Quantum optics
& quantum information

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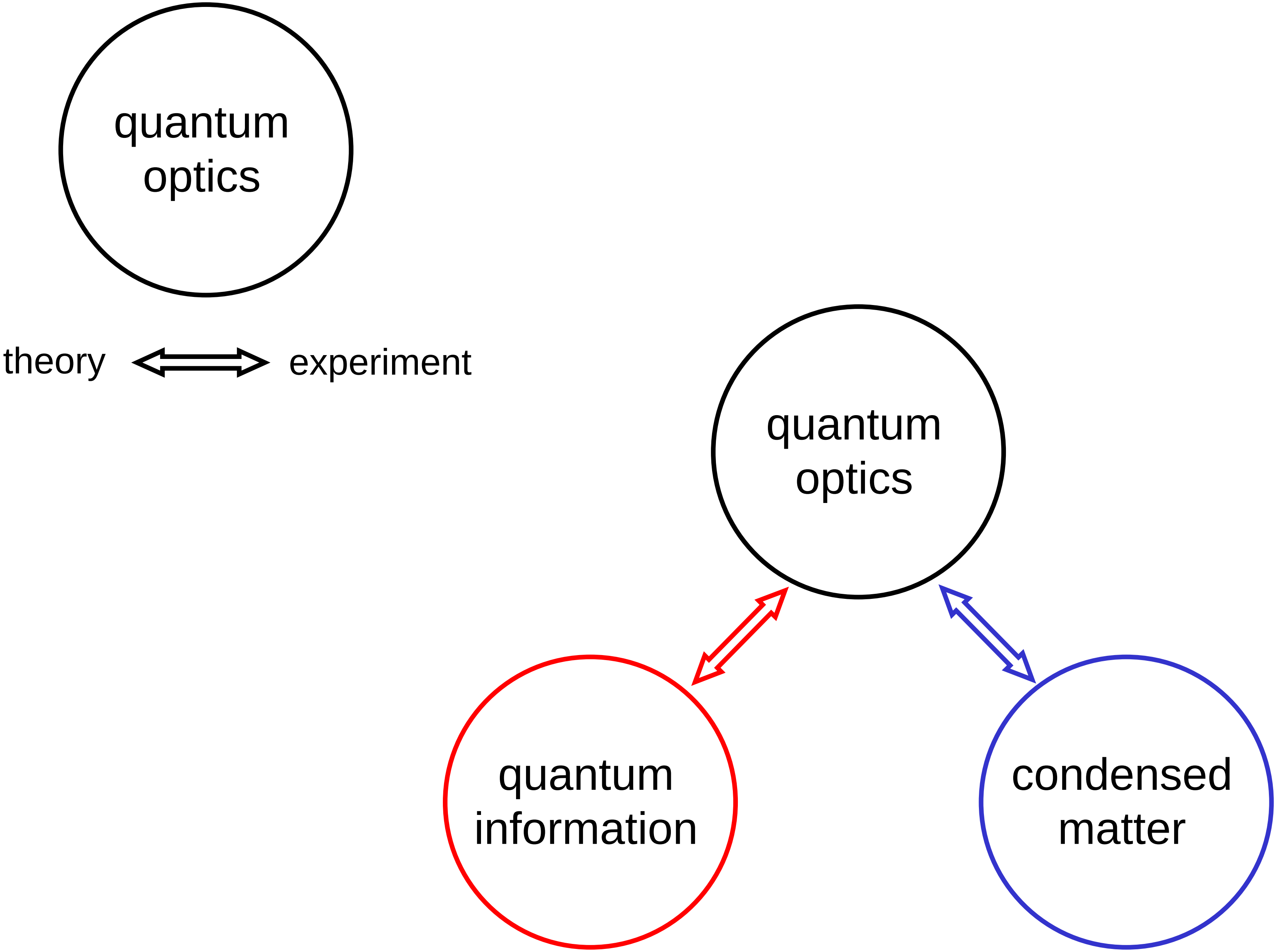
SFB
Coherent Control of
Quantum Systems

€U networks



quantum optics

theory ⟺ experiment

quantum optics

quantum information

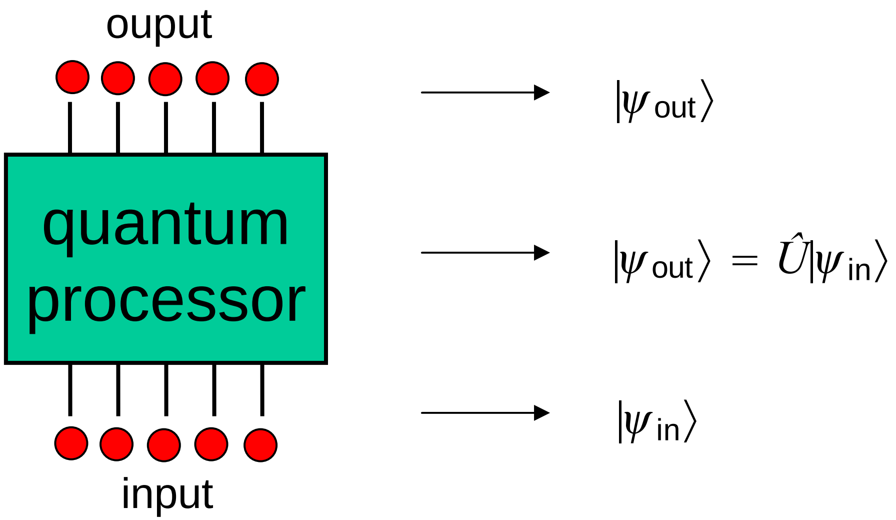
condensed matter

Introduction / Motivation / Overview

- Quantum information
  - quantum computing, quantum communication etc.
- Zoo of quantum optical systems
  - ions, neutral atoms, CQED, atomic ensembles
- Theoretical Tools of Quantum Optics
  - quantum optical systems as open quantum systems

1.1 Quantum information processing

- quantum computing



ouput

quantum processor

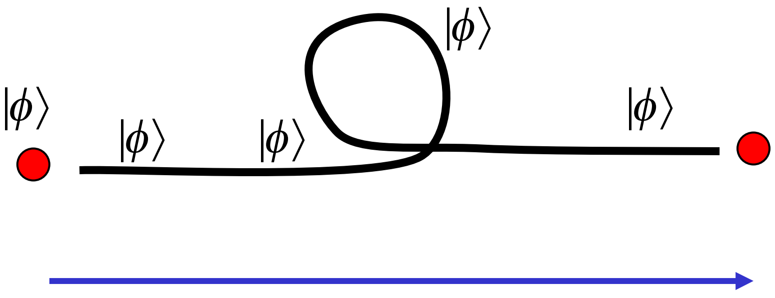
input

$|\psi_{\text{out}}\rangle$

$|\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$

$|\psi_{\text{in}}\rangle$

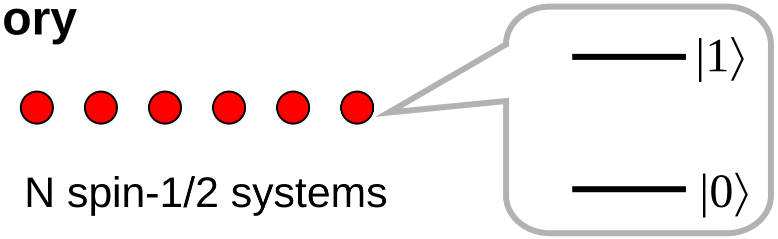
- quantum communication



transmission of a quantum state

Quantum computing

- **quantum memory**



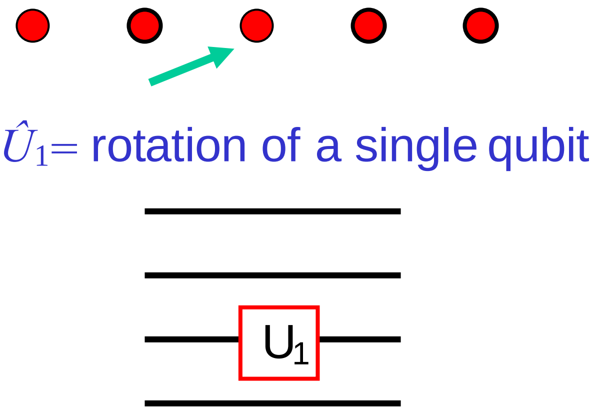
N spin-1/2 systems

quantum register

qubit

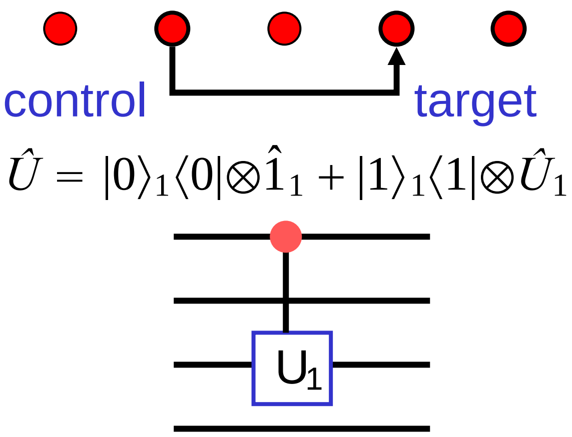
- **quantum gates**

single qubit gate:



$\hat{U}_1$= rotation of a single qubit

two-qubit gate:

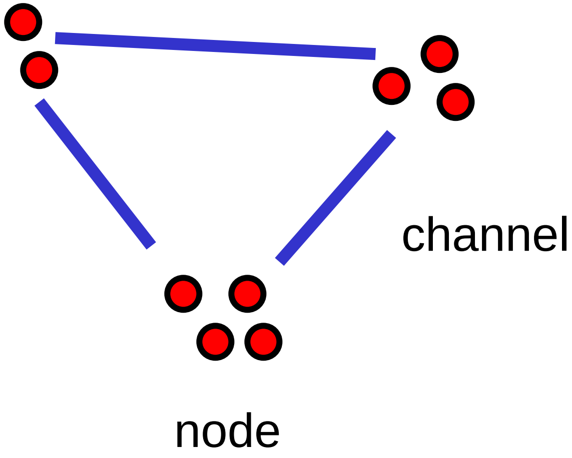


$$\hat{U} = |0\rangle_1\langle 0|\otimes\hat{1}_1 + |1\rangle_1\langle 1|\otimes\hat{U}_1$$

- **read out**
- **[no decoherence]**

Our goal ... implement quantum networks

- quantum network



- Nodes: local quantum computing
  - store quantum information
  - local quantum processing
  - measurement

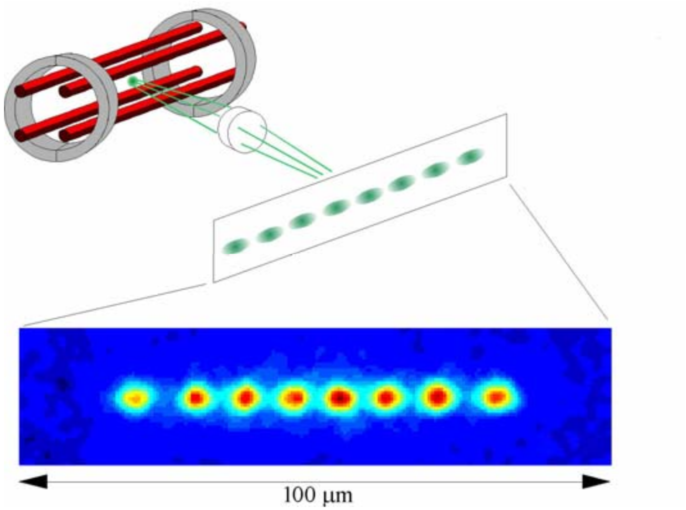
- Channels: quantum communication
  - transmit quantum information
  - local / distant

