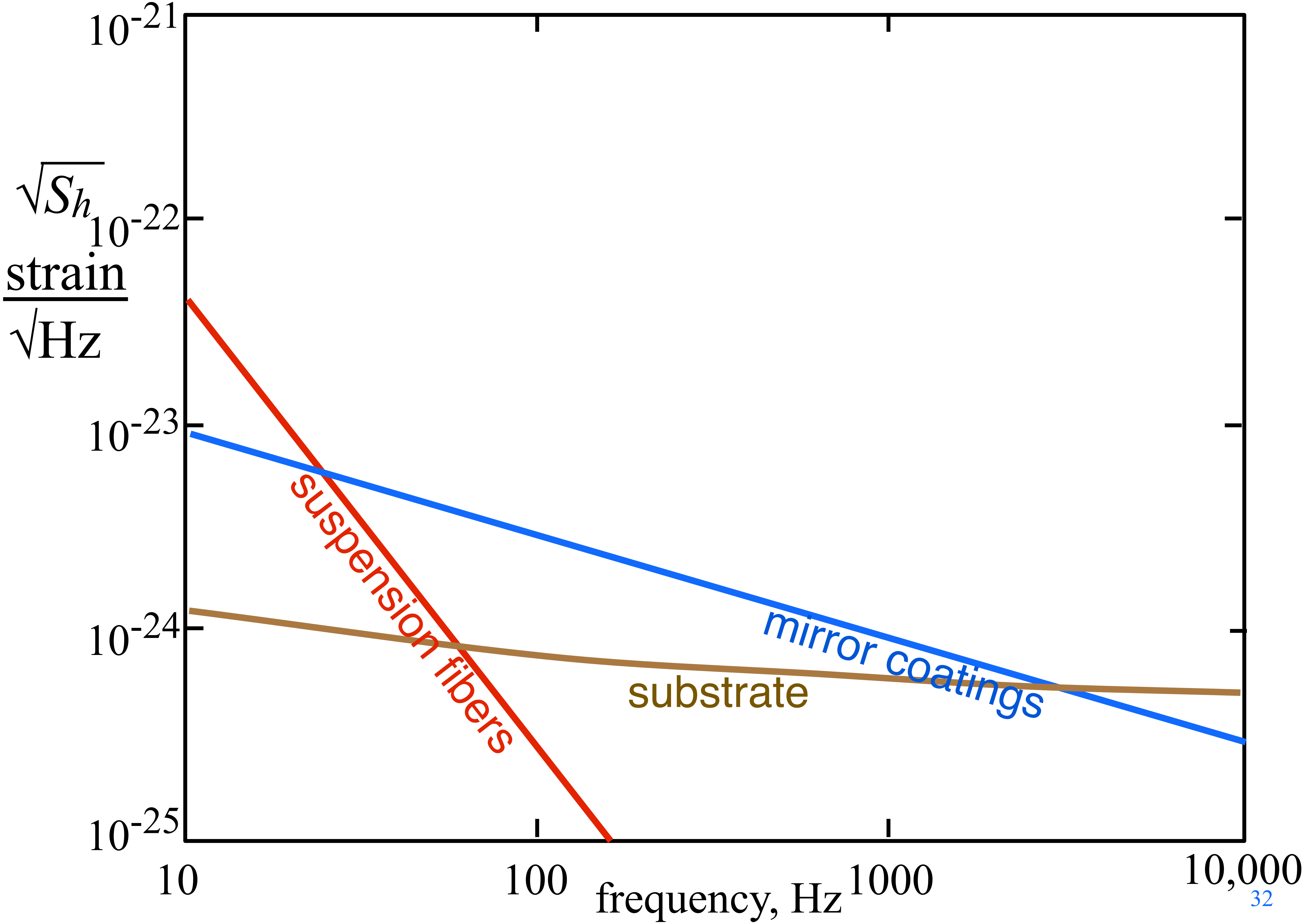


- Previous paradigm: sum over normal modes
- **Yuri Levin's** thought experiment (variant of fluctuation dissipation theorem)
- To compute spectral density of noise at frequency f=2πω:
- Apply an oscillating force F_o with frequency f and cross-sectional profile same as light beam
- Compute total rate of dissipation $W_{diss} = T\ dS/dt$
- $S_x(\omega) = (4kT/\omega^2)(W_{diss}/F_o^2)$

- **CONSEQUENCES:**
- Previous paradigm is gives wrong answers if dissipation inhomogeneous
- Classify noise by dissipation location & mechanism
- Mirror coating dangerous!

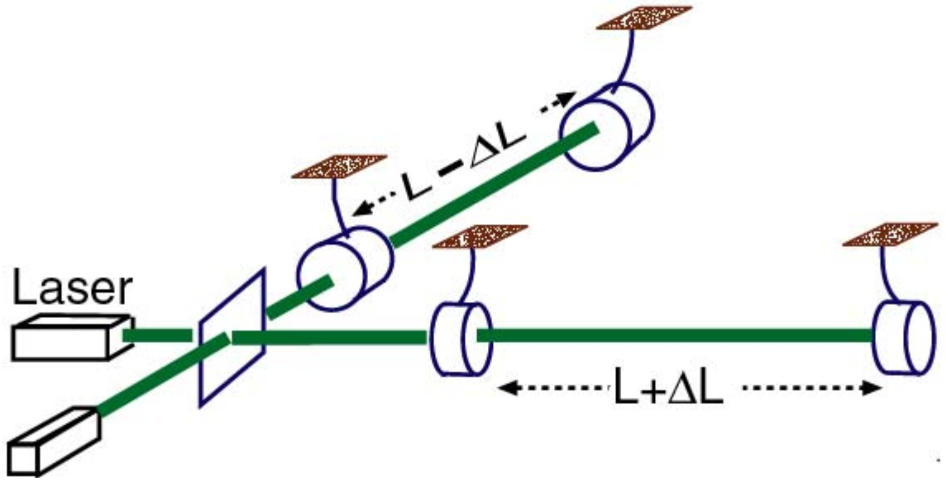
Huge effort has gone into exploring this and optimizing design

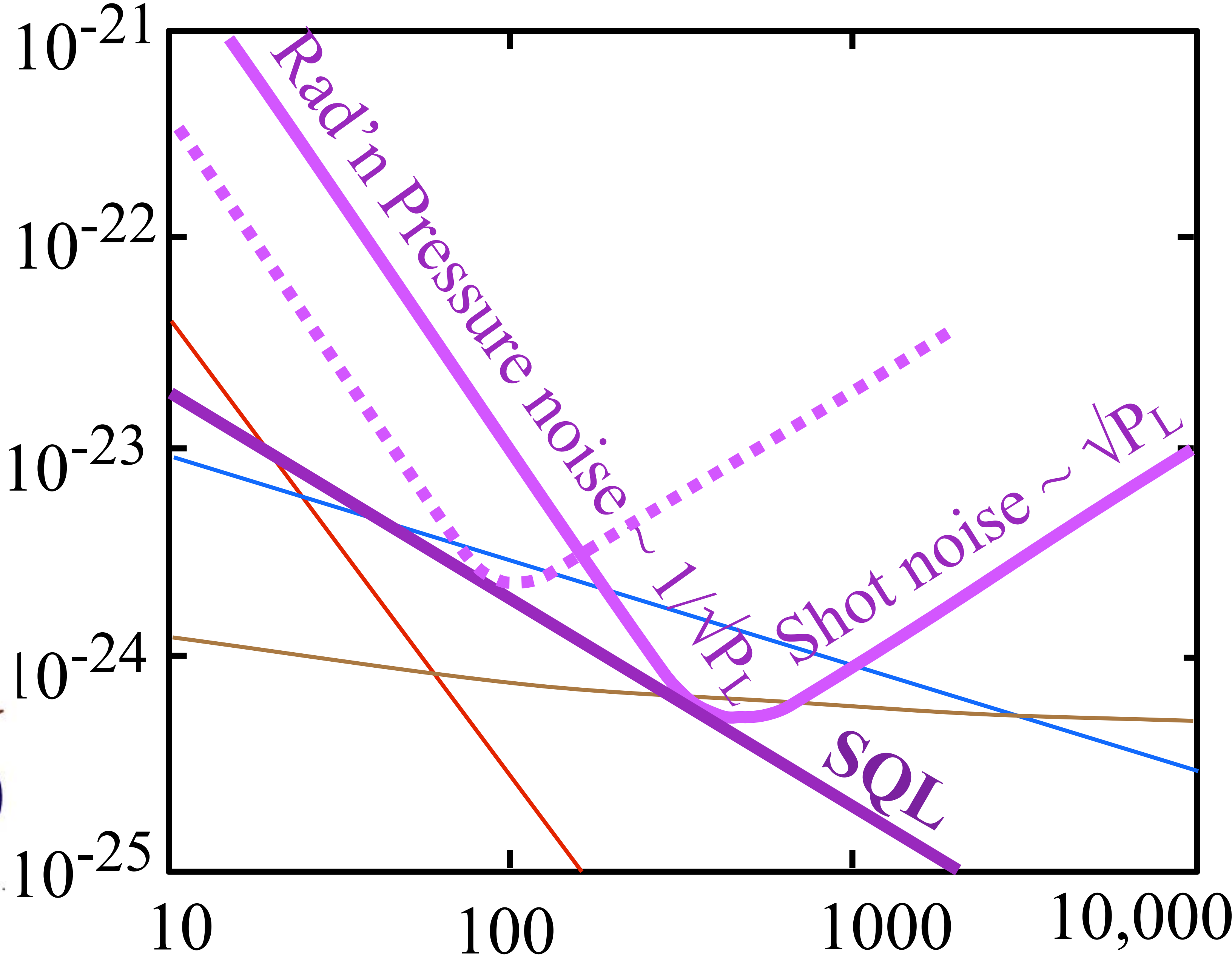
Thermal Noises in Advanced LIGO



Optical and Quantum Noise Paradigm Shift

- Standard Quantum Limit [Braginsky, Caves]
  - Like Heisenberg microscope
  - SQL enforced by radiation pressure





Optical and Quantum Noise Paradigm Shift

- Standard Quantum Limit [Braginsky, Caves]
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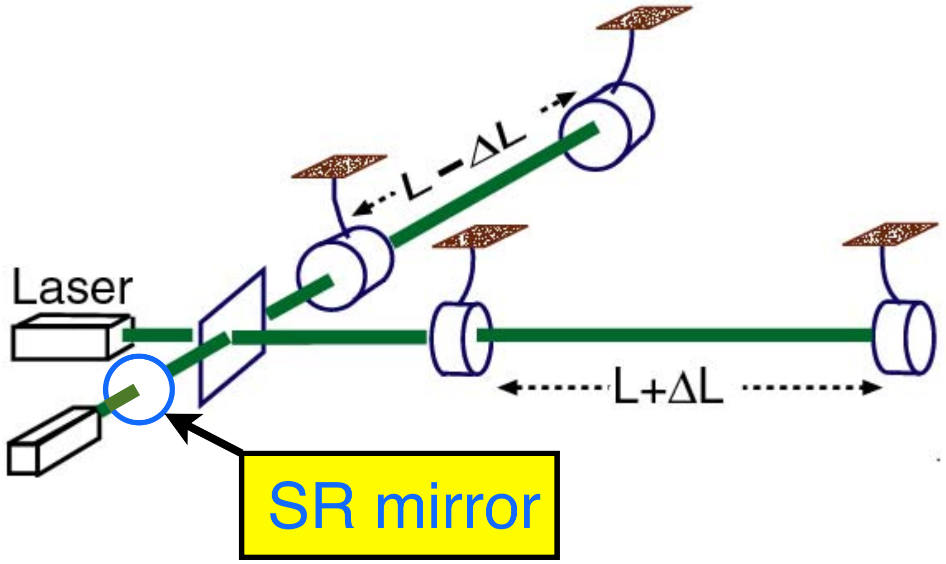
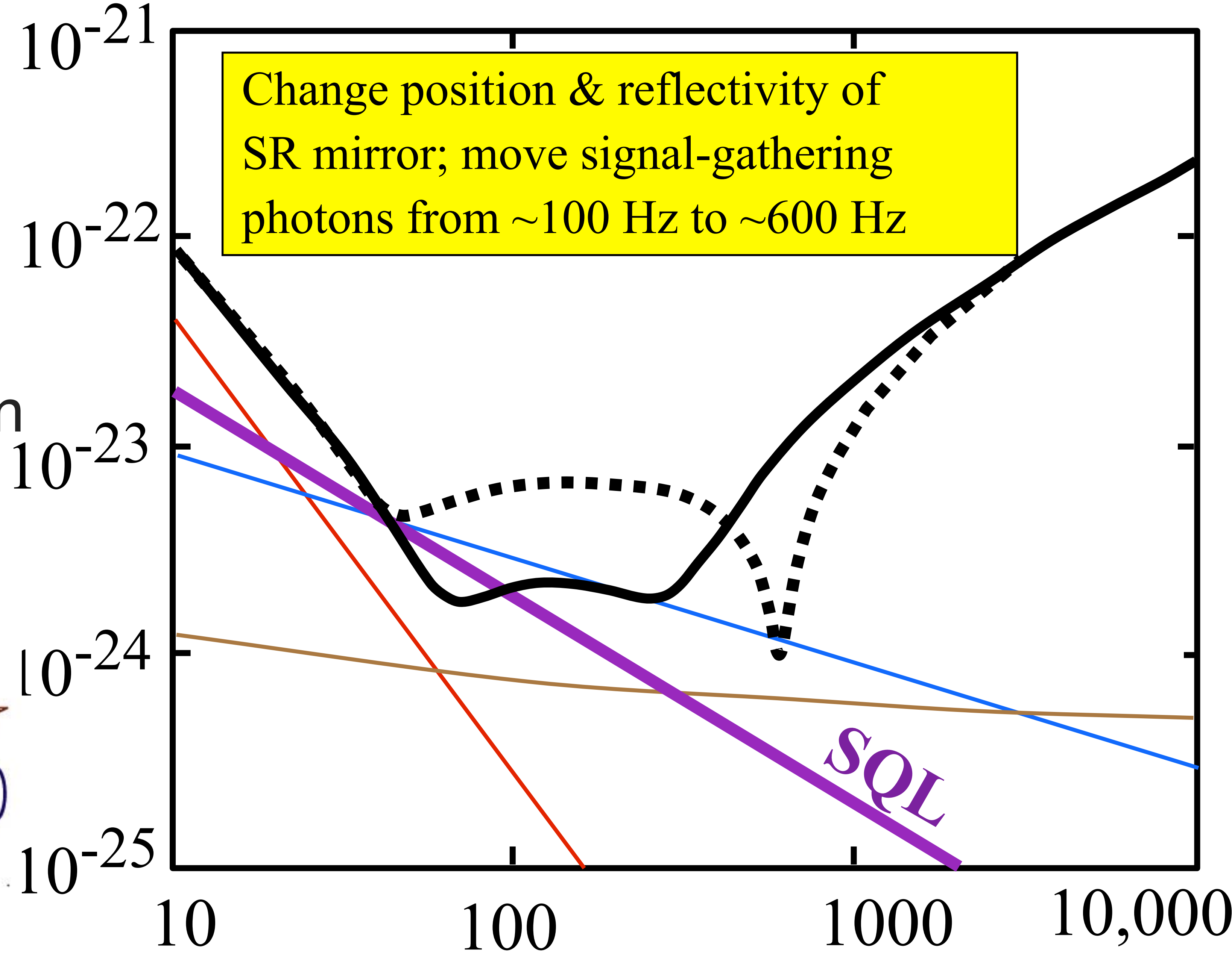
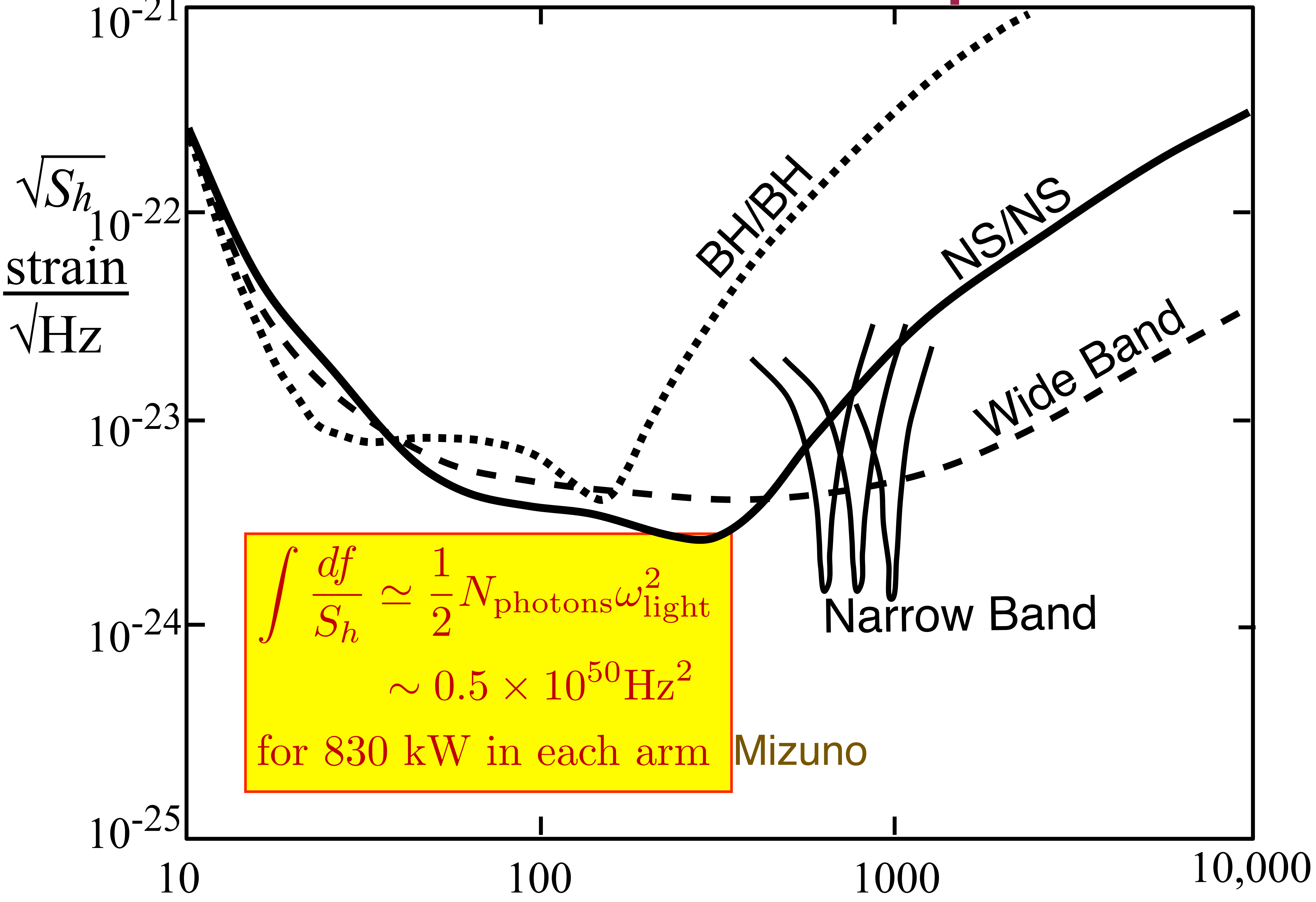