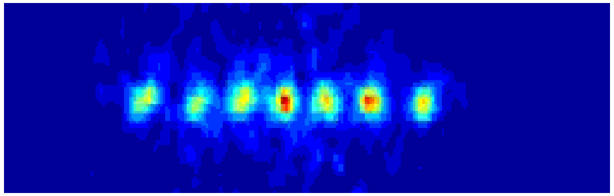
Goals:

- map to physical (quantum optical) system
- map quantum information protocols to physical processes

1.2 Zoo of quantum optical systems

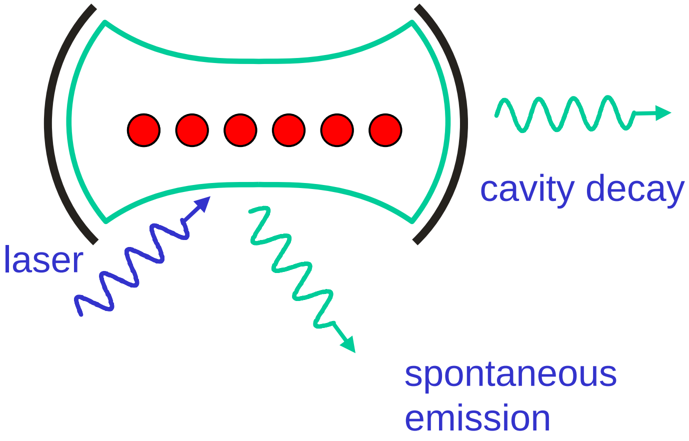
- trapped ions





collective modes

- CQED



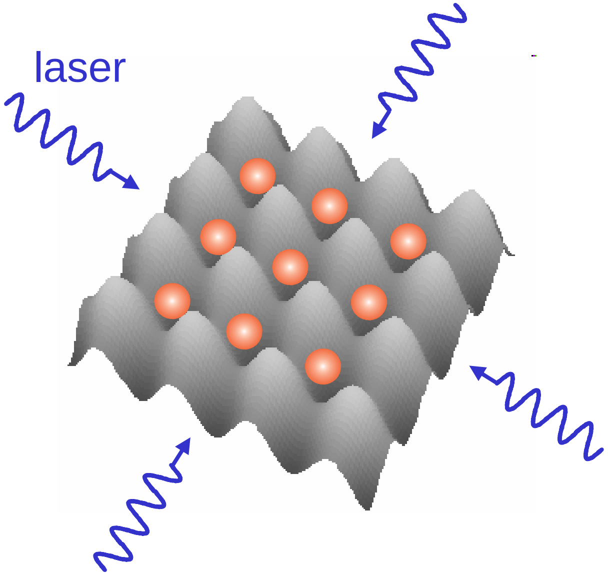
Few particle system with complete quantum control:

spin-1/2s coupled to harmonic oscillator(s)

- quantum state engineering:
  quantum computing
- state preparation & measurement

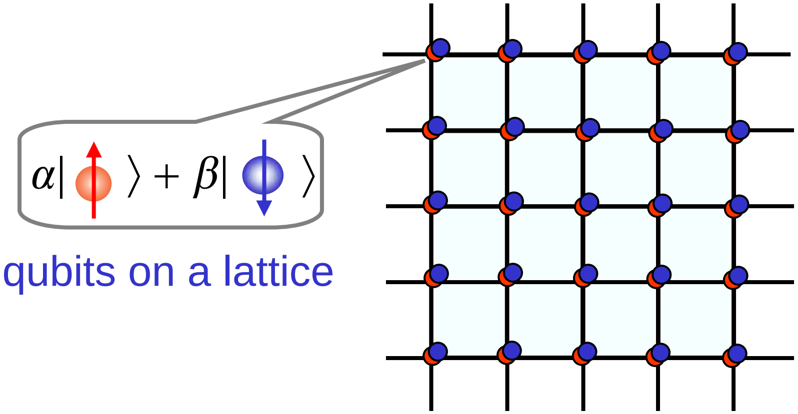
- **from BEC to Hubbard models**
  - strongly correlated systems
  - time dependent, e.g. quantum phase transitions
  - …
  - exotic quantum phases (?)

optical lattice as a regular array of microtraps for atoms



- **quantum information processing**
  - new quantum computing scenarios, e.g. "one way quantum computer"

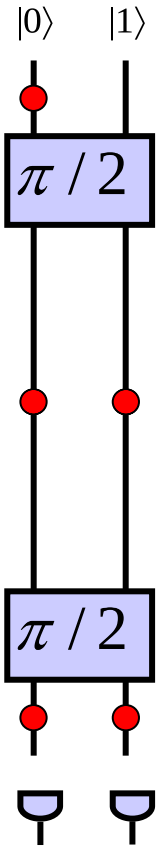
"quantum simulator"



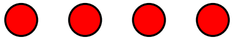
qubits on a lattice

entangling qubits via "Ising"
(cluster state)

… measurements beyond standard quantum limit



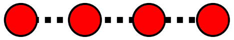
- N independent atoms



$$\Delta\omega_{\text{SQL}} = \frac{1}{T\sqrt{n_{\text{rep}}}}\,\frac{1}{\sqrt{N}}$$

↑ standard quantum noise limit

- N entangled atoms

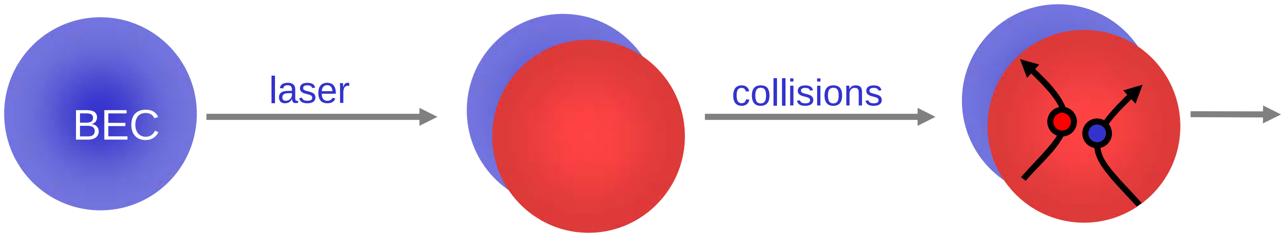


$$\Delta\omega_{\text{ent}} = \frac{1}{T\sqrt{n_{\text{rep}}}}\,\frac{1}{f(N)} \geq \frac{1}{T\sqrt{n_{\text{rep}}}}\,\frac{1}{N}$$

↑ Heisenberg limit:
maximally entangled state

$$|0000\rangle + |1111\rangle$$

- Entanglement via collisions: spin squeezing



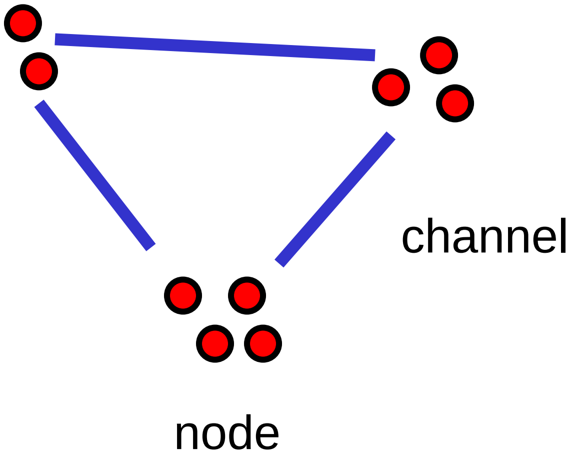
$|0\rangle^{\otimes N}$

product state

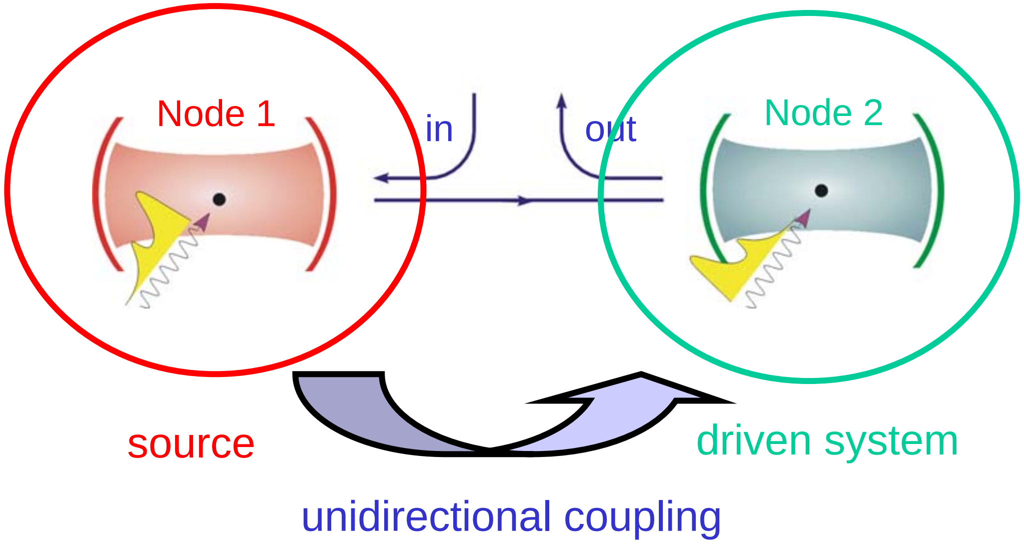
$[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)]^{\otimes N}$

product state

$\sum c_n |0\rangle^n |1\rangle^{N-n}$

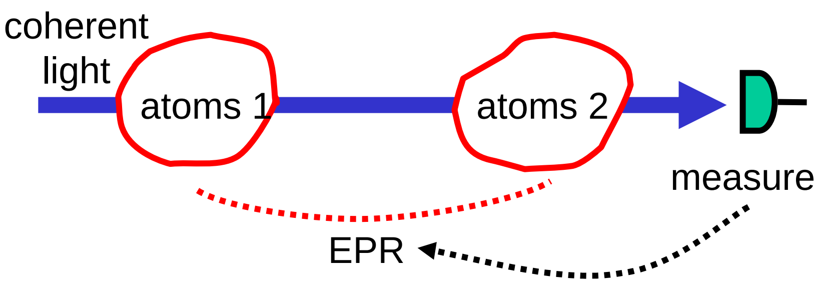


- **cascaded quantum system: transmission in a quantum network**



- **atomic ensembles**

atomic / spin squeezing; quantum memory for light;
continuous variable quantum states