Change position & reflectivity of SR mirror; move signal-gathering photons from ~100 Hz to ~600 Hz

- Buonanno & Chen: Signal recycling (SR) mirror feeds position signal back onto mirrors as a back-action force ⇒
- Mirrors & light behave like coupled oscillators with **f**-dependent spring constants ⇒ correlations in shot noise & radn pressure noise; beat SQL
- Richer possibilities for reshaping noise than previously realized

Advanced LIGO Modes of Operation



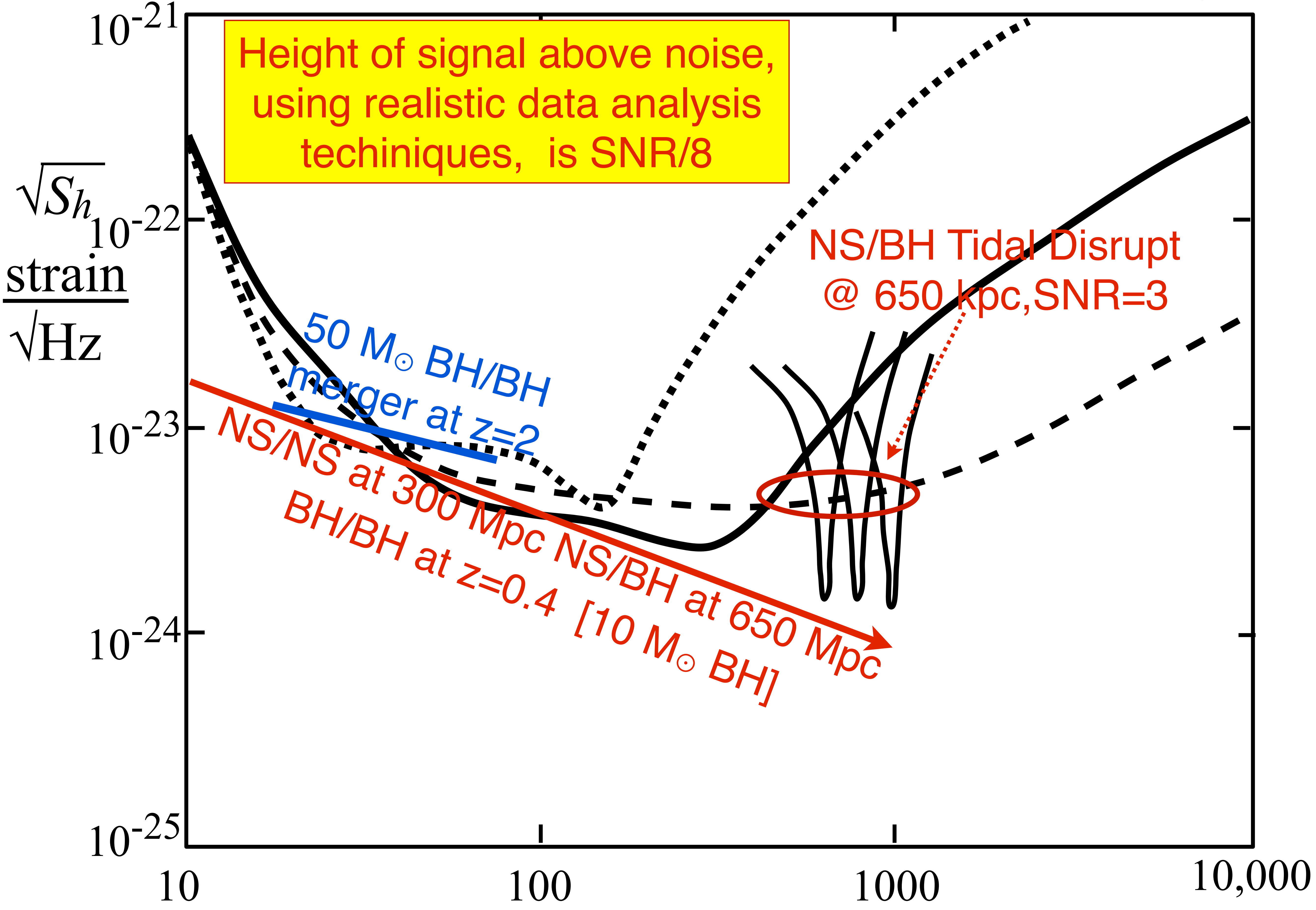
$$\int \frac{df}{S_h} \simeq \frac{1}{2} N_{\rm photons}\omega_{\rm light}^2$$

$$\sim 0.5 \times 10^{50}{\rm Hz}^2$$

for 830 kW in each arm

Mizuno

Some Sources for Advanced LIGO



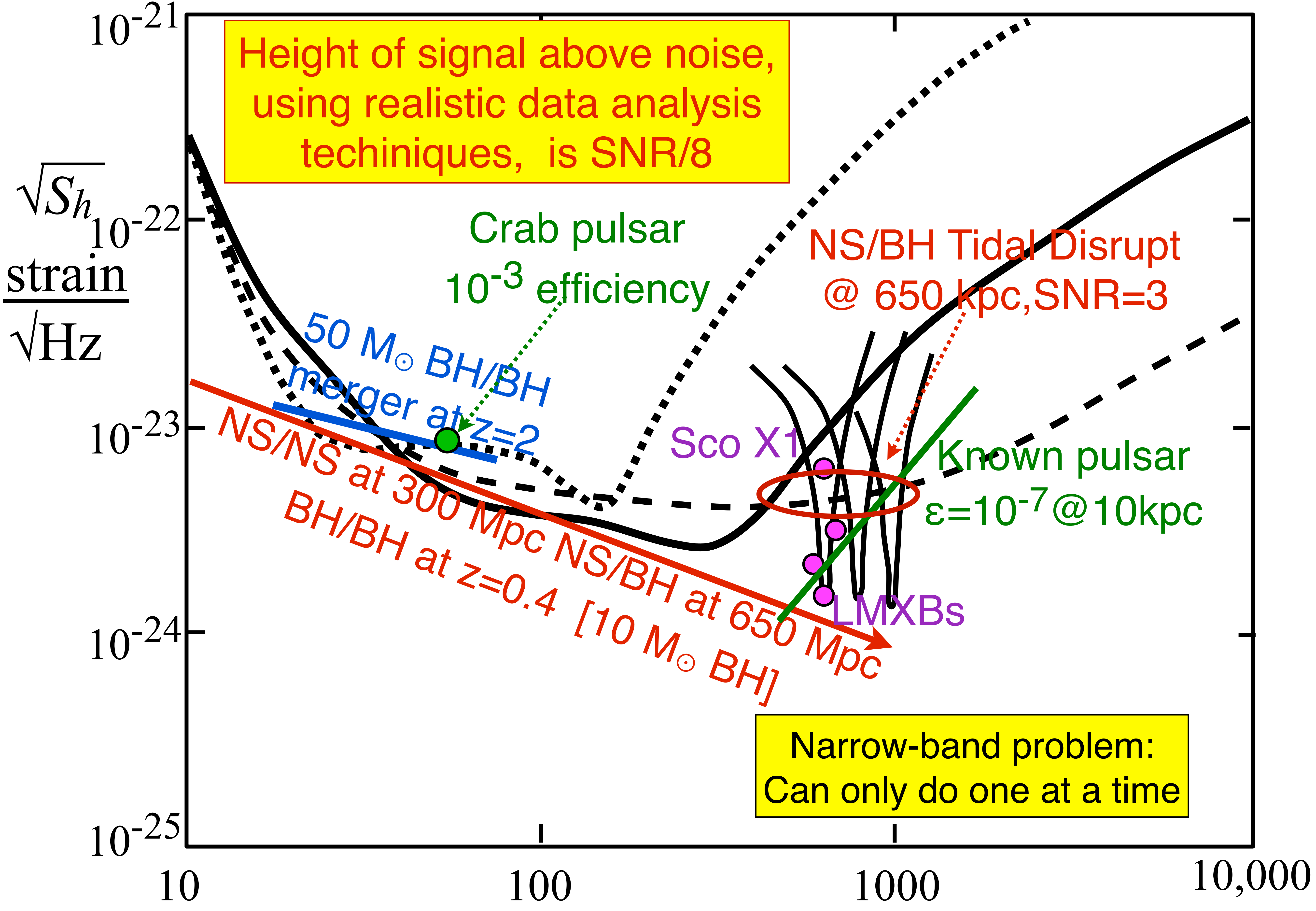
Height of signal above noise, using realistic data analysis techiniques, is SNR/8

Estimated Compact Binary Rates in Advanced LIGO

[from recent unpublished compilation by Ilya Mandel]

- **NS/NS:** ~ 40/yr. [~ 0.4 to ~ 400/yr]
  - extrapolating from observed NS/NS in our galaxy; also population synthesis
- **NS/BH:** ~ 10/yr [~0.2 to ~300/yr]
  - population synthesis
- **BH/BH:** ~ 20/yr [~0.5 to ~1000/yr]
  - population synthesis

Some Sources for Advanced LIGO

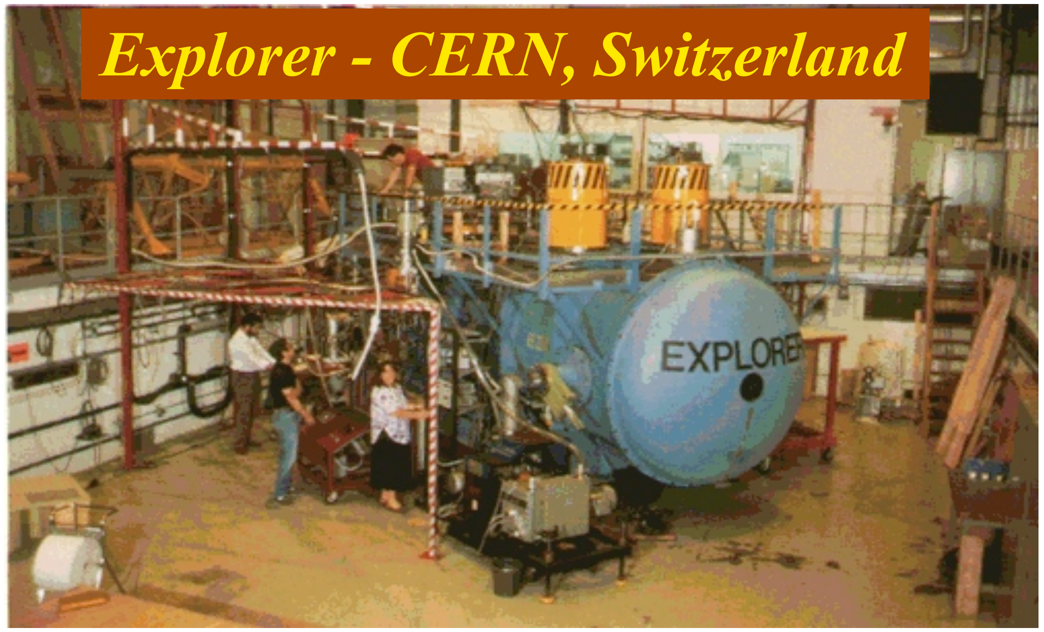


Height of signal above noise, using realistic data analysis techiniques, is SNR/8

Narrow-band problem: Can only do one at a time

Resonant-Mass GW Detectors

- Network in 1990s - 2000s


Explorer - CERN, Switzerland

Pioneered by
Joseph Weber
(U Maryland)
1960s & 70s


Niobe - Perth Australia


Nautilus - Rome, Italy


Auriga - Lugarno, Italy


Allegro - Louisiana USA

Resonant-Mass GW Detectors

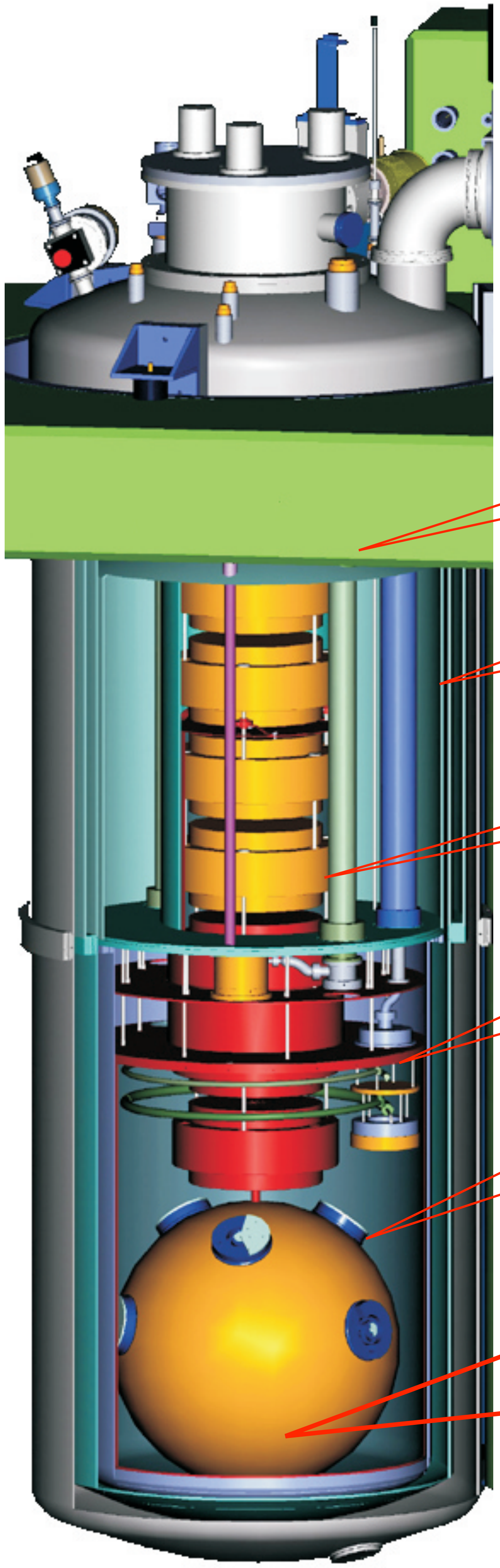
- **The most promising future: Spherical Masses - GRAIL Georgio Frossati et al here in Leiden**

- Significantly higher sensitivity per unit mass than cylinders
- Omnidirectional; optimal directional resolution
- Far less expensive than interferometers [a few million Euros vs hundreds of million Euros
- But far less mature

Minigrail: A prototype for GRAIL

MiniGRAIL

Frossati, de Waard, Gottardi
Usenko, Vinante.