coherent light

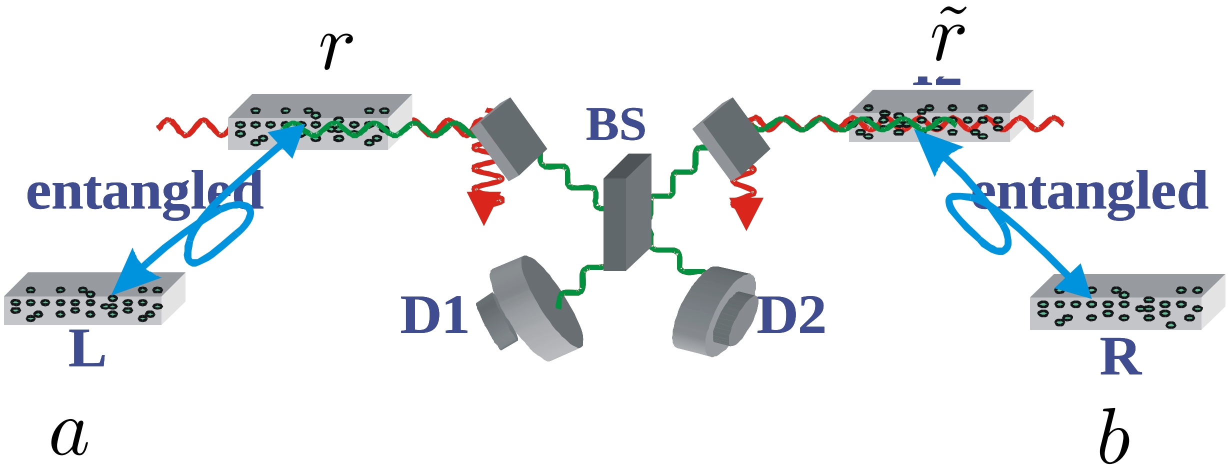
atoms 1

atoms 2

measure

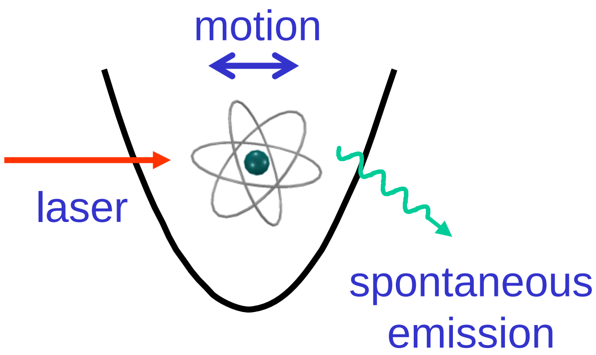
EPR

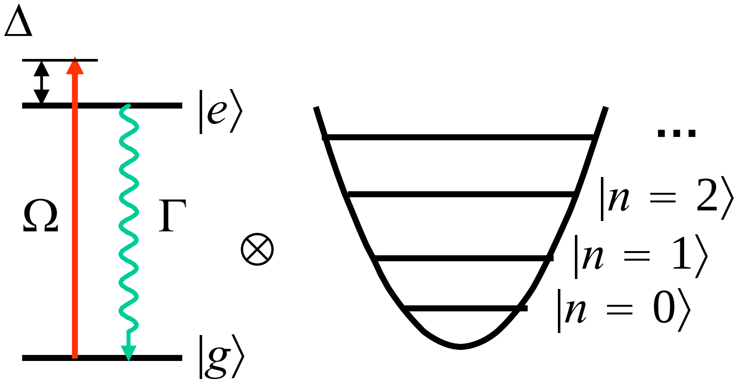
quantum repeater: establishing long distance EPR pairs
for quantum cryptography and teleportation

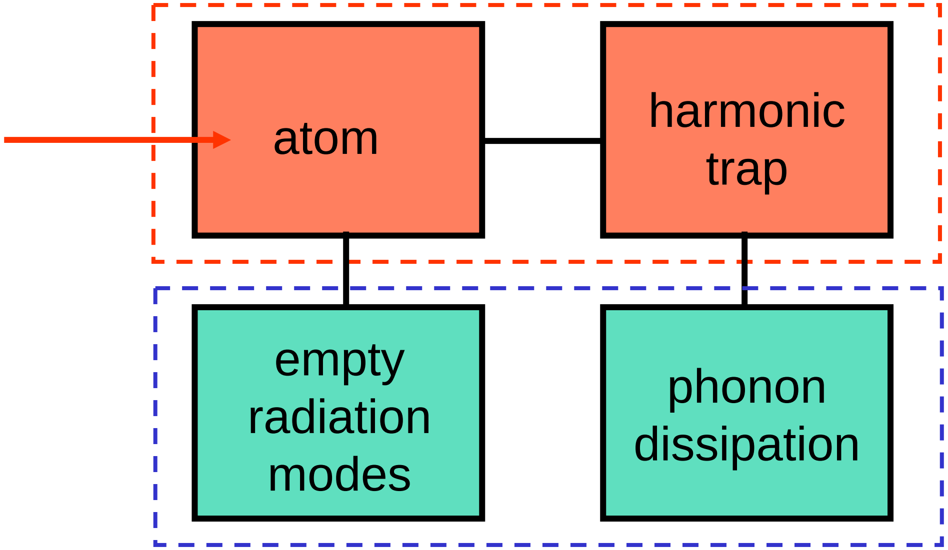


1.3 Quantum optical systems as *open* quantum systems

- example: trapped ion







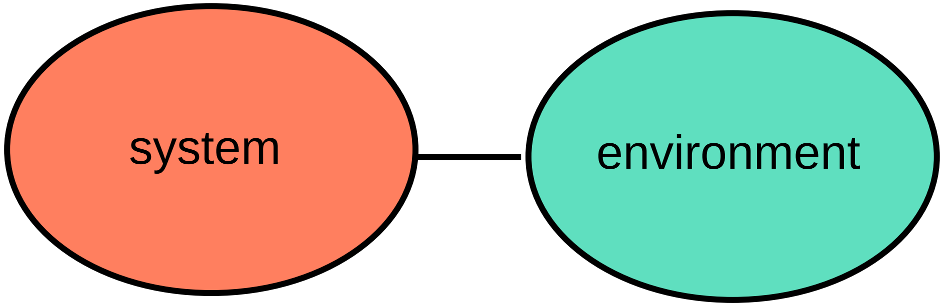
atom

harmonic trap

empty radiation modes

phonon dissipation

1.3 ... *Open* Quantum System



role of coupling to environment:

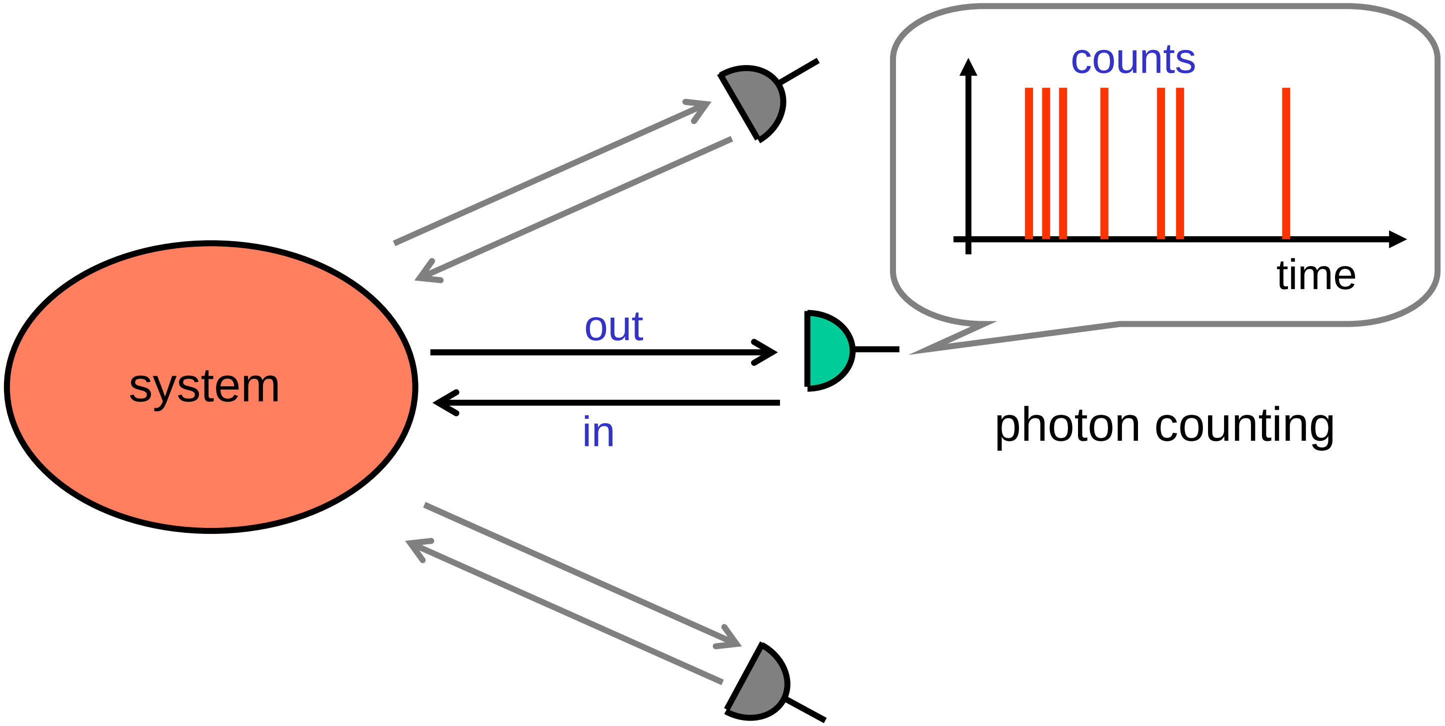
- noise / dissipation (decoherence)
- quantum optics … state preparation (e.g. laser cooling)

this *is* valid
in quantum optics

Quantum Markov processes:

- quantum stochastic Heisenberg and Schrödinger equations
- master equations etc.

1.3 ... *Open* Quantum System



photon counting

role of coupling to environment:

- continuous observation:

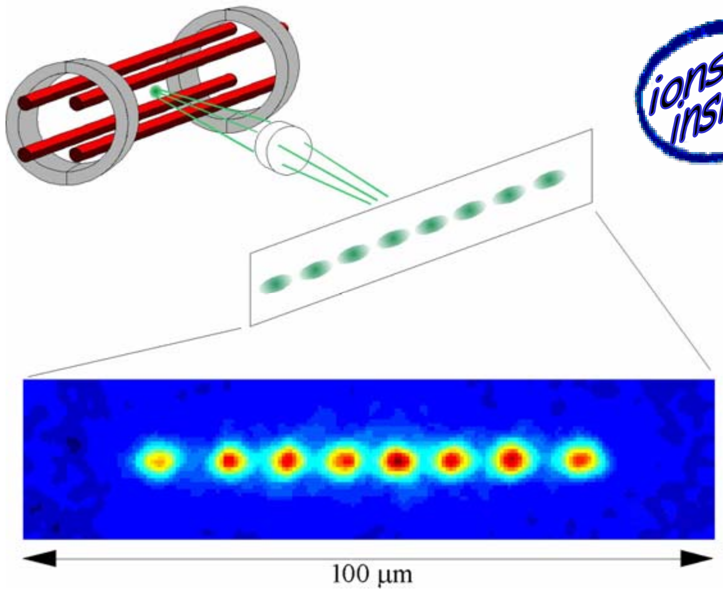
clicks ↔ quantum jumps

& preparation

Outline: Quantum Computing & Communication with …

1. trapped ions
    - a tour: the *1995 2-qubit gate* … the *2003 / 2004 „best“ coherent control gate*
2. neutral atoms
    - optical lattices, cold collisions, Rydberg gates etc.
3. atomic ensembles
    - quantum repeater with atoms / qdots
    - teleportation with ensembles