Mechanically insulated concrete block

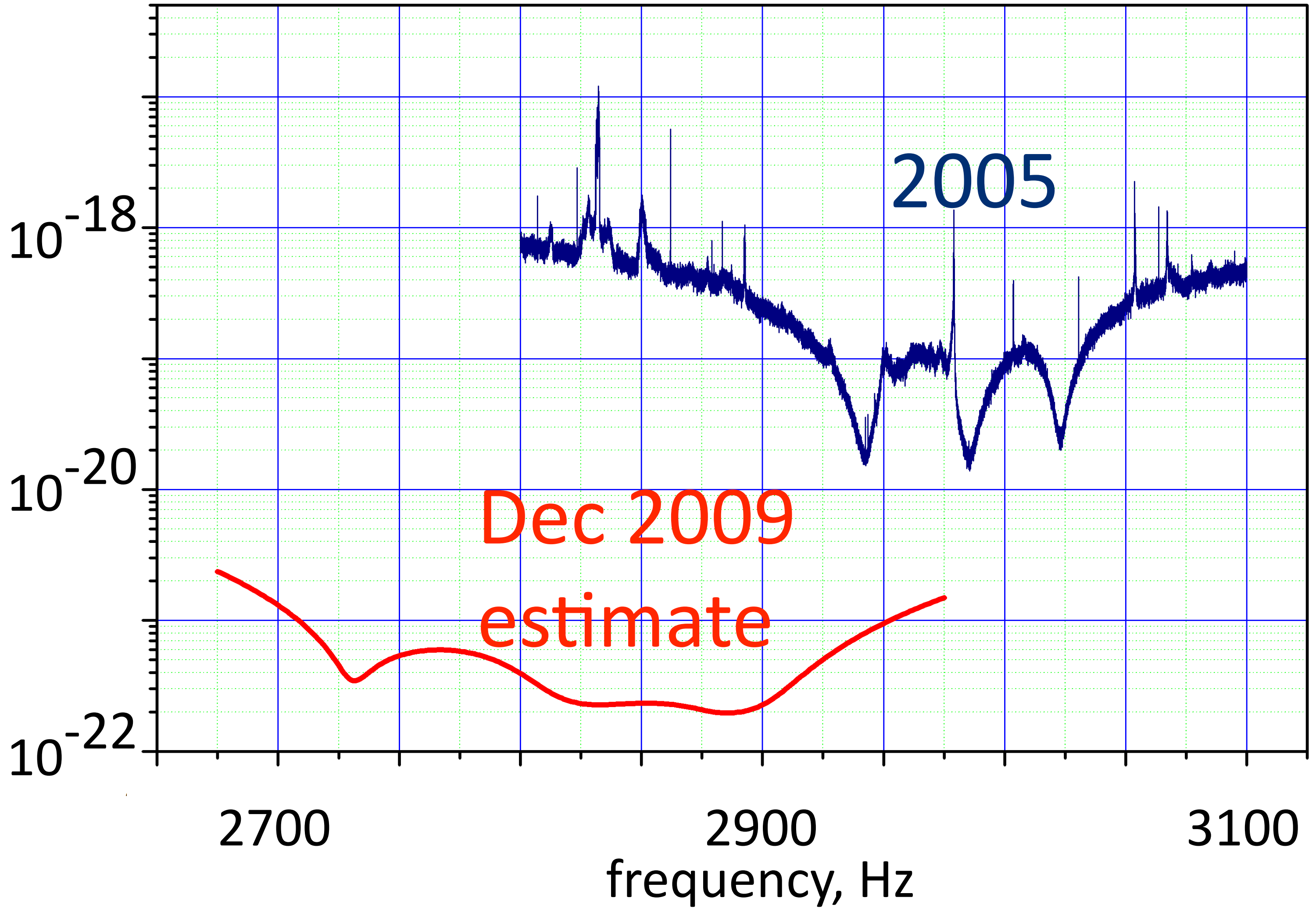
Vacuum chamber

Vibration insulation
Total attenuation ~350 dB

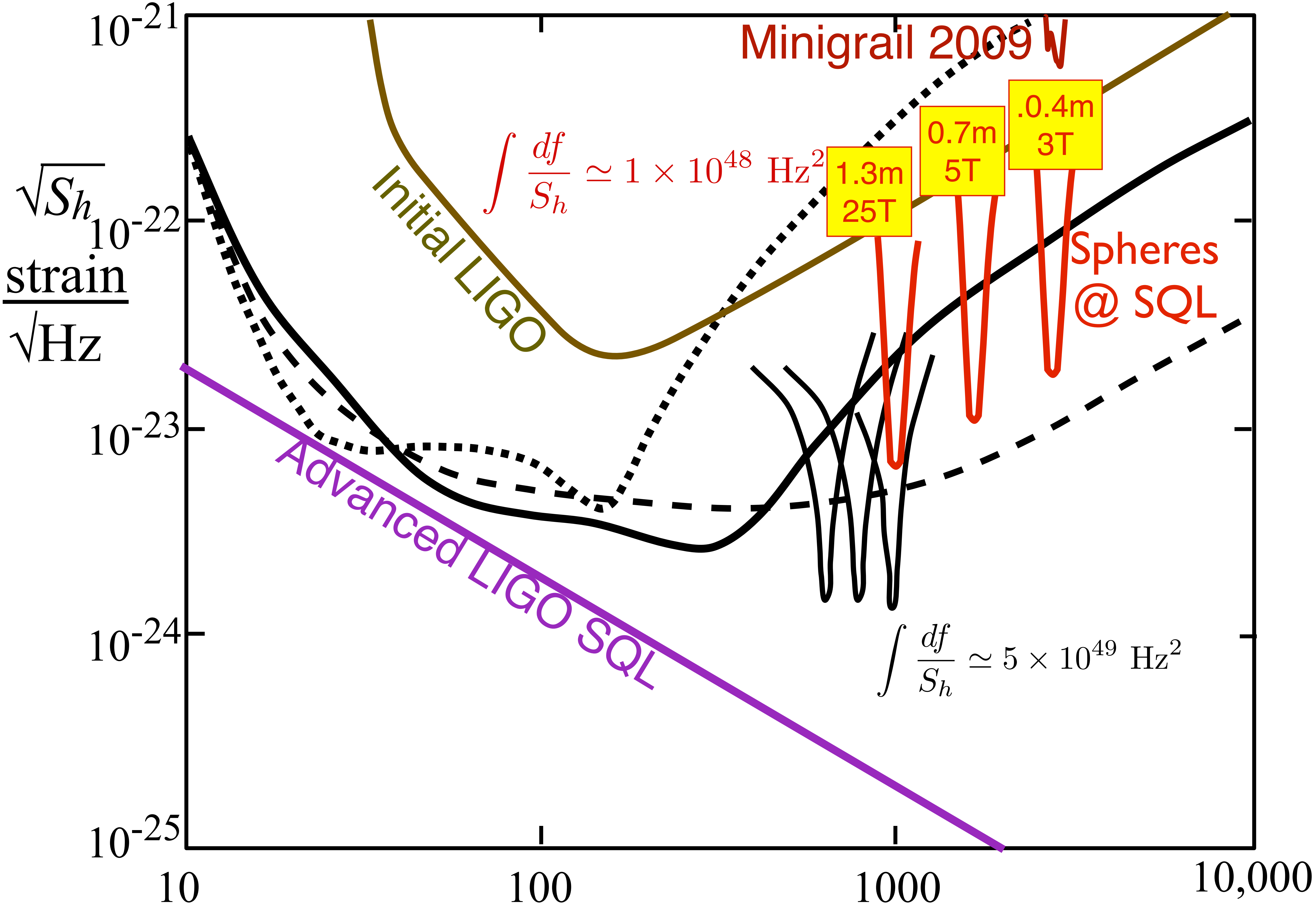
Dilution refrigerator
T < 10 mK

6 Capacitive transducers

1.4 ton, 0.68 m CuAl sphere
f ~ 2.9 kHz, Q ~ 10^6
T ~ 20 mK

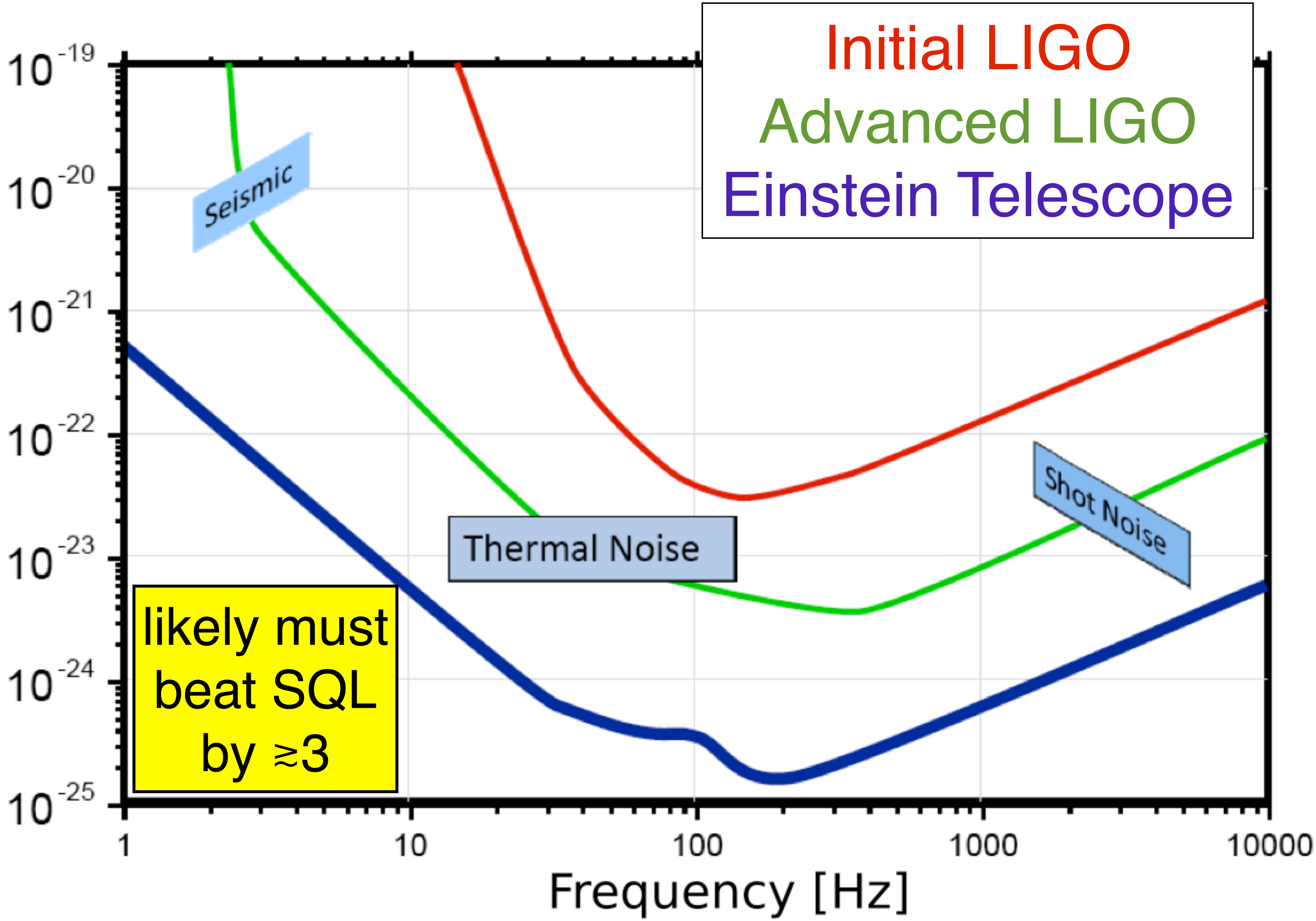


Spherical Detectors Compared with LIGO



Einstein Telescope

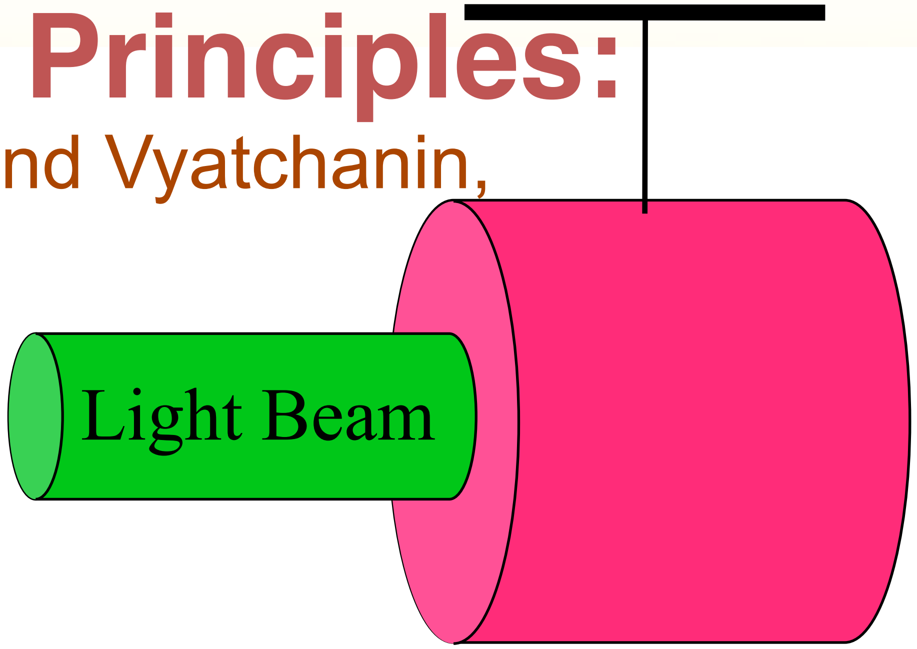
Design study underway by European consortium including NIKHEF



likely must beat SQL by ≳3

Two Independent Uncertainty Principles:

Braginsky, Gorodetsky, Khalili, Matsko, Thorne, and Vyatchanin,
Physical Review D, **67**, 082001 (2003)



- *Quantum-State Uncertainty Principle:*
  $[x, p]=ih$ =>
  independent of state of x, $\Delta x\, \Delta p \geq h/2$
- *Back-Action Uncertainty Principle:* Light superposes shot noise x_{SH} on output signal so $x_{OUT} = x + x_{SH}$, & kicks back at mirrors via radiation-pressure fluctuations to produce momentum change p_{RP}, so $p_{AFTER} = p + p_{RP}$
  - $[x_{SH}, p_{RP}]=-ih \Rightarrow \Delta x_{SH}\, \Delta p_{RP} \geq h/2$ *[Heisenberg Microscope]*
  - $[x_{OUT}, p_{AFTER}]=0$ and p_{AFTER} influences subsequent measurements => $[x_{OUT}(t), x_{OUT}(t')]=0$ for all t, t' =>
- *Collapse of wave function in one measurement cannot influence result of future measurements! And*
- *Quantum state uncertainty principle can be evaded by data analysis. LIGO GW signal independent mirror quantum state.*
- *Only Back-Action Uncertainty Principle is Dangerous*

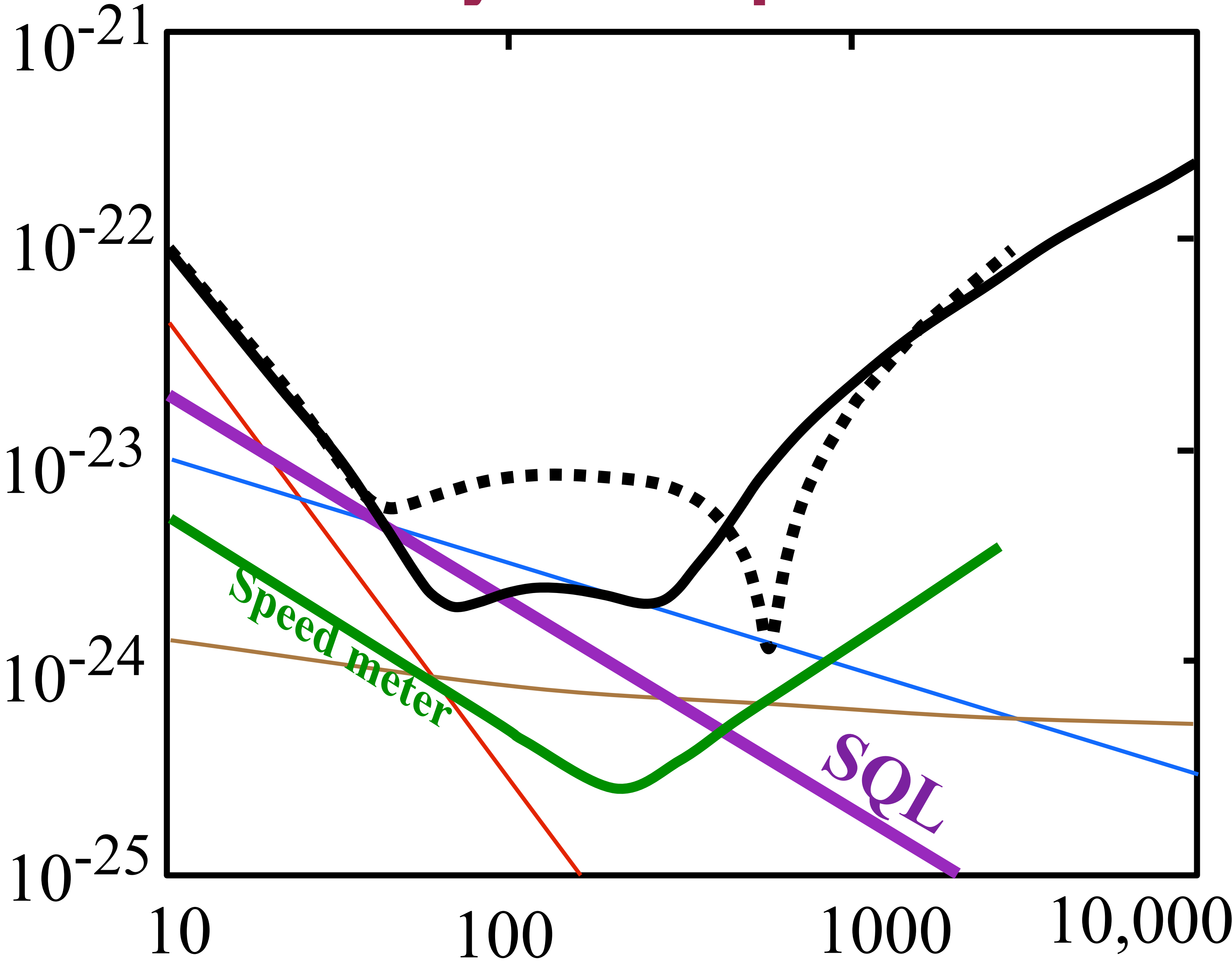
Evading Back-Action Uncertainty Principle in LIGO

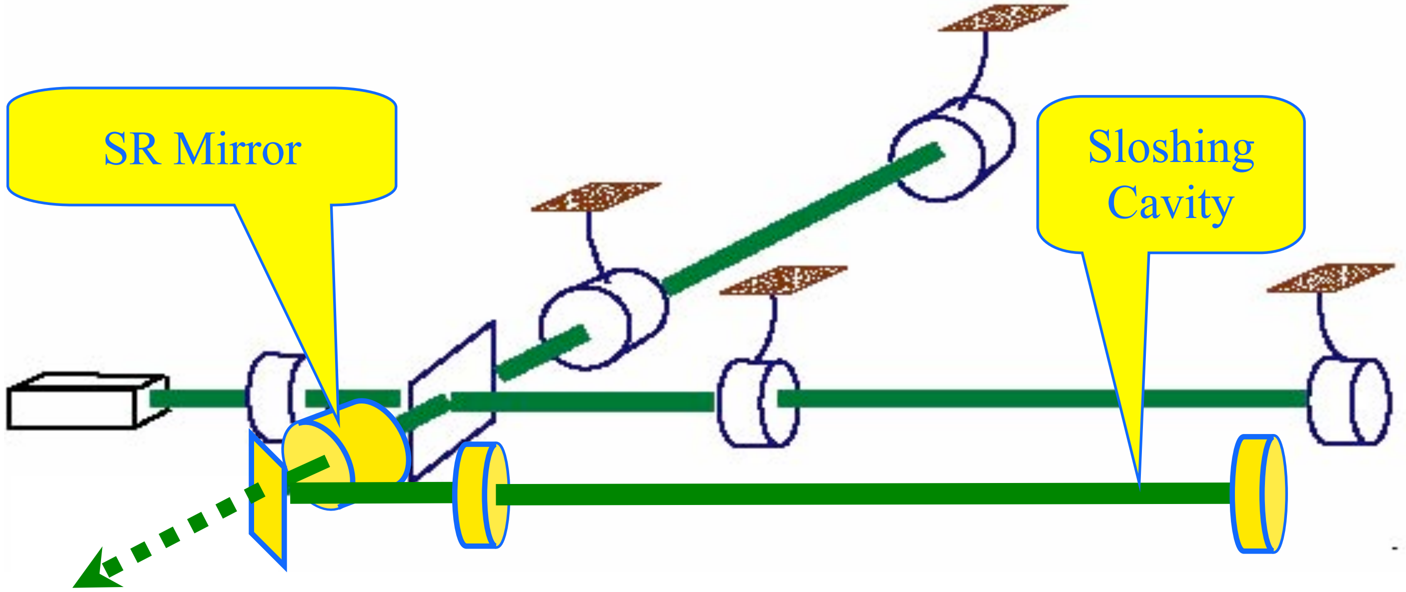
- **Several methods**

- **Example:**

  Monitor momentum $p(t)$ instead of position $x(t)$ ("speedmeter")

  Yanbei Chen





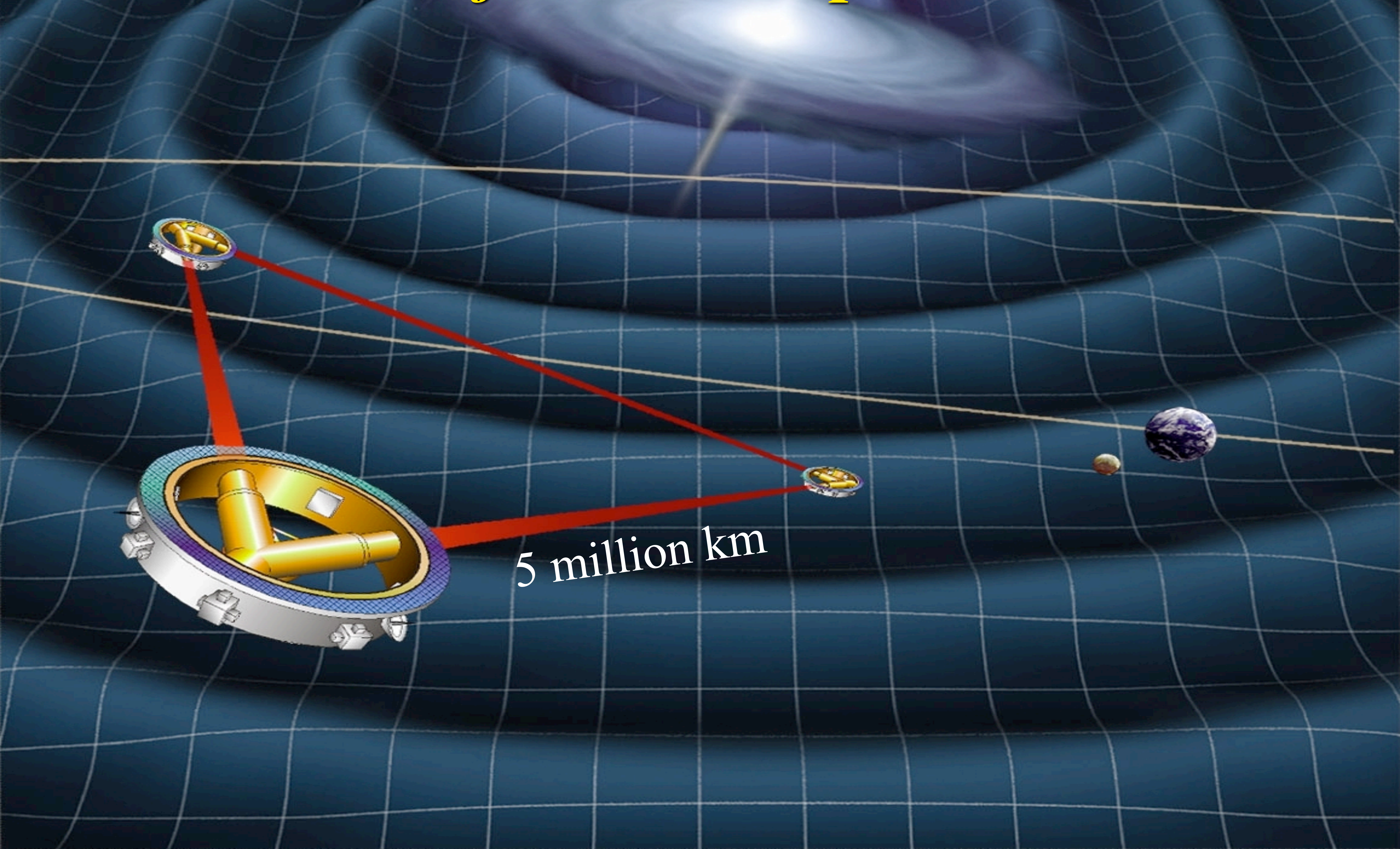
Space-Based GW Detectors:

Low-Frequency Band (LF)