- Theoretical Tools: Quantum noise
    - decoherence, state preparation (by “quantum jumps, read out
    - from quantum operations to stochastic Schrödinger equations, continuous measurement and all that

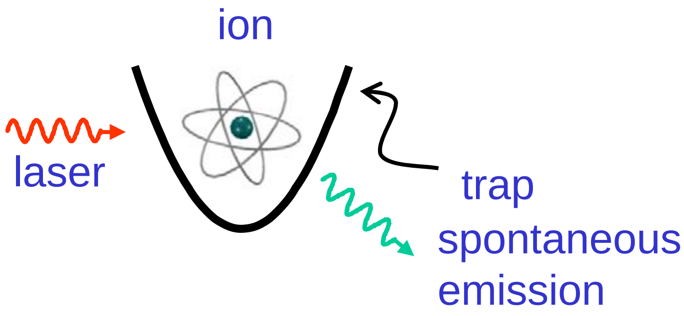


Quantum Computing with Trapped Ions

- basics: quantum optics of single ions & many ions
  - develop toolbox for quantum state engineering
- 2-qubit gates
  - from first 1995 gate proposals and realizations
  - ... geometric and „best“ coherent control gates
- spin models

1. A single trapped ion

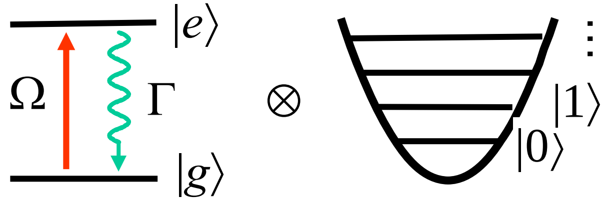
- a single laser driven trapped ion



✓ system: atom + motion in trap:
goal: quantum engineering

✓ [open quantum system]

- system: two-level atom + harmonic oscillator



two-level system

(= qubit)

phonons

$\nu \sim 10$ MHz

$$H = H_{0T} + H_{0A} + H_1$$

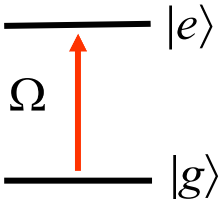
trap $\quad H_{0T} = \frac{\hat{P}^2}{2M} + \frac{1}{2}M\nu^2\hat{X}^2 \equiv \hbar\nu(a^\dagger a + \frac{1}{2})$

atom $\quad H_{0A} = -\hbar\Delta|e\rangle\langle e|$

laser $\quad H_1 = -\frac{1}{2}\hbar\Omega e^{ik_L\hat{X}}|e\rangle\langle g| + \text{h.c.}$

$$H = \frac{\hat{p}^2}{2M} + \frac{1}{2}M\nu^2\hat{x}^2 + \hbar\omega_{eg}|e\rangle\langle e| - \hbar(\frac{1}{2}\Omega e^{ik\hat{x} - i\omega t}|e\rangle\langle g| + \text{h.c.})$$

- laser absorption & recoil



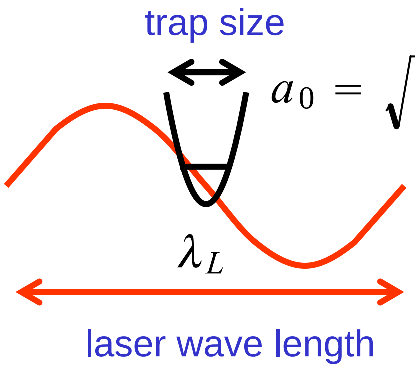
$$|g\rangle|\text{motion}\rangle \rightarrow |e\rangle e^{ik_L\hat{X}}|\text{motion}\rangle$$

photon recoil kick

interaction $H_1 = -\frac{1}{2}\hbar\Omega e^{ik_L\hat{X}}|e\rangle\langle g| + \text{h.c.}$

laser photon recoil:
couples internal dynamics and center-of-mass

- Lamb-Dicke limit



$$a_0 = \sqrt{\frac{\hbar}{2M\nu}}$$

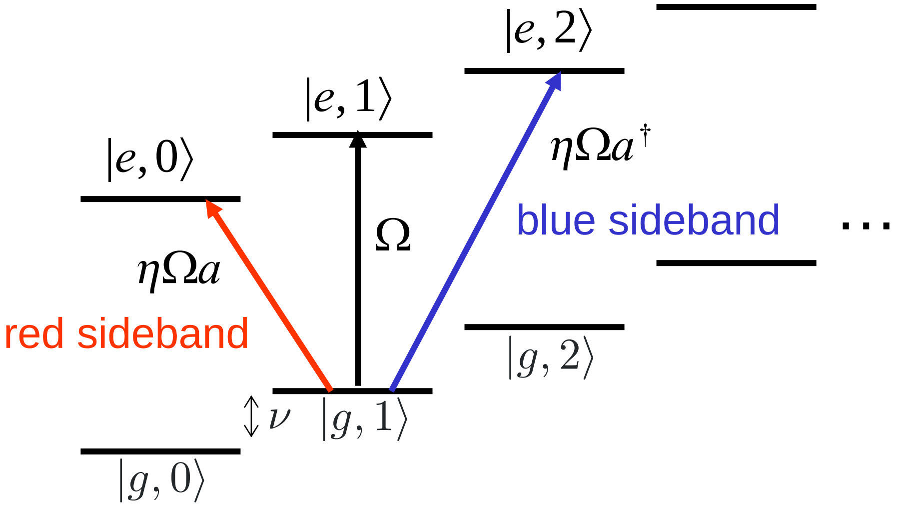
Lamb-Dicke expansion

$$e^{ik_L\hat{X}} = e^{i\eta(a+a^\dagger)}$$

$$= 1 + i\eta(a + a^\dagger) + \ldots$$

$$\eta = 2\pi\frac{a_0}{\lambda_L} \equiv \sqrt{\frac{\epsilon_R}{\hbar\nu}} \quad \sim 0.1$$

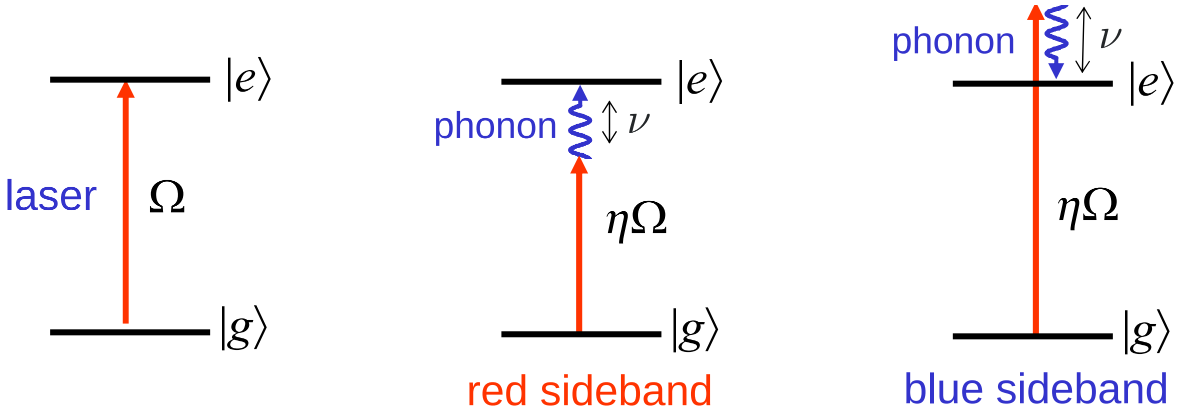
- spectroscopy: atom + trap



laser interaction

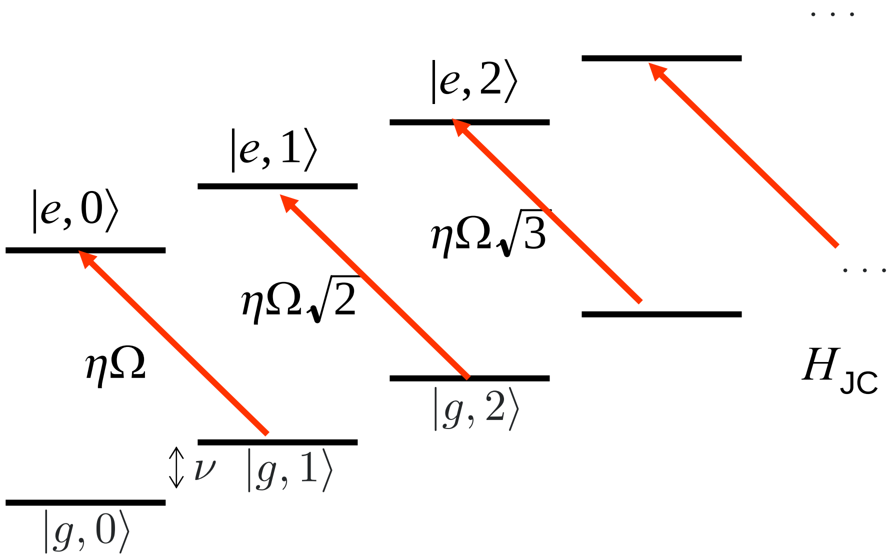
$$\frac{1}{2}\Omega e^{ik_L\hat{X}}|e\rangle\langle g| = \frac{1}{2}\Omega|e\rangle\langle g|$$
$$+ i\frac{1}{2}\Omega\eta a\,|e\rangle\langle g|$$
$$+ i\frac{1}{2}\Omega\eta a^\dagger|e\rangle\langle g|$$
$$+\ldots$$

- processes: "Hamiltonian toolbox for phonon-state engineering"



laser assisted phonon absorption and emission

- example: "laser tuned to red sideband"



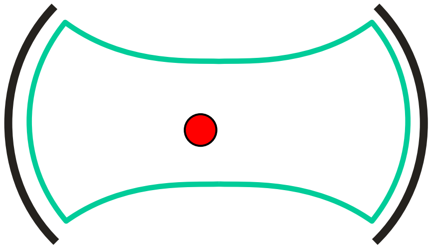
Jaynes-Cummings model

$$H_{\text{JC}} = \hbar\nu a^\dagger a - \hbar\Delta|e\rangle\langle e| - \tfrac{1}{2} i\hbar(\Omega\eta)|e\rangle\langle g|a + \text{h.c.}$$

↑ trap

↑ vacuum Rabi frequency
~ laser (switchable)

- Remark: CQED



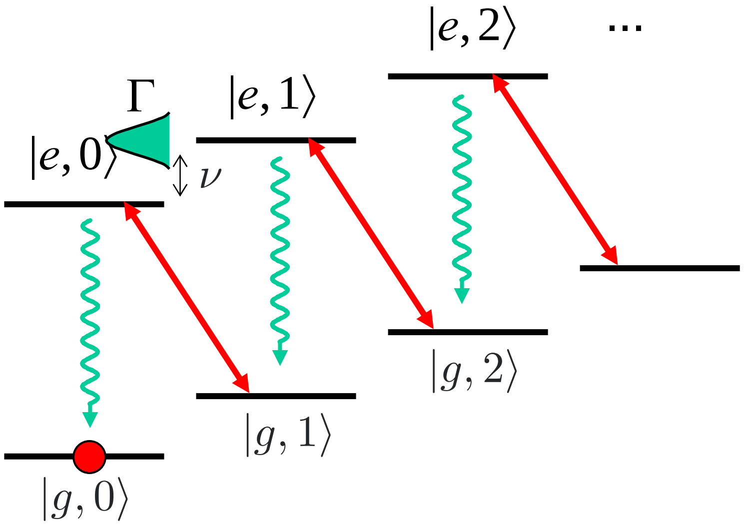
$$H_{\text{JC}} = \nu a^\dagger a + \omega_{eg}|e\rangle\langle e| - ig|e\rangle\langle g|a + \text{h.c.}$$