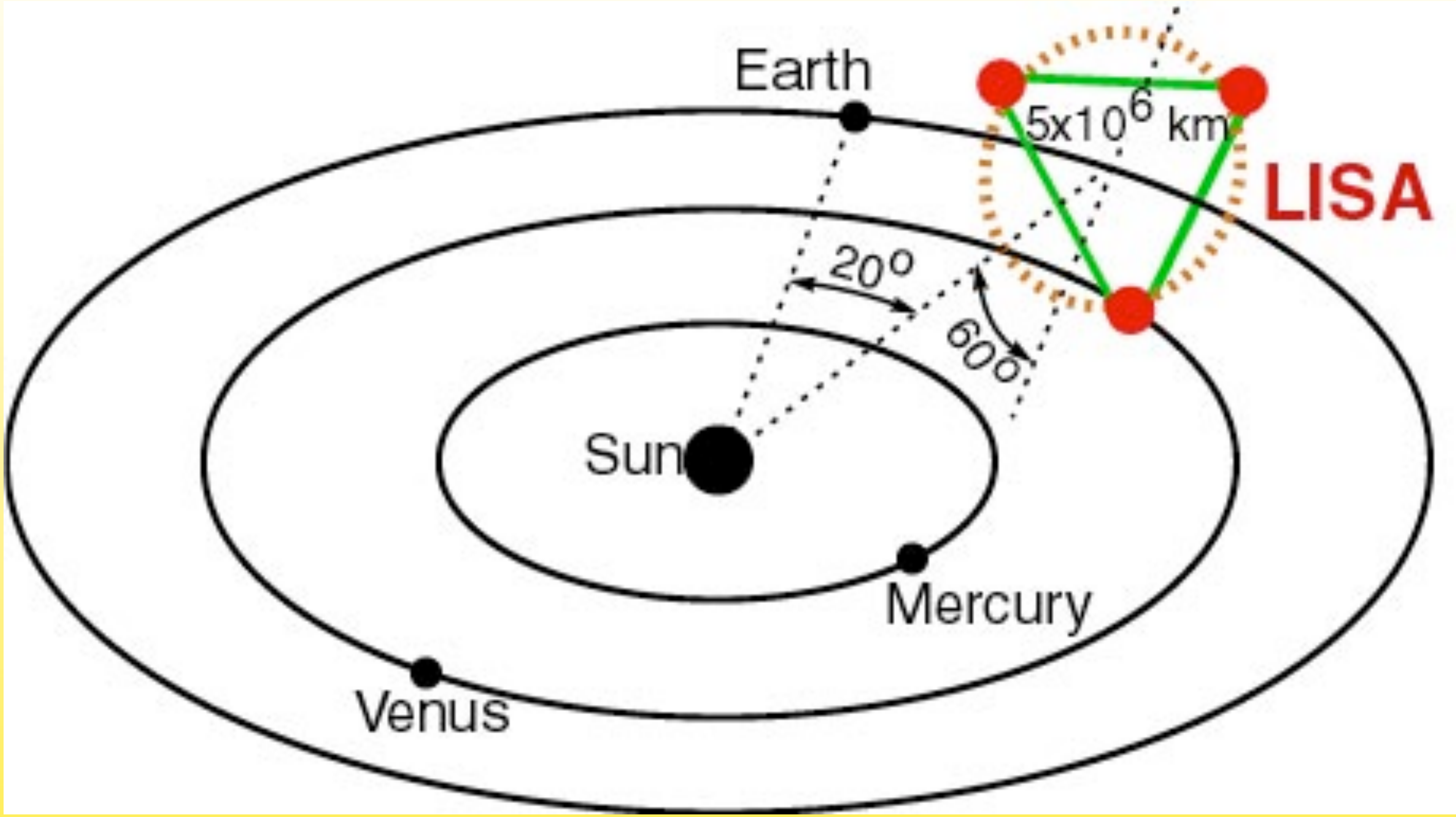
10^{-5}Hz - 0.1 Hz

LISA

Laser Interferometer Space Antenna

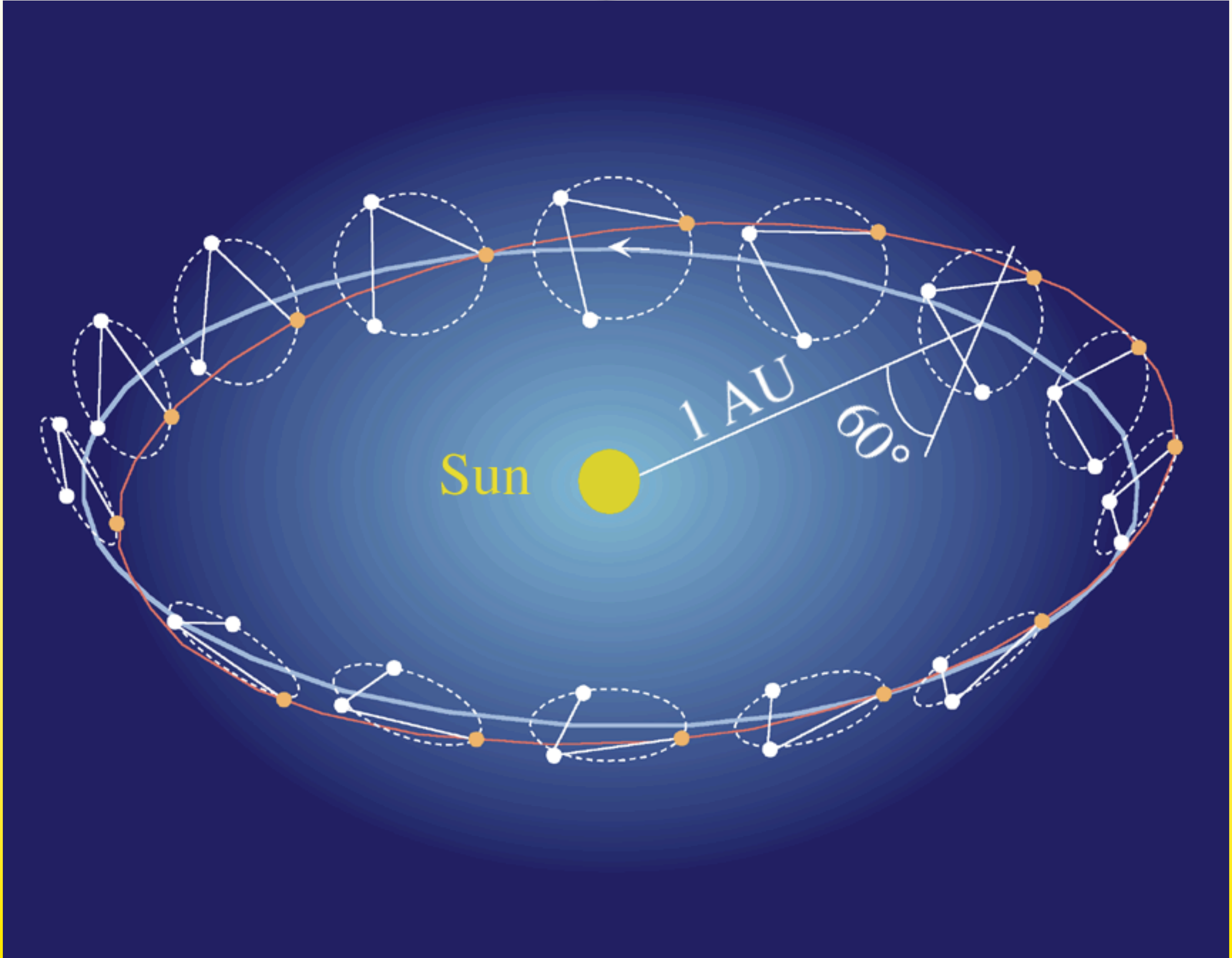


LISA: Joint ESA/NASA Mission

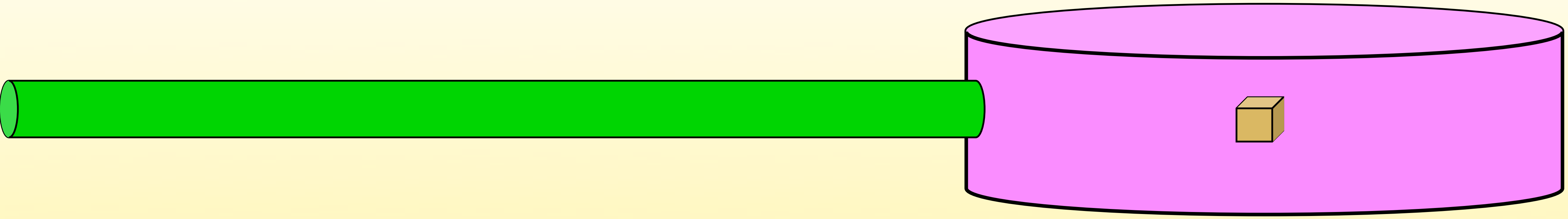


- **Launch: ~2018 or later**

LISA

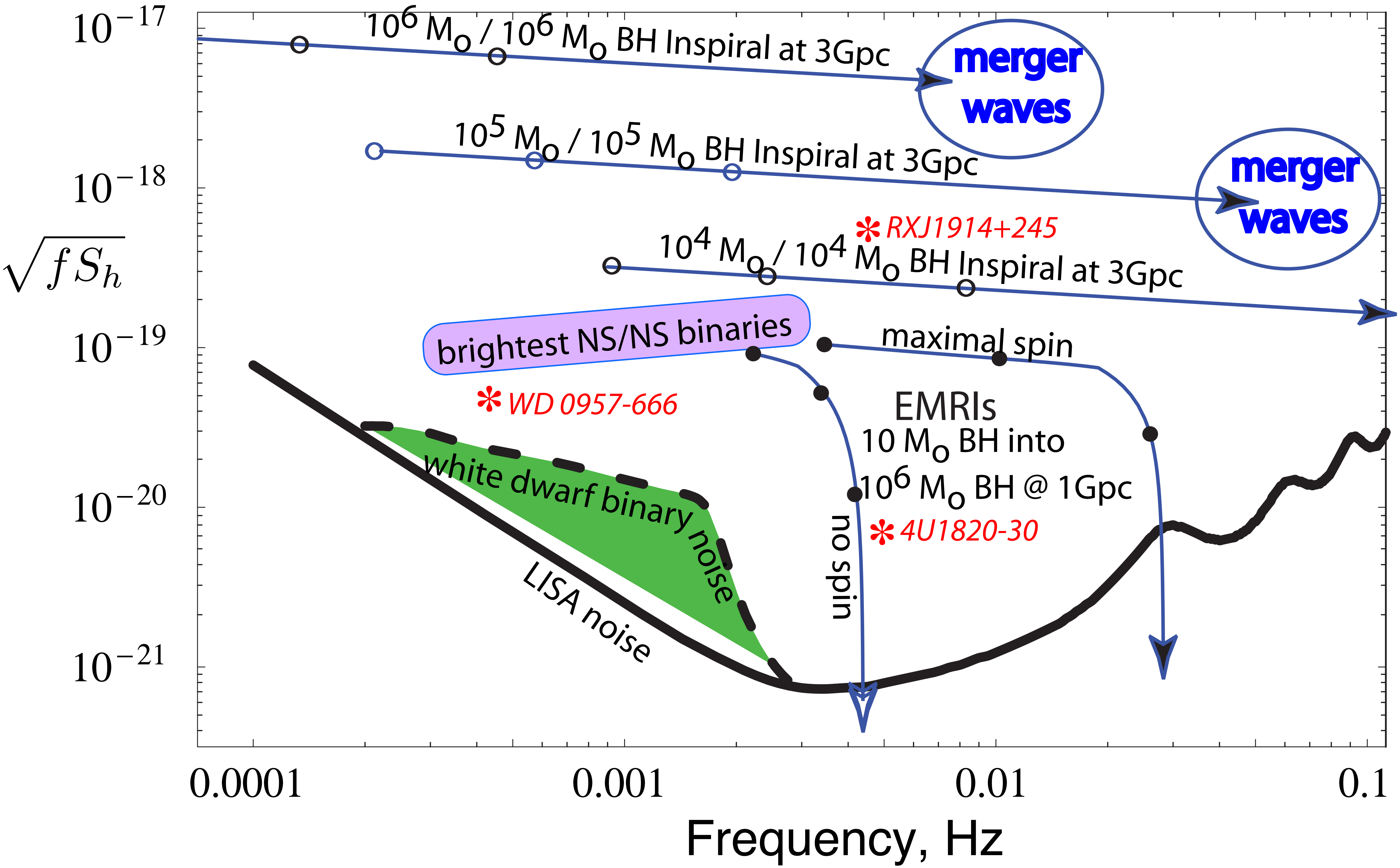


LISA: The Technical Challenge



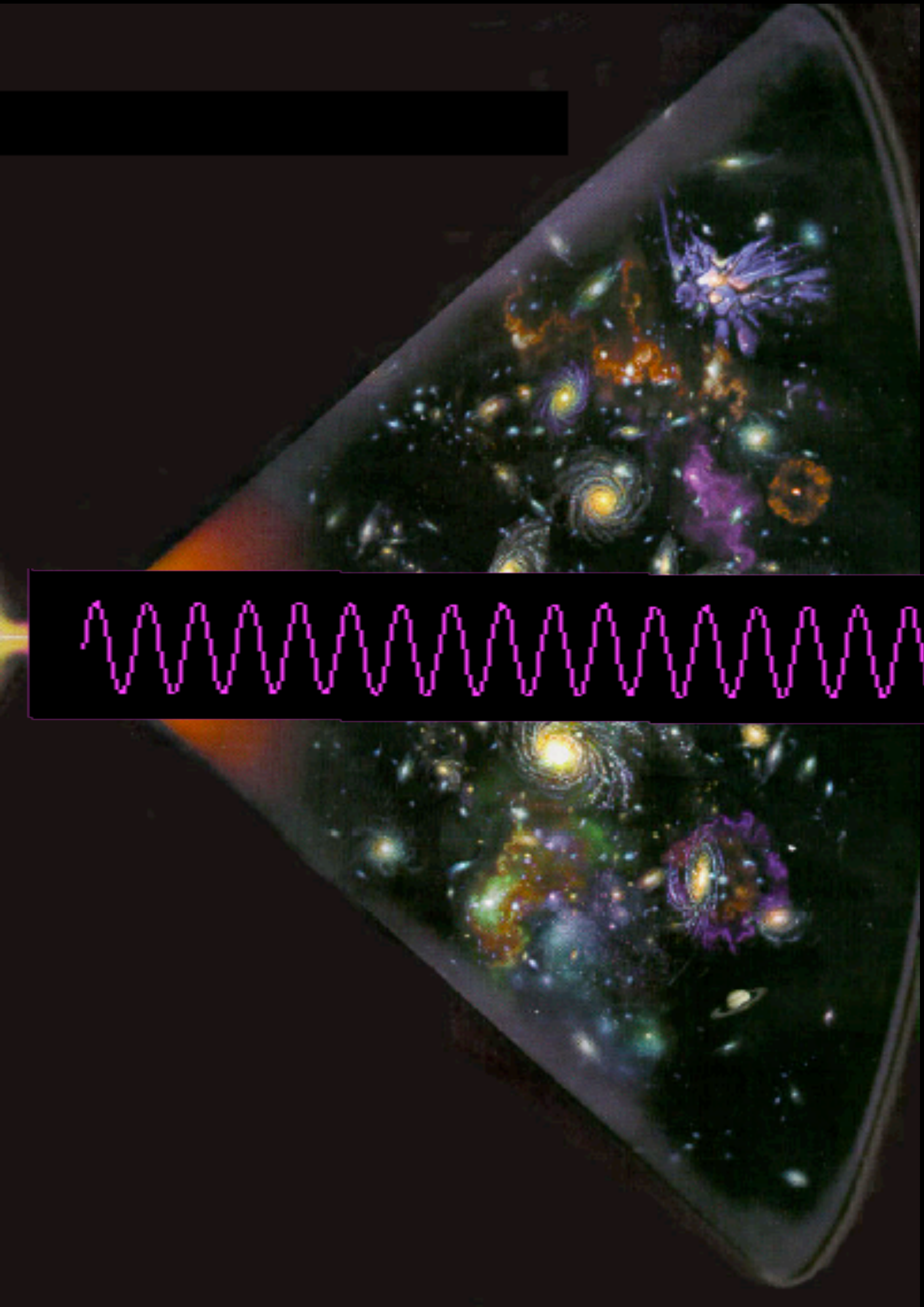
- Monitor the relative motion of the satellites' "Proof Masses", 5 million kilometers apart, to a precision
  - ~ 10^{-10} cm [in frequency band f ~ 0.1 - 10^{-4} Hz]
  - ~ 10^{-6} of the wavelength of light
- With Proof-Mass relative speeds ~ few million wavelengths/ second
- Guarantee that the only accelerations acting on the proof masses at level ~10^{-16} g are gravitational, from outside the spacecraft

LISA Sensitivity and Sources



BBO: Big Bang Observer

Launch ~2030 or later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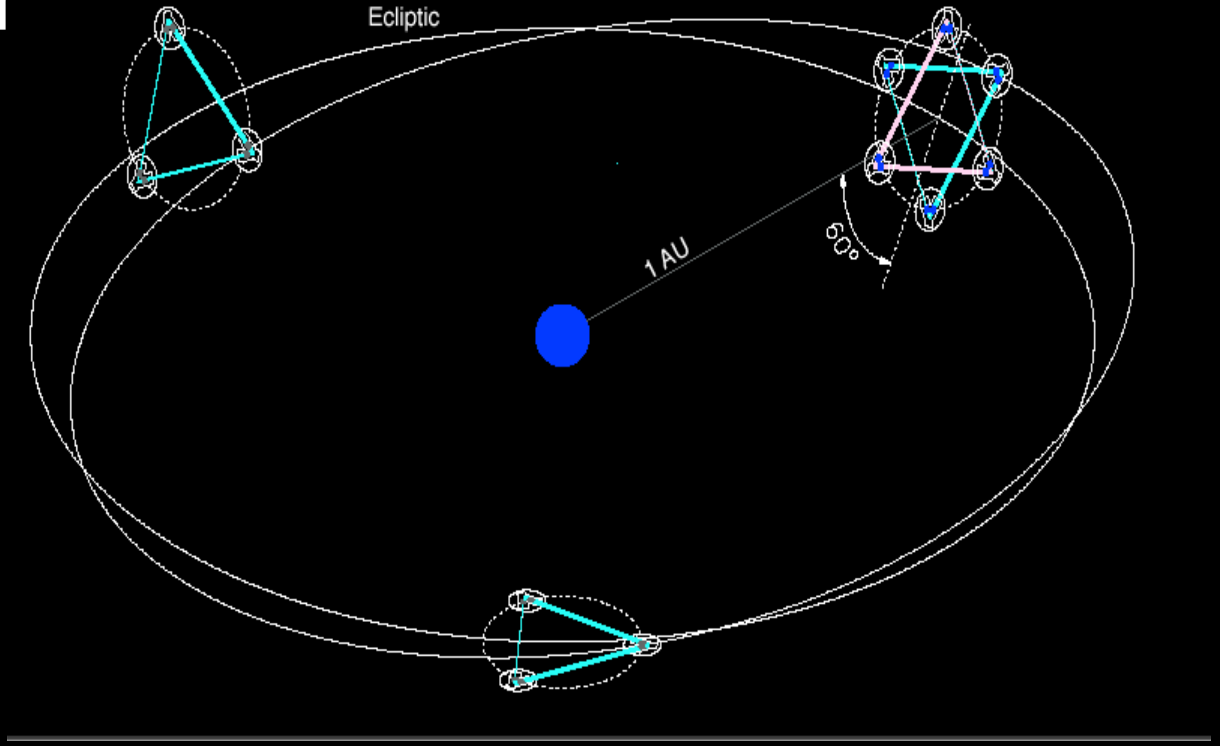


Based on 2005 study by
BBO Team of 56 - chair: **Sterl Phinney**

- slides from Phinney

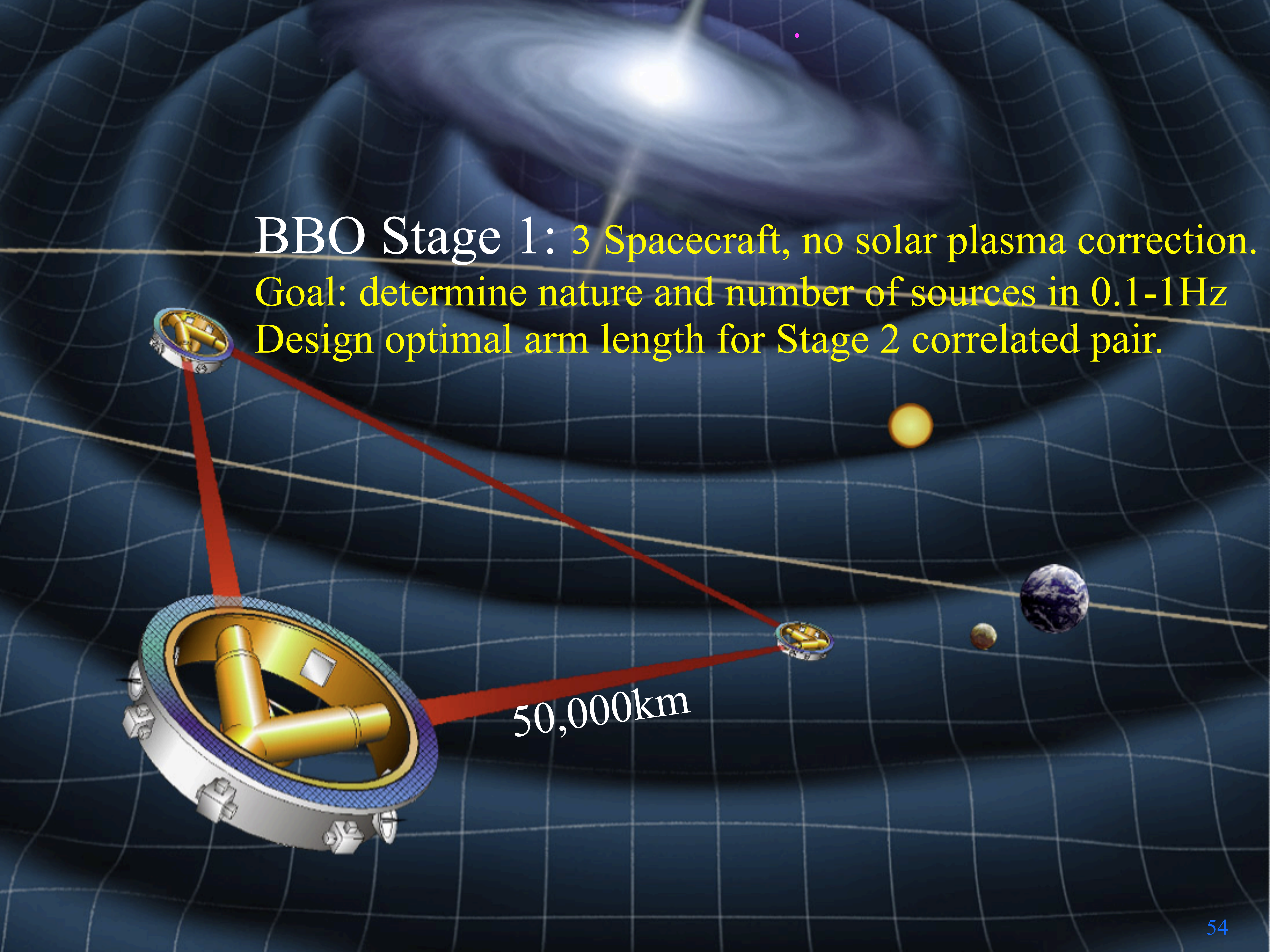
BBO vs LISA - Instrumentation

	BBO	LISA
Arm length L (km)	5×10^4	5×10^6
Laser Power L (W)	300	1
Laser λ (nm)	355	1065
Mirror diam D (m)	2.5	0.3
Accel. noise (m $s^{-2}Hz^{-1/2}$)	3×10^{-17} at 0.1 Hz	3×10^{-15} at 1mHz
Proof Mass M (kg)	10 Al_2O_3	1.6 Au/Pt
Interferometer op	dark fringe	fringe counting
Proof mass accel	3×10^{-10}m s^{-2}	0
$c/(2\pi L)$ (Hz)	1	0.01
Position shot noise (m $Hz^{-1/2}$)	1.5×10^{-17}	1.1×10^{-11}
Pointing stability req	10^{-12}rad $Hz^{-1/2}$	10^{-8}rad $Hz^{-1/2}$

BBO Stage 1: 3 Spacecraft, no solar plasma correction.

Goal: determine nature and number of sources in 0.1-1Hz

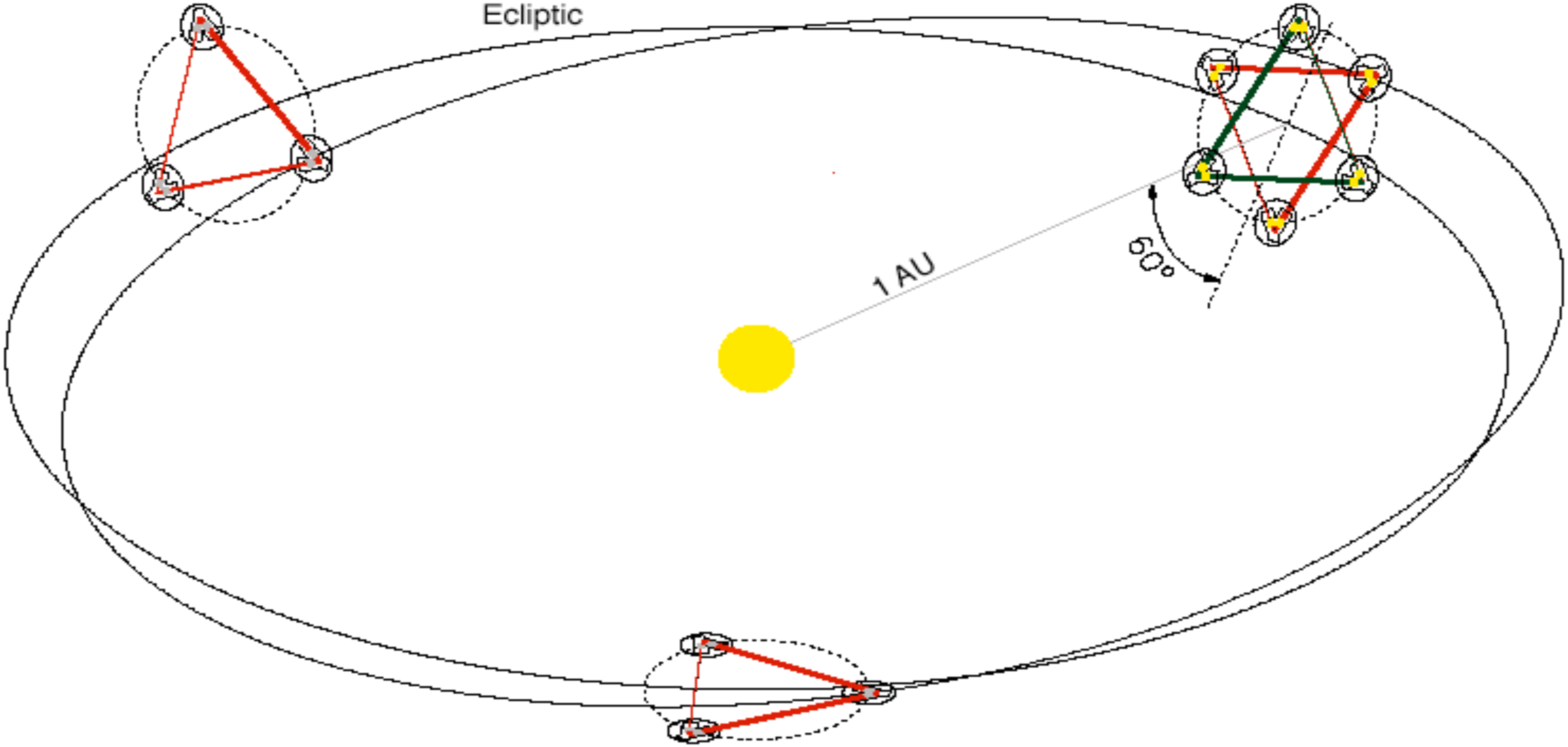
Design optimal arm length for Stage 2 correlated pair.



BBO Stage 1: Science

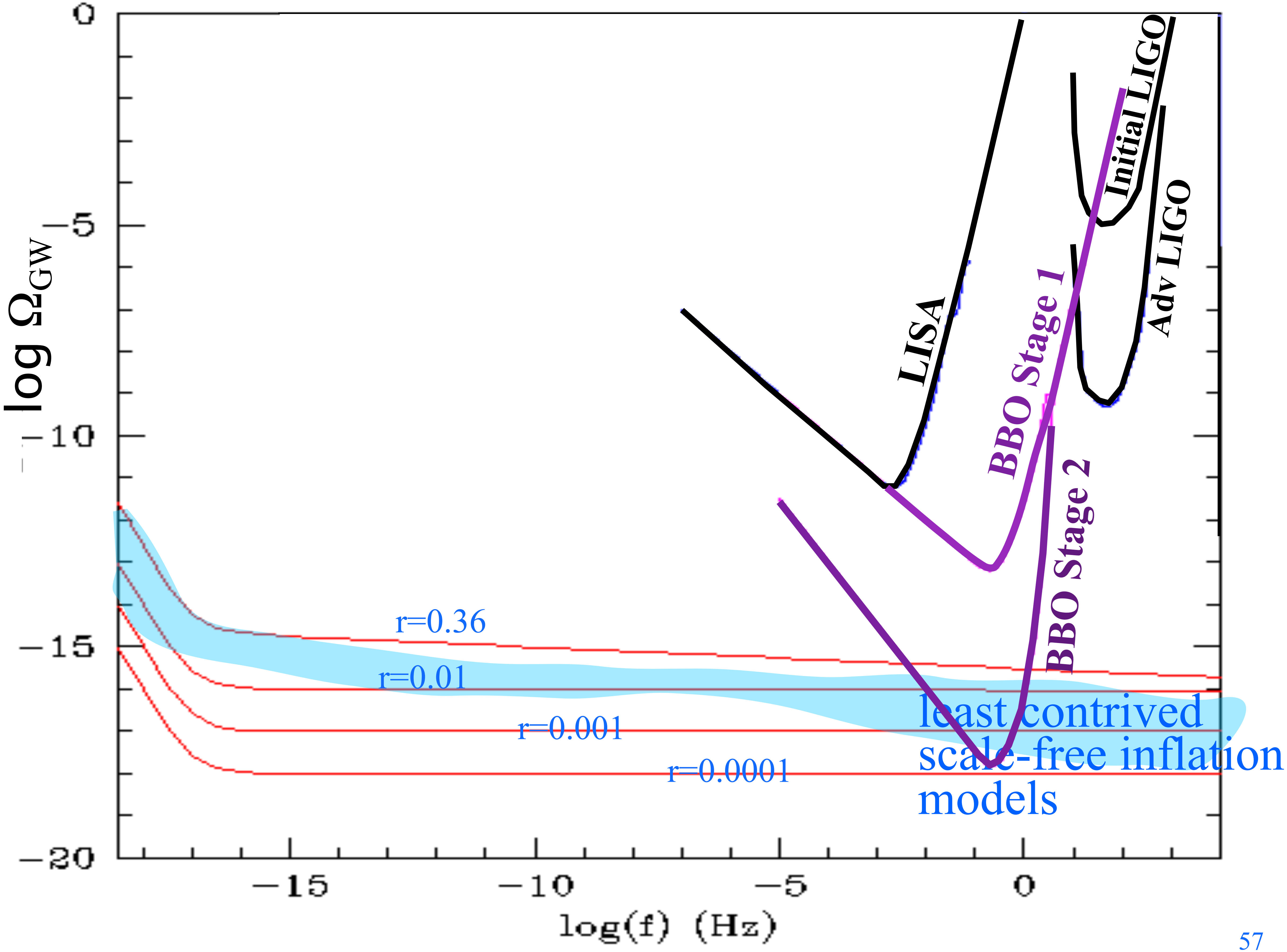
- Last year of every merging NS-NS, NS-BH, BH-BH of stellar mass at $z<8$. ~1 arcmin positions.
- Luminosity distances for these: ~10^4-10^5 sources, accurate to < 1%
- All mergers of intermediate mass BHs at any z.
- Cosmic strings over entire range $G\mu/c^2 > 10^{-14}$

BBO Stage 2



- *Triangulate on foreground sources: positions to subarcsecond*
- *Colocated IFOs: Stochastic Background*

BBO & Stochastic Background



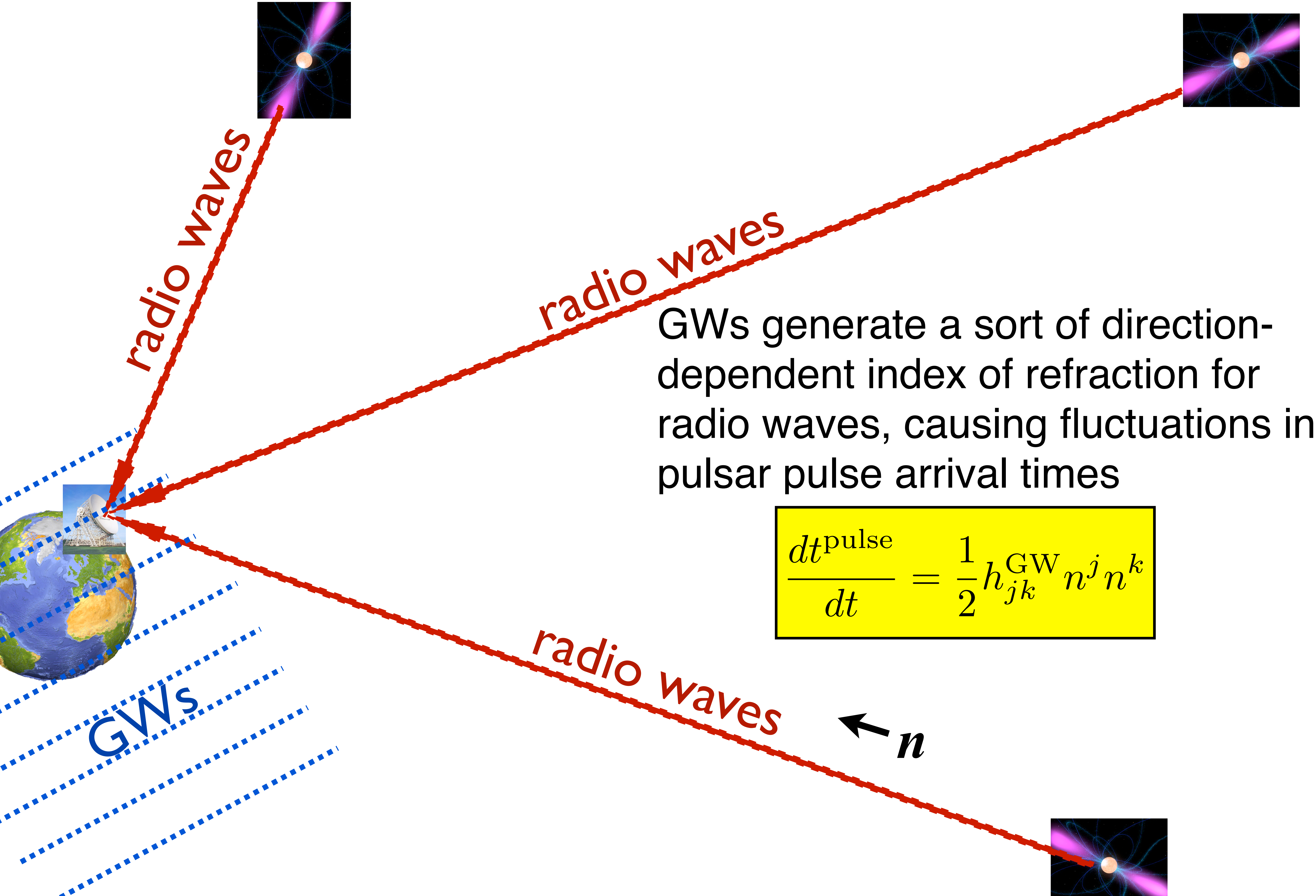
Array of pulsars

Pulsar Timing Arrays:

Very-Low-Frequency Band (VLF)