↑ optical

↑ vacuum Rabi frequency

[Dissipation: spontaneous emission]

- sideband cooling... as optical pumping to the ground state



preparation of pure states

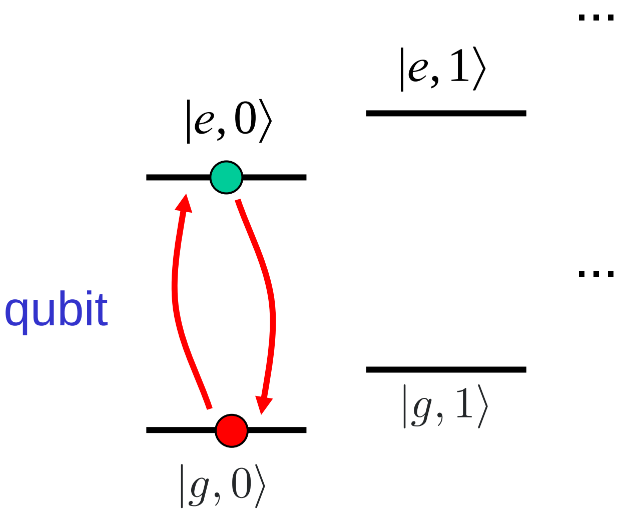
$$\rho_{\text{atom}} \otimes \rho_{\text{motion}} \to |g\rangle\langle g| \otimes |0\rangle\langle 0|$$

- measurement of internal states: quantum jumps …

qubit read out

Excercises in quantum state engineering

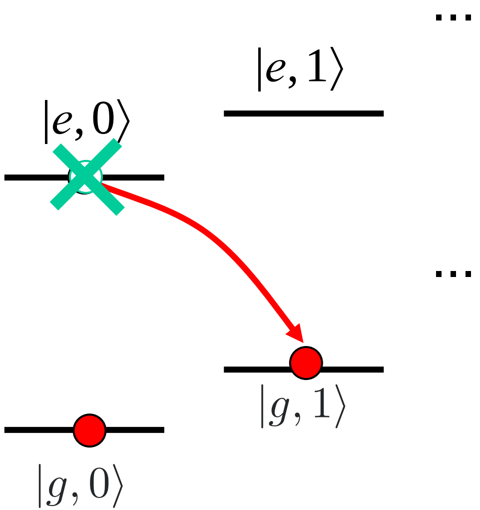
- **Example 1:** single qubit rotation



$$(\alpha|g\rangle + \beta|e\rangle) \otimes |0\rangle \overset{U_1}{\to} (\alpha'|g\rangle + \beta'|e\rangle) \otimes |0\rangle$$

(1) we can rotate the qubit without touching the phonon state

- **Example 2:** swapping the qubit to the phonon mode



$$(\alpha|g\rangle + \beta|e\rangle) \otimes |0\rangle \longrightarrow |g\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$$

ion qubit phonon qubit

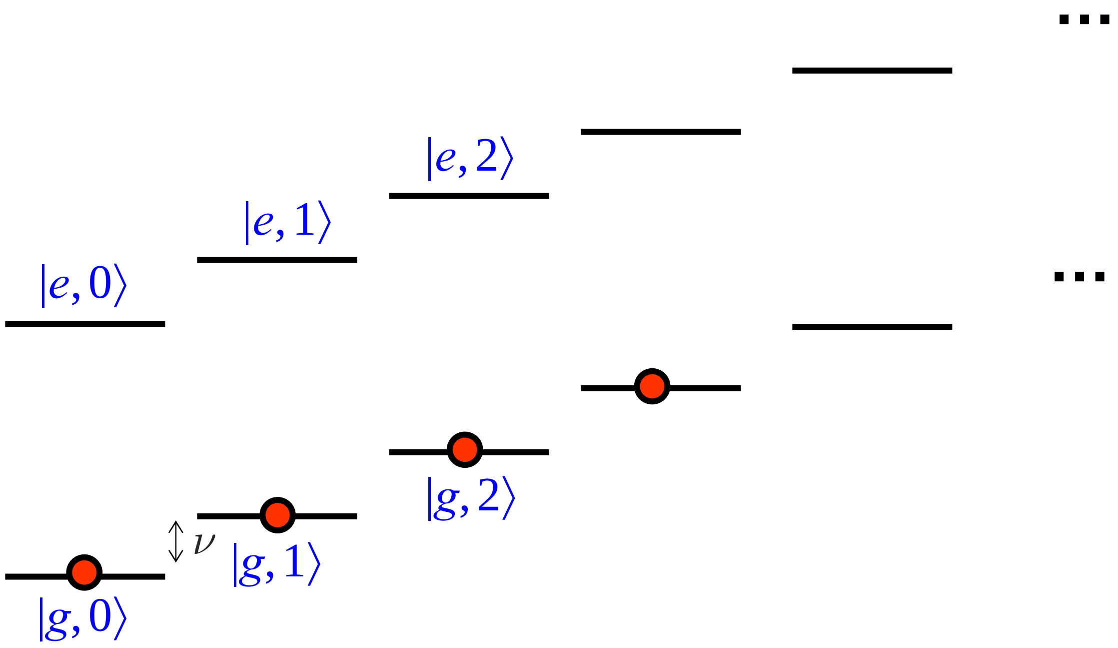
(2) Using a laser pulse we can swap qubits stored in ions to the phonon modes (and vice versa)

- **Example 3:** engineering arbitrary phonon superposition states

$$|g\rangle \otimes |0\rangle \xrightarrow{U} |\Psi\rangle = |g\rangle \otimes \sum\nolimits_{n=0}^{N} c_n |n\rangle$$

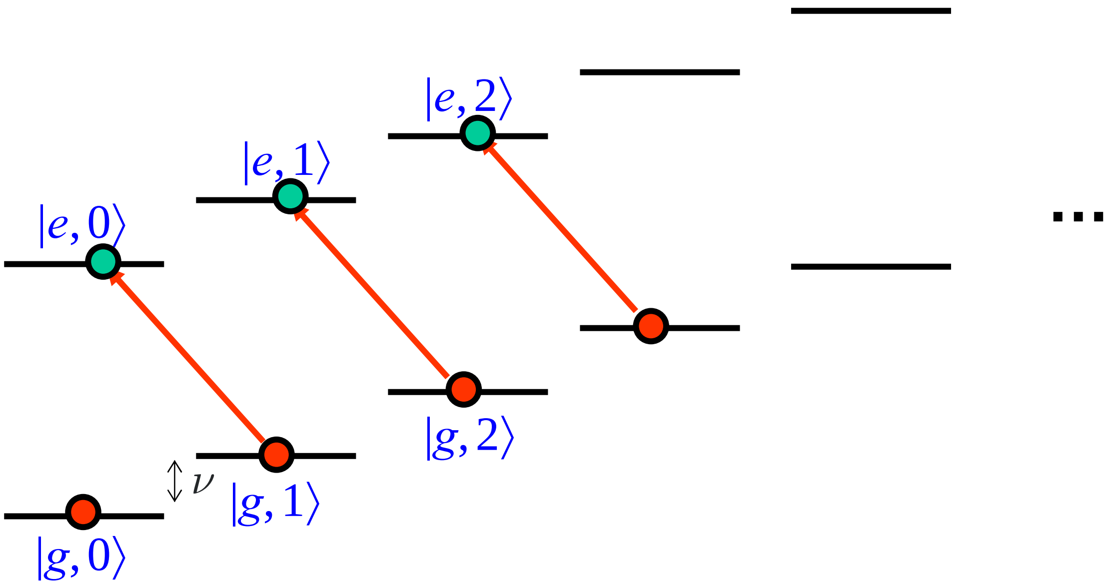
given coefficients c_n

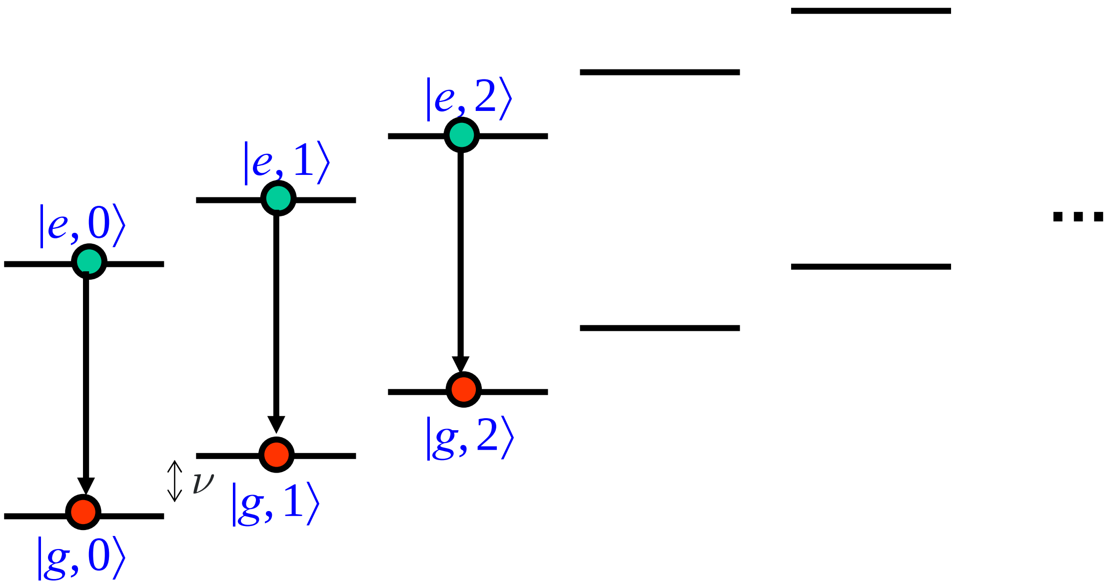
✓ Fock states
✓ squeezed & coherent states
✓ Schrödinger cat states
✓ ...

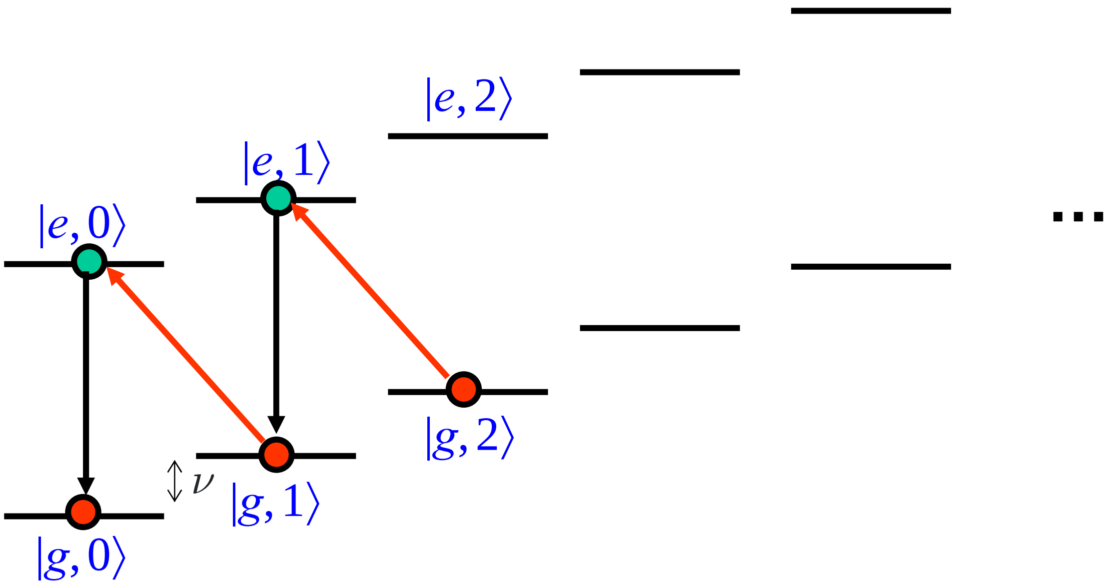


- Idea: we will look for the inverse U which transforms $|\Psi\rangle$ to $|g\rangle \otimes \sum_{n=0}^{n_{\text{max}}} c_n |n\rangle$

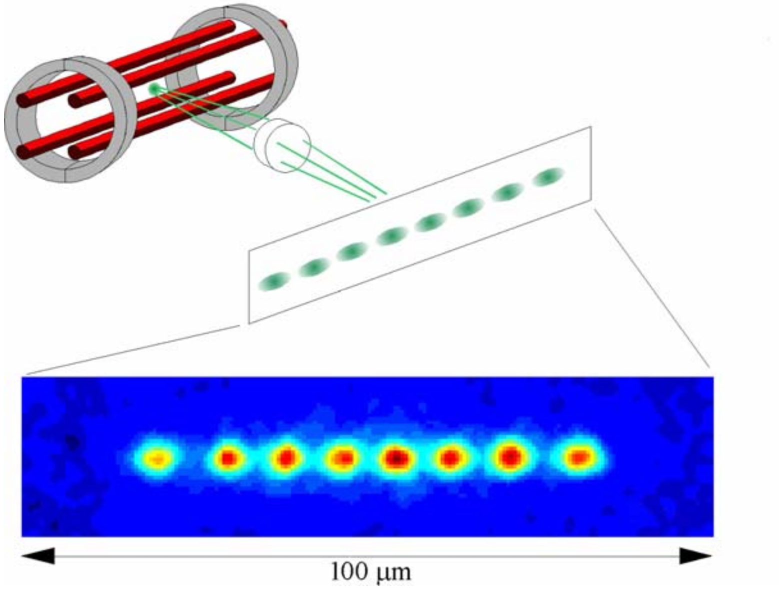
Law & Eberly, Gardiner et al., Wineland et al.


$|e,0\rangle$
$|e,1\rangle$
$|e,2\rangle$
ν
$|g,0\rangle$
$|g,1\rangle$
$|g,2\rangle$
...

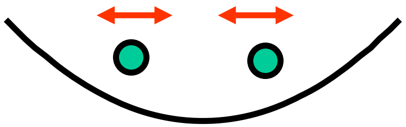

$|e,0\rangle$
$|e,1\rangle$
$|e,2\rangle$
$|g,0\rangle$
$|g,1\rangle$
$|g,2\rangle$
ν
...


$|e,2\rangle$
$|e,1\rangle$
$|e,0\rangle$
$|g,2\rangle$
$|g,1\rangle$
$|g,0\rangle$
ν
...

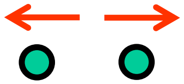
2. Many Ions



- 2 ions & collective phonon modes

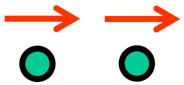


stretch mode



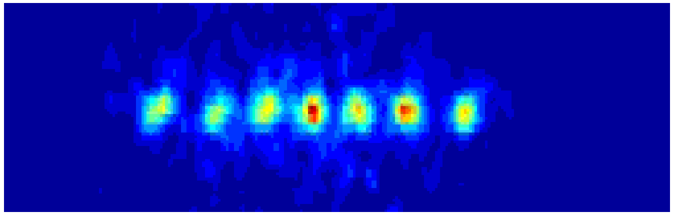
$$\nu_r = \sqrt{3}\,\nu_c$$

center-of-mass



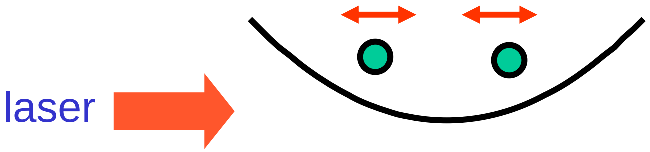
$$\nu_c = \nu$$

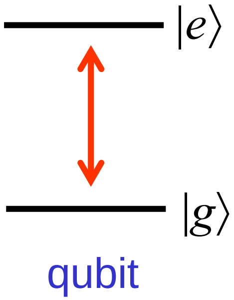
- example: classical ion motion



(3) We can swap a qubit to a *collective* mode via laser pulse

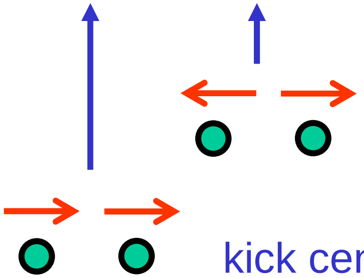
- **Example:** 2 ions in a 1D trap kicked by laser light





$$H = \nu_c a^\dagger a + \nu_r b^\dagger b$$

$$+ \frac{1}{2}\Omega(t)\sigma_1^+ e^{i\eta_c(a^\dagger+a)+\frac{1}{2}\eta_r(b^\dagger+b)} + \frac{1}{2}\Omega(t)\sigma_2^+ e^{i\eta_c(a^\dagger+a)-\frac{1}{2}\eta_r(b^\dagger+b)} + \text{h.c}$$



kick stretch mode

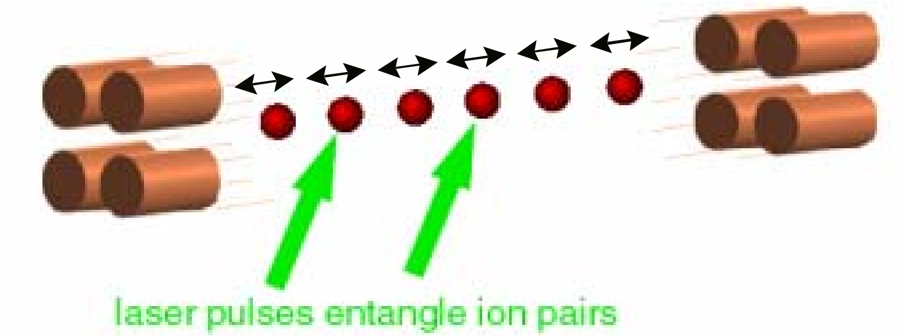
$\nu_c = \nu$

kick center-of-mass

$\nu_r = \sqrt{3}\,\nu_c$

Ion Trap Quantum Computer '95

- Cold ions in a linear trap



Qubits: internal atomic states

1-qubit gates: addressing ions with a laser

2-qubit gates: entanglement via exchange of phonons of quantized collective mode

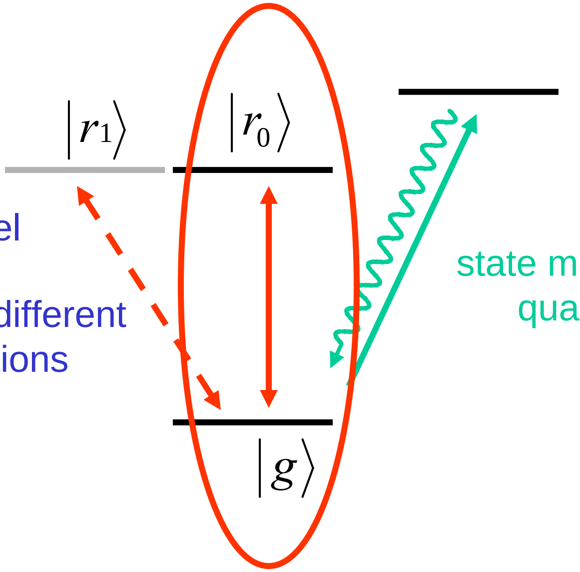
- State vector

$$|\Psi\rangle = \sum c_x |x_{N-1}, \ldots , x_0\rangle_{\text{atom}} \quad |0\rangle_{\text{phonon}}$$

quantum register databus

- QC as a time sequence of laser pulses
- Read out by quantum jumps

Level scheme



auxiliary level

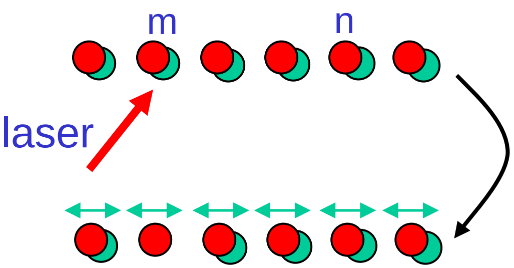
addressing with different
light polarizations

state measurement via
quantum jumps

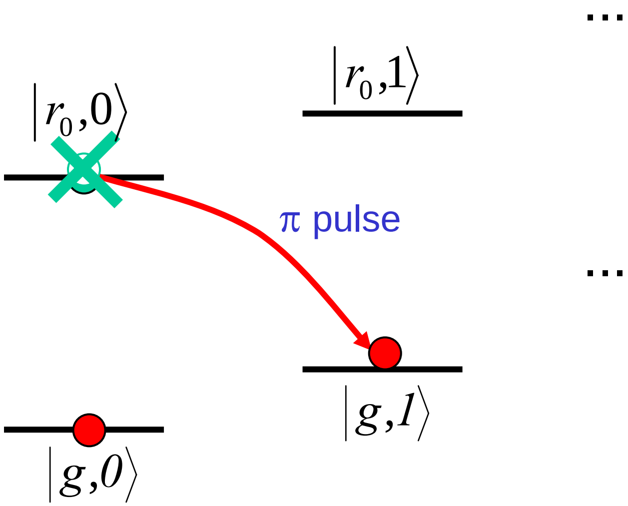
qubit

Two-qubit phase gate

- step 1: swap first qubit to phonon



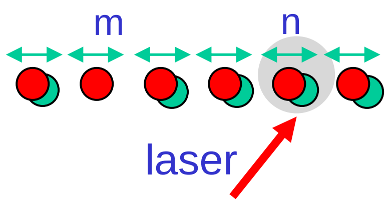
first atom: m

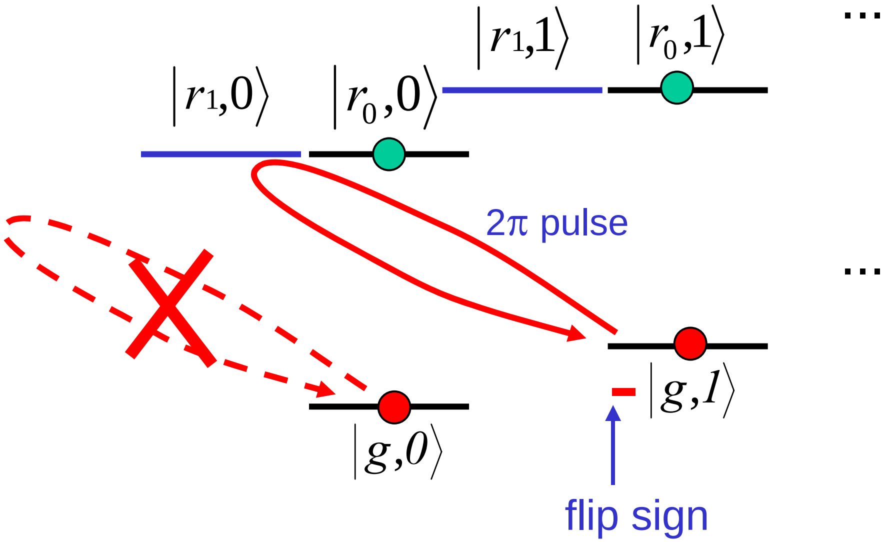


$$\begin{array}{ccc} & \hat{U}_m^{\pi,0} & \\ |g\rangle_m|0\rangle & \longrightarrow & |g\rangle_m|0\rangle \\ |r\rangle_m|0\rangle & \longrightarrow & -i|g\rangle_m|1\rangle \end{array}$$