10^{-7} Hz - 10^{-5} Hz

PTA Detection of Gravitational Waves

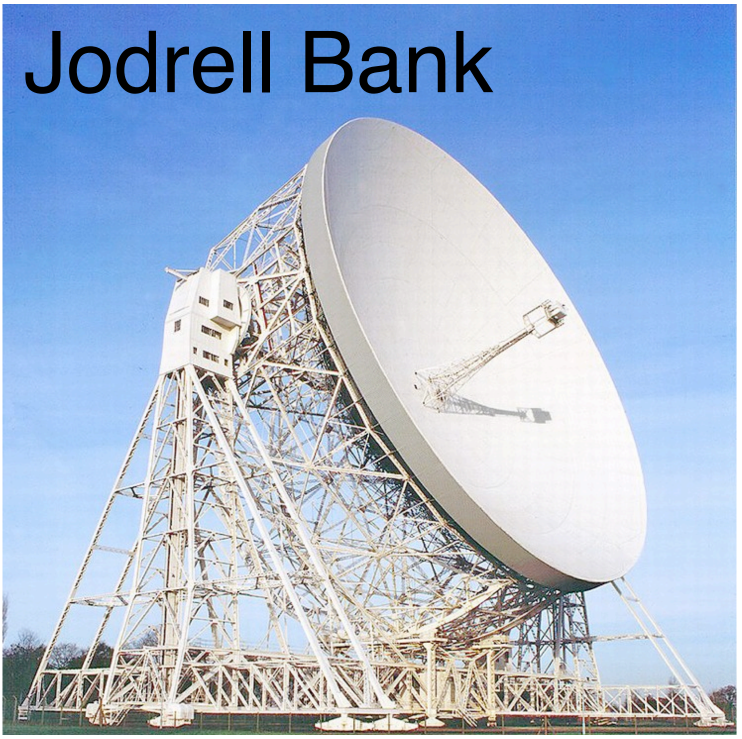


GWs generate a sort of direction-dependent index of refraction for radio waves, causing fluctuations in pulsar pulse arrival times

$$\boxed{\frac{dt^{\mathrm{pulse}}}{dt} = \frac{1}{2} h_{jk}^{\mathrm{GW}} n^j n^k}$$

PTA Collaborations

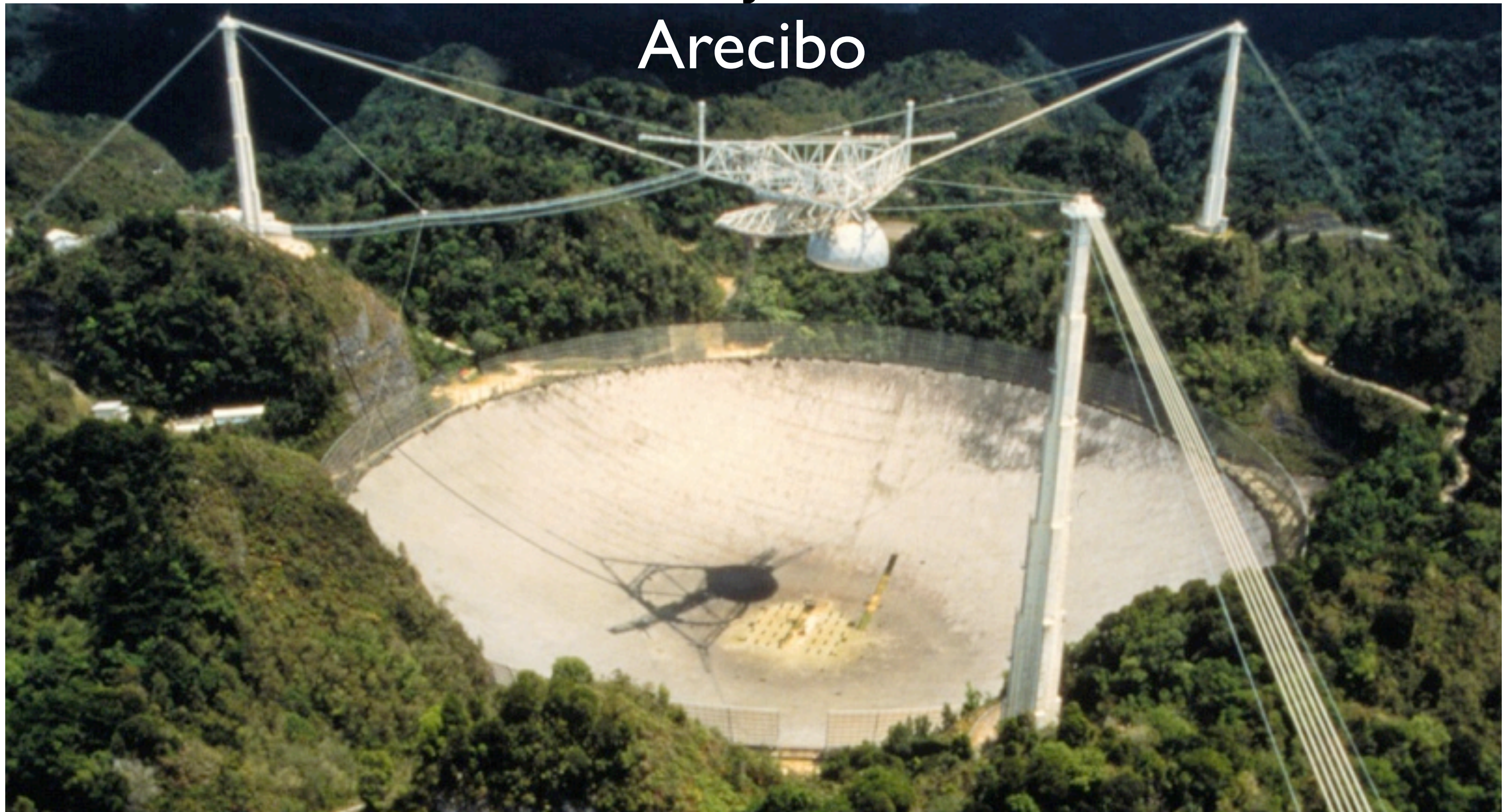
European PTA: Effelsberg, Nancay, Sardinia, Westerbork, Jodrell Bank



Parkes [Australia]

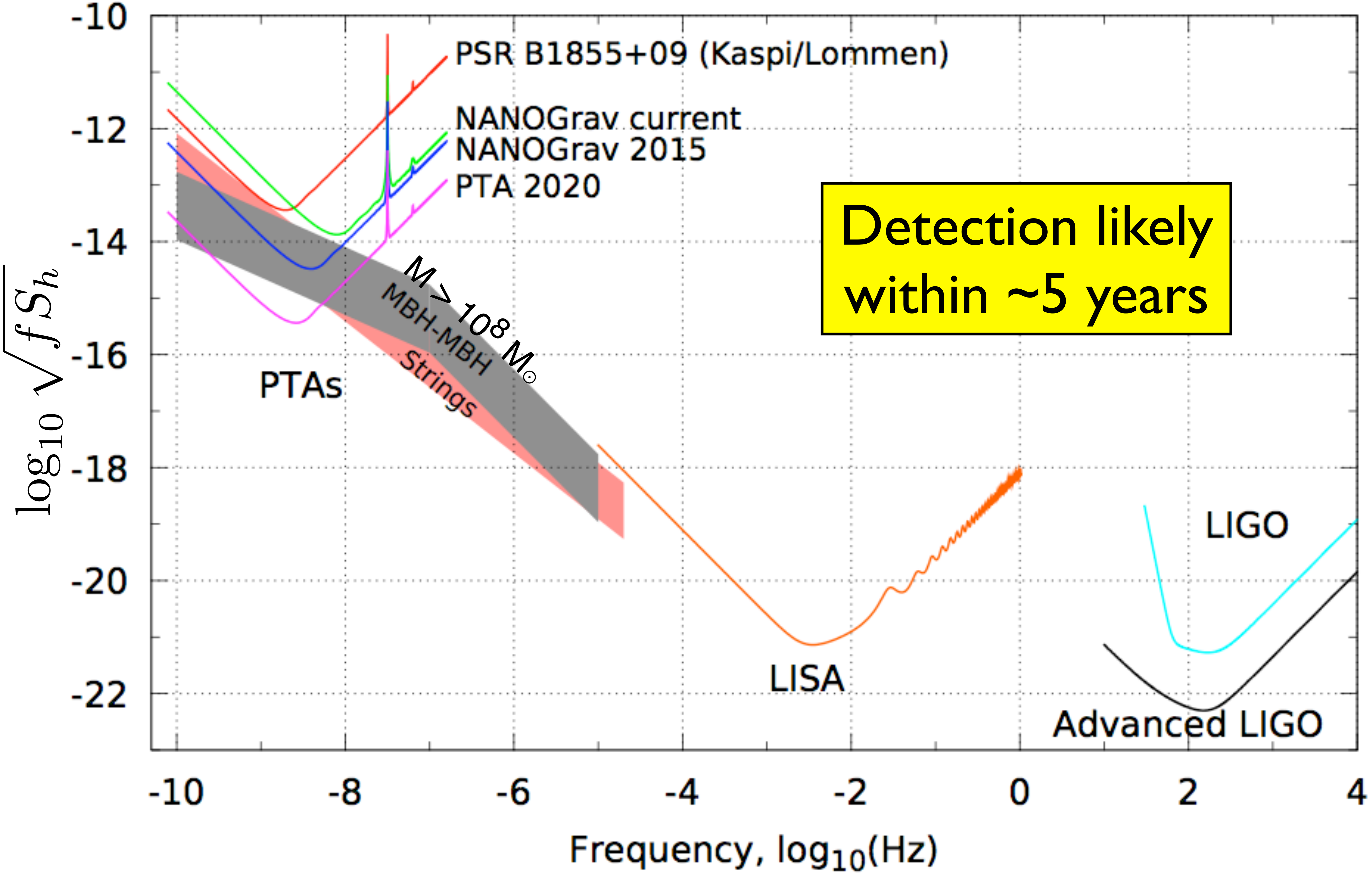
NANOGrav: North American Nanohertz Observatory for Gravitational Waves

Green Bank



Arecibo

PTA Noise Levels & Sources



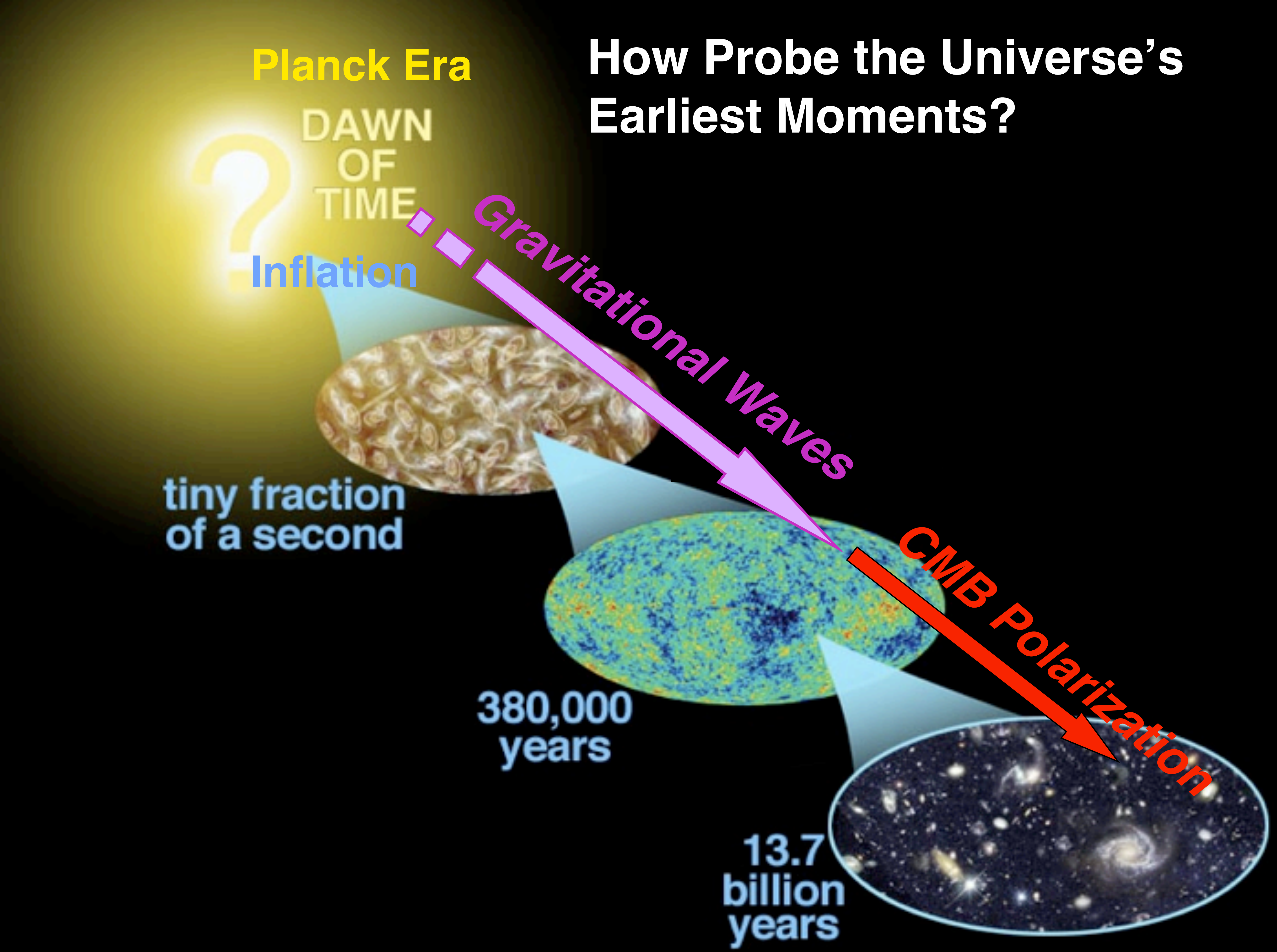
Detection likely within ~5 years

CMB Polarization

Extremely-Low -Frequency Band (ELF)

10^{-18}-10^{-16} Hz

How Probe the Universe's Earliest Moments?