- step 2: conditional sign change

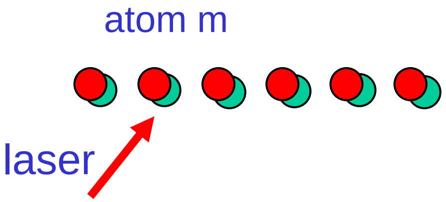
second atom: n





$$
\begin{array}{rcr}
 & \hat{U}_n^{2\pi,1} & \\
|g\rangle_m|g\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|g\rangle_n|0\rangle \\
|g\rangle_m|r\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|r\rangle_n|0\rangle \\
-i|g\rangle_m|g\rangle_n|1\rangle & \longrightarrow & i|g\rangle_m|g\rangle_n|1\rangle \\
-i|g\rangle_m|r\rangle_n|1\rangle & \longrightarrow & -i|g\rangle_m|r\rangle_n|1\rangle
\end{array}
$$

- step 3: swap phonon back to first qubit



$$|g\rangle_m \otimes \begin{array}{rcl} & \hat{U}_m^{\pi,0} & \\ |g\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|g\rangle_n \\ |r\rangle_n|0\rangle & \longrightarrow & |g\rangle_m|r\rangle_n \\ i|g\rangle_n|1\rangle & \longrightarrow & |r\rangle_m|g\rangle_n \\ -i|r\rangle_n|1\rangle & \longrightarrow & -|r\rangle_m|r\rangle_n \end{array} \otimes |0\rangle$$

- summary: we have a phase gate between atom m and n

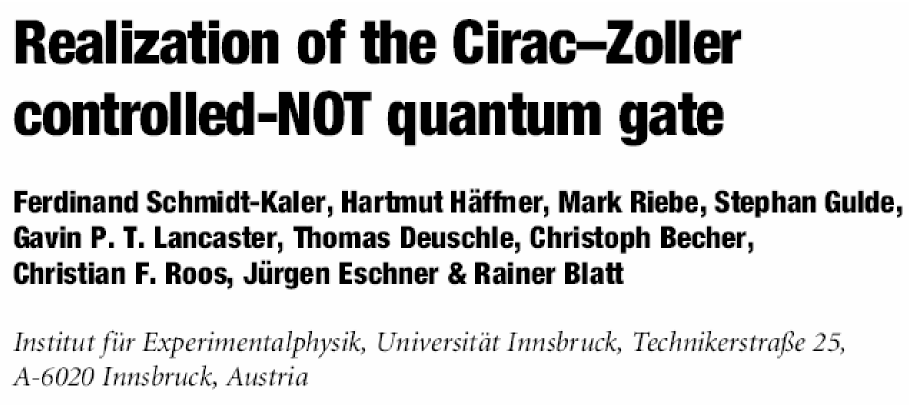
$$\begin{array}{lcrl} |g\rangle|g\rangle\ |0\rangle & \longrightarrow & & |g\rangle|g\rangle\ |0\rangle, \\ |g\rangle|r_0\rangle\ |0\rangle & \longrightarrow & & |g\rangle|r_0\rangle\ |0\rangle, \\ |r_0\rangle|g\rangle\ |0\rangle & \longrightarrow & & |r_0\rangle|g\rangle\ |0\rangle, \\ |r_0\rangle|r_0\rangle|0\rangle & \longrightarrow & - & |r_0\rangle|r_0\rangle\ |0\rangle. \end{array}$$

phonon mode returned to initial state

$$|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow (-1)^{\epsilon_1\,\epsilon_2}|\epsilon_1\rangle|\epsilon_2\rangle \quad (\epsilon_{1,2}=0,1)$$

Rem.: this idea translates immediately to CQED

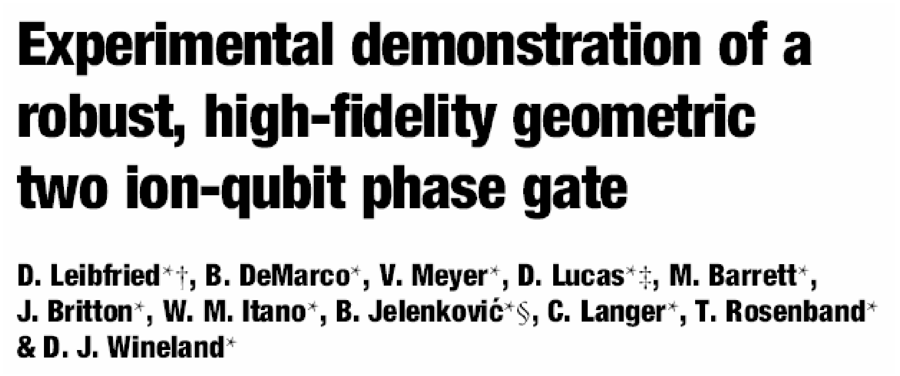
- **(addressable) 2 ion controlled-NOT + tomography**



Realization of the Cirac–Zoller controlled-NOT quantum gate

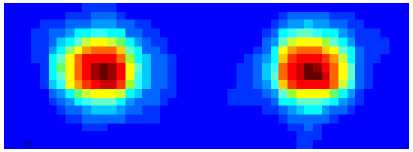
Ferdinand Schmidt-Kaler, Hartmut Häffner, Mark Riebe, Stephan Gulde, Gavin P. T. Lancaster, Thomas Deuschle, Christoph Becher, Christian F. Roos, Jürgen Eschner & Rainer Blatt

Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria

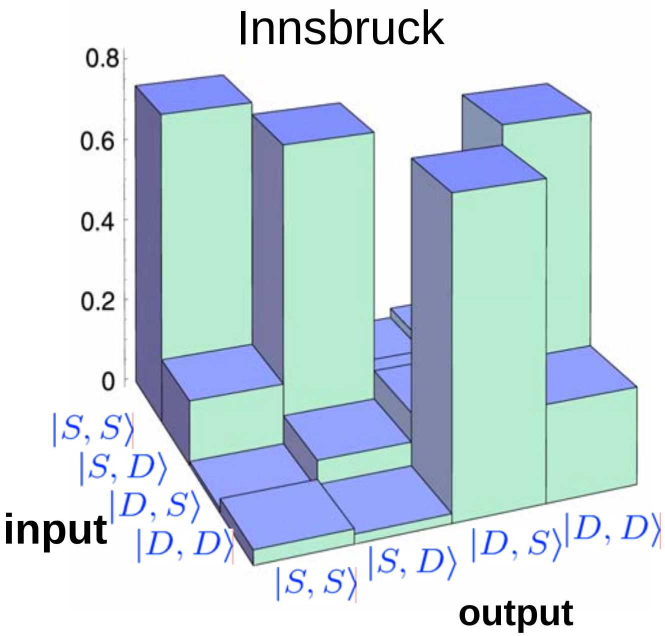


Experimental demonstration of a robust, high-fidelity geometric two ion-qubit phase gate

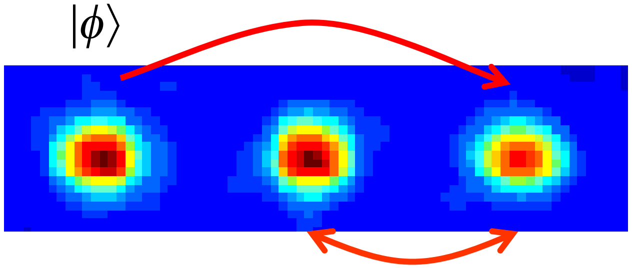
D. Leibfried*†, B. DeMarco*, V. Meyer*, D. Lucas*‡, M. Barrett*, J. Britton*, W. M. Itano*, B. Jelenković*§, C. Langer*, T. Rosenband* & D. J. Wineland*



truth table CNOT



- **teleportation Innsbruck / Boulder**

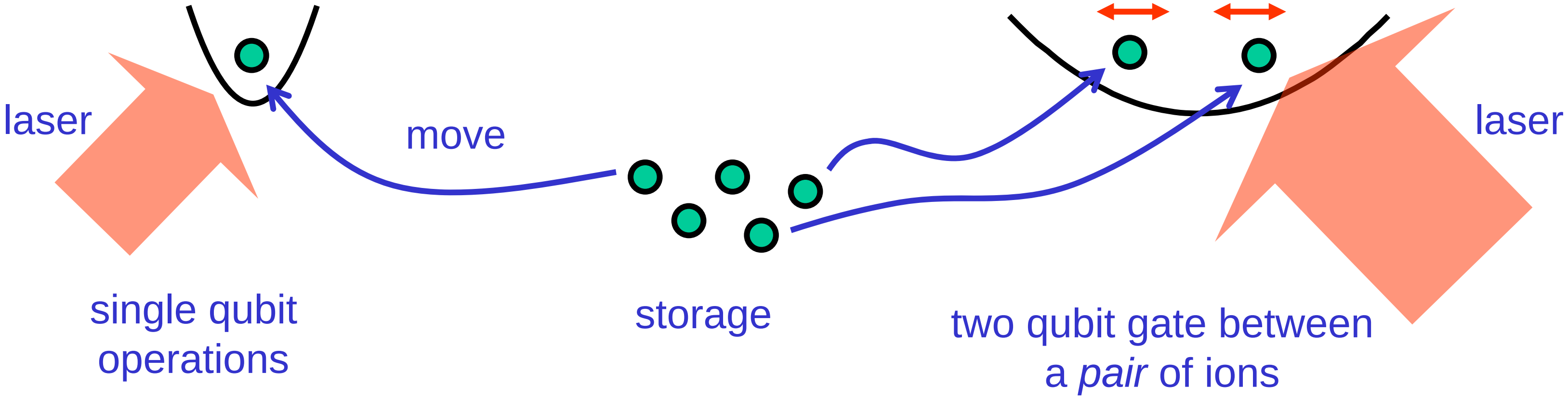


EPR pair

- **decoherence: quantum memory DFS 20 sec**

Scalability

- key idea: moving ions … without destroying the qubit



Two-qubit gate … the “wish list”

- fast: max # operations / decoherence [what are the limits?]
- NO temperature requirement: “hot” gate, i.e. NO ground state cooling