Planck Era

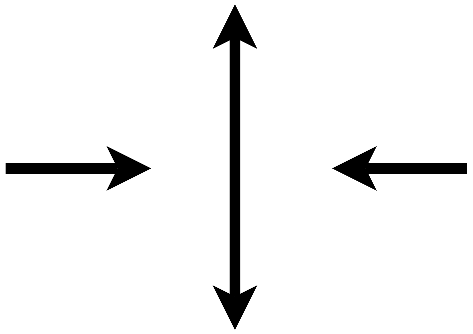
Inflation

Gravitational Waves

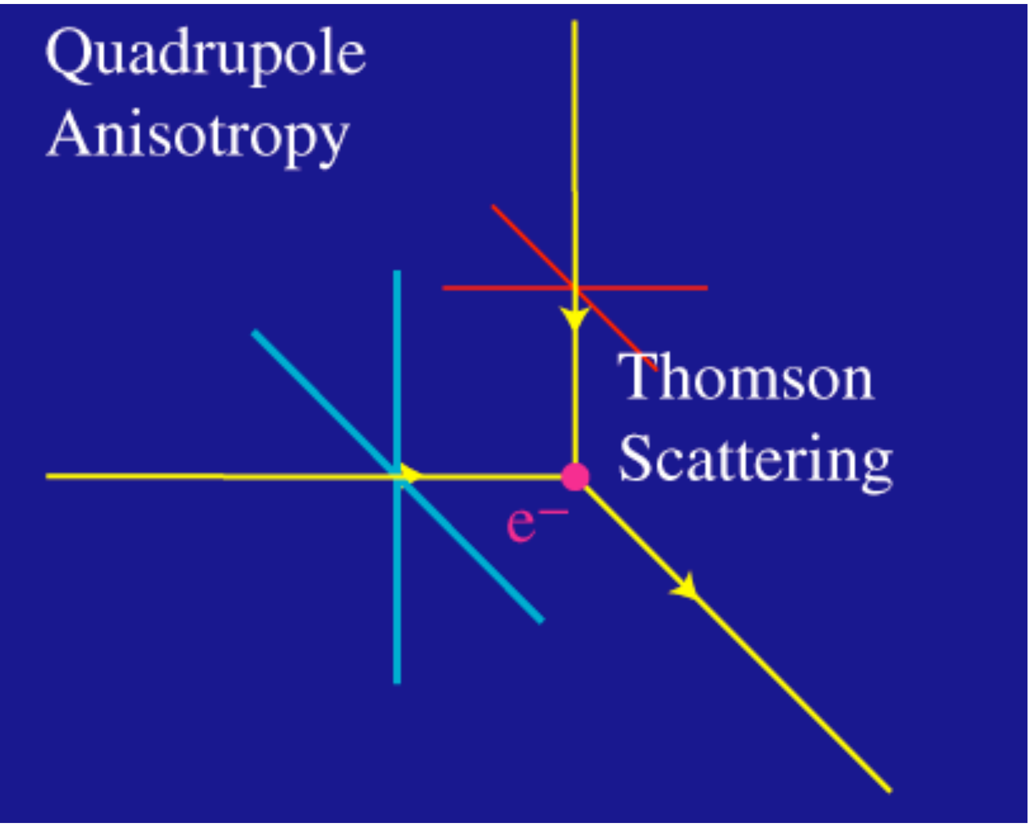
CMB Polarization

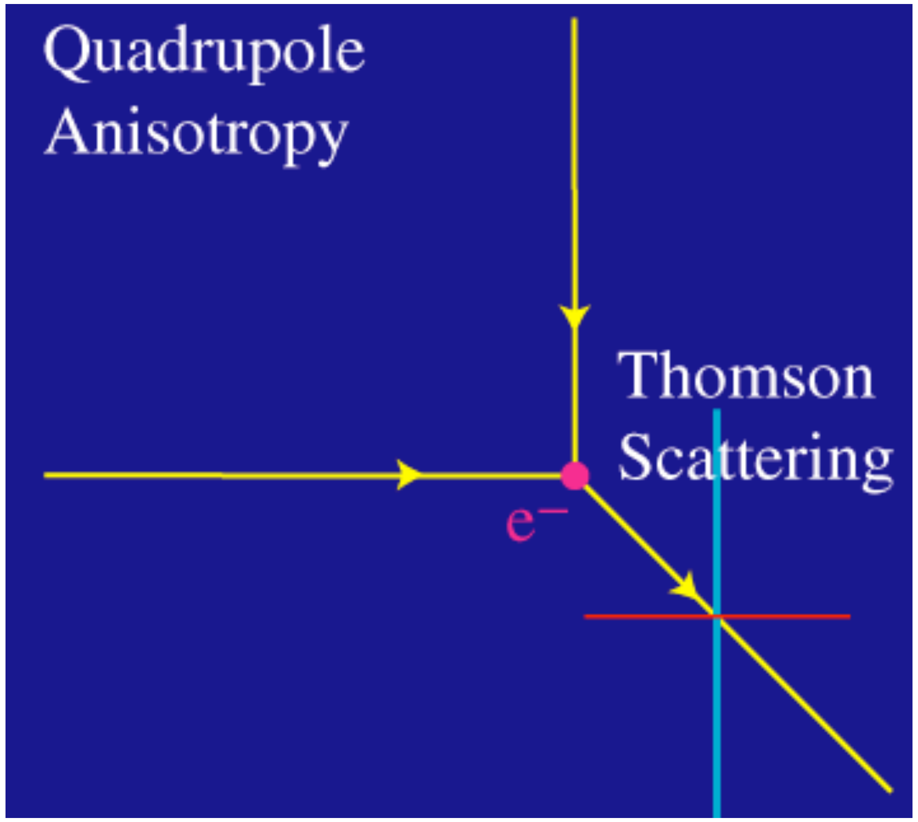
How GWs Produce CMB Polarization

- **Gravitational waves from big-bang singularity**
  - amplified by inflation
- **At era of recombination,** age ~ 380,000 yrs (redshift 1090)
  - GWs with wavelength ~ size of universe stretch and squeeze plasma



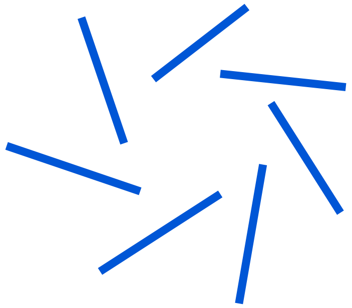
- Along squeeze direction, electrons see CMB photons blue shifted; along stretch direction, redshifted
- Scattering produces linear polarization





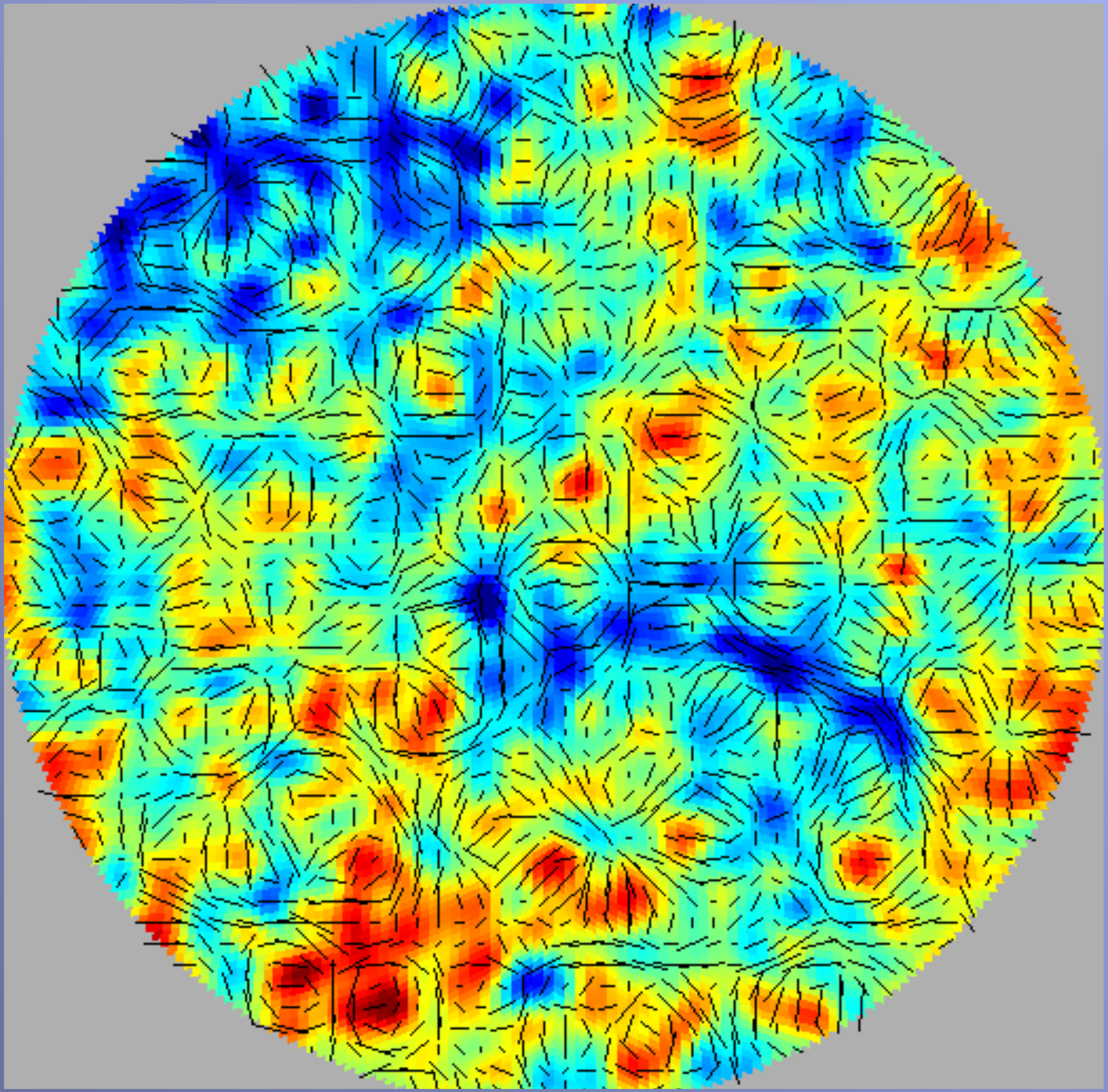
W. Hu

Polarization pattern is curl (“B-mode”)

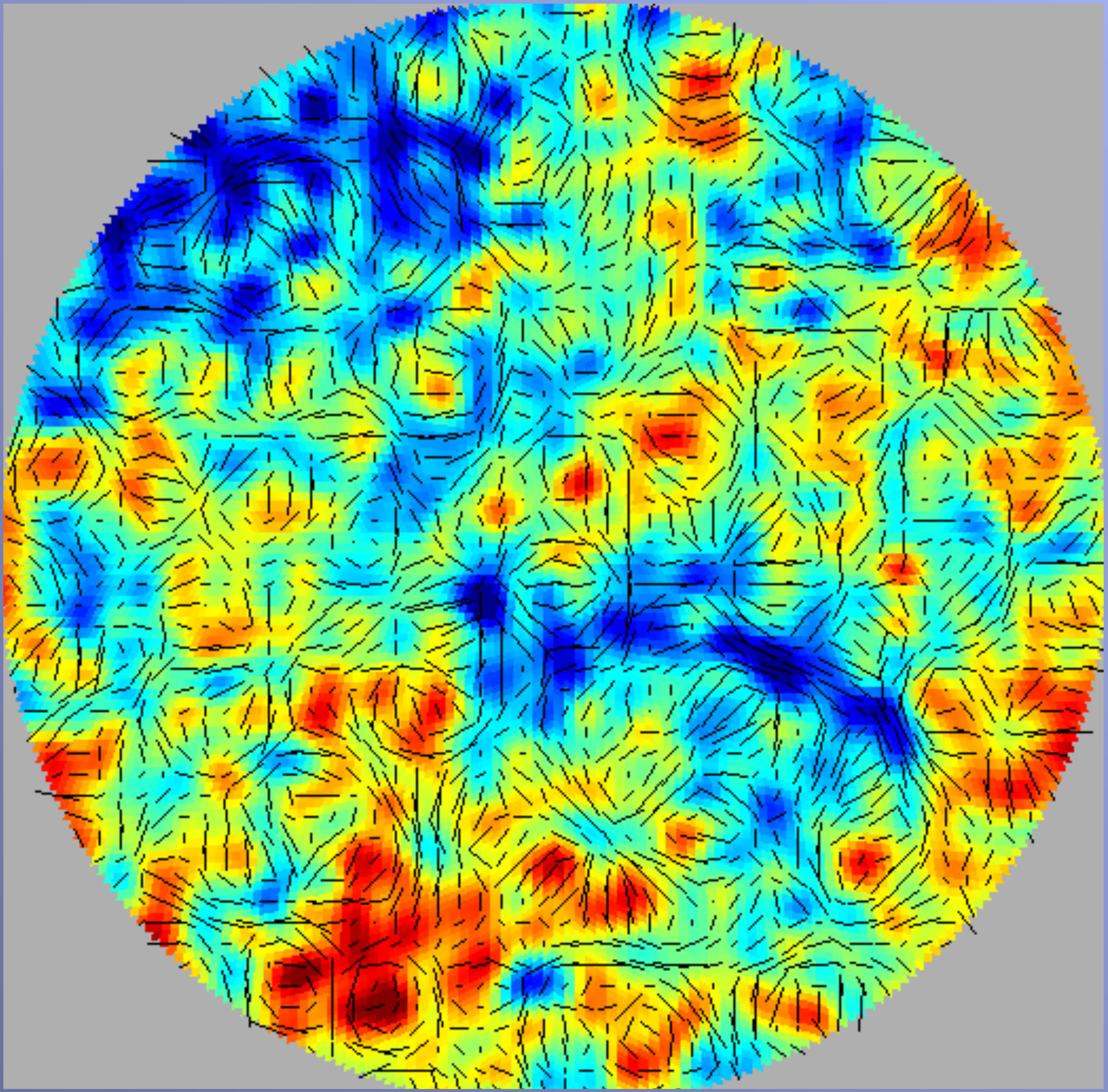


Most other processes produce gradient pattern (“E-mode”)

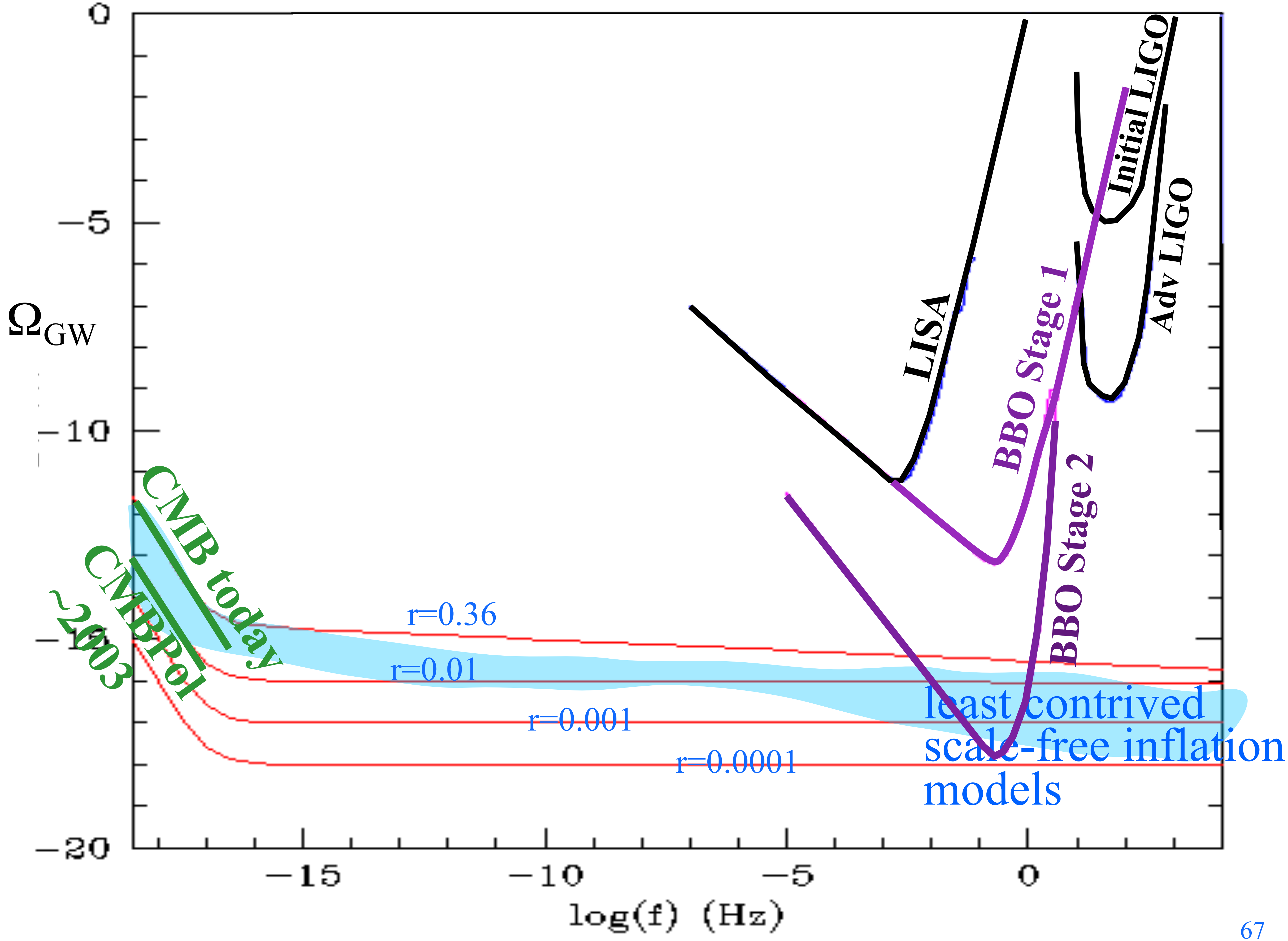
No Gravity Waves



Gravity Waves



CMB Sensitivities to Primordial GWs



Conclusions

- **1956 John Archibald Wheeler: The second Lorentz Professor:**
  - His transition from nuclear physics to relativity
  - Joseph Weber, his postdoc, came with him
  - Beginning of modern era of relativity research - both theory and GW experiment
- **Great honor and pleasure to follow in Wheeler's footsteps as the 54th Lorentz Professor**

Relativity and GW Science: Amazing Transformation