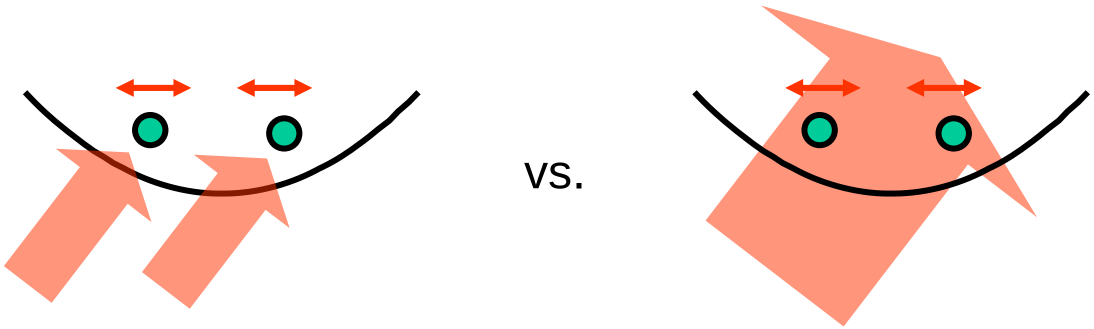
$$|\psi\rangle\langle\psi|\otimes\rho_{\text{motion}} \rightarrow \text{ entangle qubits via motion } \rightarrow |\psi\rangle\langle\psi|\otimes\rho'_{\text{motion}}$$

qubits

motional state: e.g. thermal

motional state factors out

- NO indivdual addressing



addressing: large distance

vs.

strong coupling: small distance

Speed limits

- In all present proposals the speed limit for the gate is given by the trap frequency

$$T_{\text{gate}} \sim 1/\eta\nu$$

trap frequency

Lamb Dicke parameter $\eta = \sqrt{\frac{\epsilon_R}{\nu}}$

⇒ $$T_{\text{gate}} \sim 1/\sqrt{\nu}$$

$\nu \sim$ 10 MHz, i.e. $T_{\text{gate}} \sim \mu$ s

limits given by trap design

The rest of the lecture …

- Push gate

J.I. Cirac & PZ

- Geometric phase gates

D. Leibfried et al.
NIST

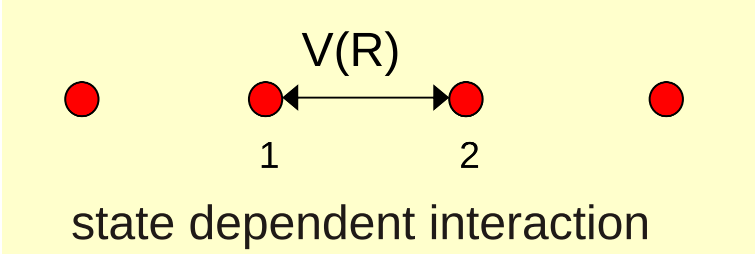
- Optimal Control Gates
  - what is the *best* gate for given resources?

J. Garcia-Ripoll
J.I. Cirac,
PZ

- [Examples]
  - fast gate with short laser pulses
  - fast gate with continuous laser pulses
  - engineering spin Hamiltonians …

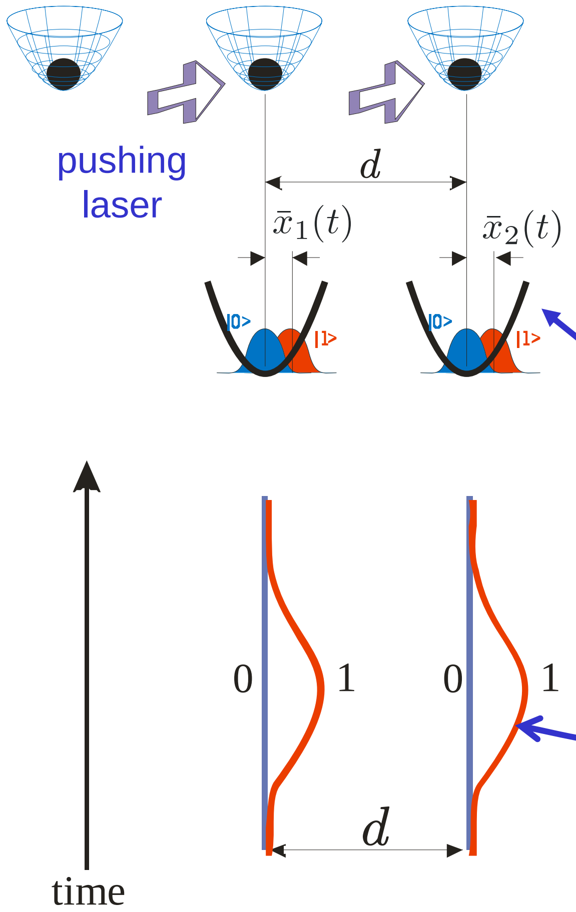
Another example for a 2-qubit gate …

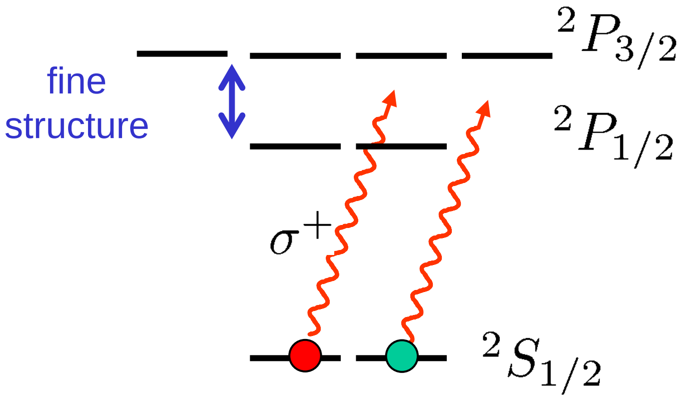
Push gate



state dependent interaction

- converting "spin to charge"
- spin dependent optical potential





different AC Stark shifts

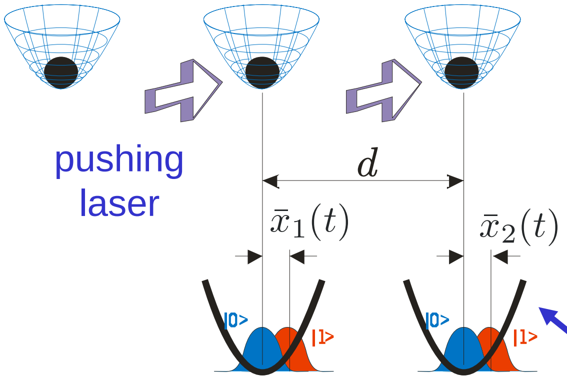
qubit dependent displacement of the ion

accumulate different energy shifts along different trajectories: 2-qubit gate

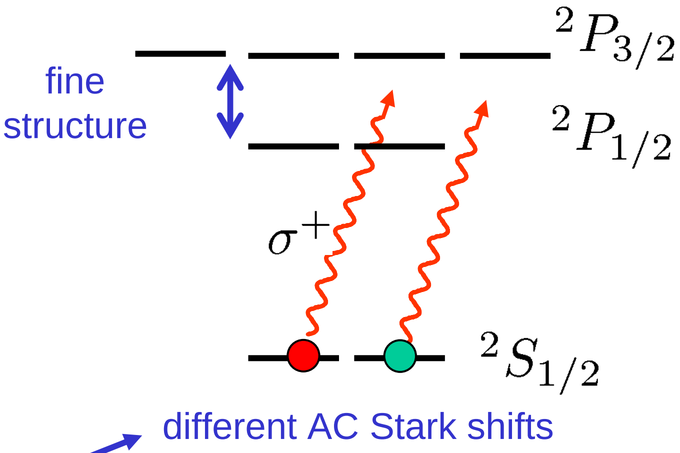
- robust: temperature insensitive ☺

Push gate

- converting "spin to charge"



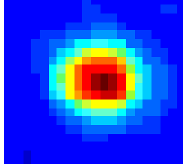
- spin dependent optical potential



qubit dependent displacement of the ion

- Hamiltonian

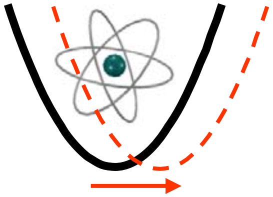
$$H = \sum_{i=1}^{N}\left[\frac{1}{2m}p_i^2 + V_{e,i}(x_i) - F_i(t)\sigma_z^i x_i\right] + \sum_{i<j}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{|x_i - x_j|}$$



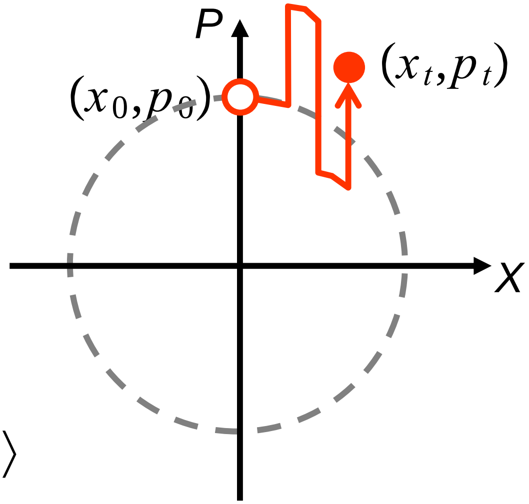
Geometric Phase [Gate]: One Ion

- Goal: geometric phase by driving a harmonic oscillator
- Hamiltonian

$$H = \tfrac{1}{2}\hbar\omega(\hat{p}^2 + \hat{x}^2) - f(t)\hat{x}$$



phase space



- Time evolution

$$|\psi_0\rangle = |z_0 \equiv x_0 + ip_0\rangle \Longrightarrow |\psi_t\rangle = e^{i\phi_t}|z_t \equiv x_t + ip_t\rangle$$

coherent state

coherent state

phase

- Solution

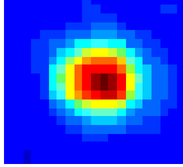
$$\frac{d}{dt}z = -i\omega z + i\frac{1}{\sqrt{2}}f(t) \Longrightarrow z_t = e^{-i\omega t}\left[z_0 + \frac{i}{\sqrt{2}}\int_0^t d\tau\, e^{i\omega\tau} f(\tau)\right]$$

$$\frac{d}{dt}\phi = \frac{1}{2\sqrt{2}}f(t)(z^* + z)$$

classical evolution

displacement

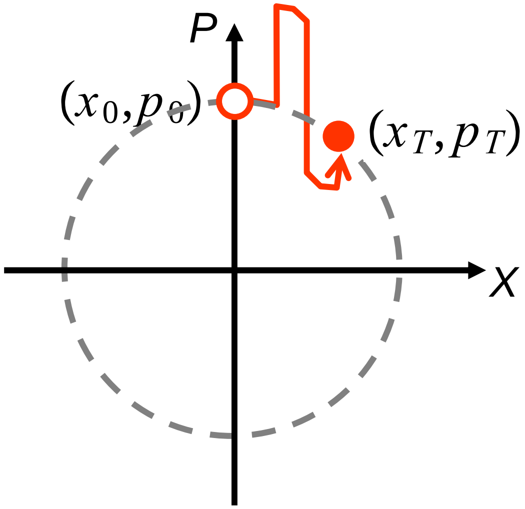
phase

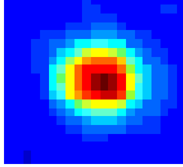


- Condition:

  After a given time *T* the coherent wavepacket is restored to the freely evolved state

$$\int_0^T d\tau\, e^{i\omega\tau} f(\tau) \overset{!}{=} 0$$

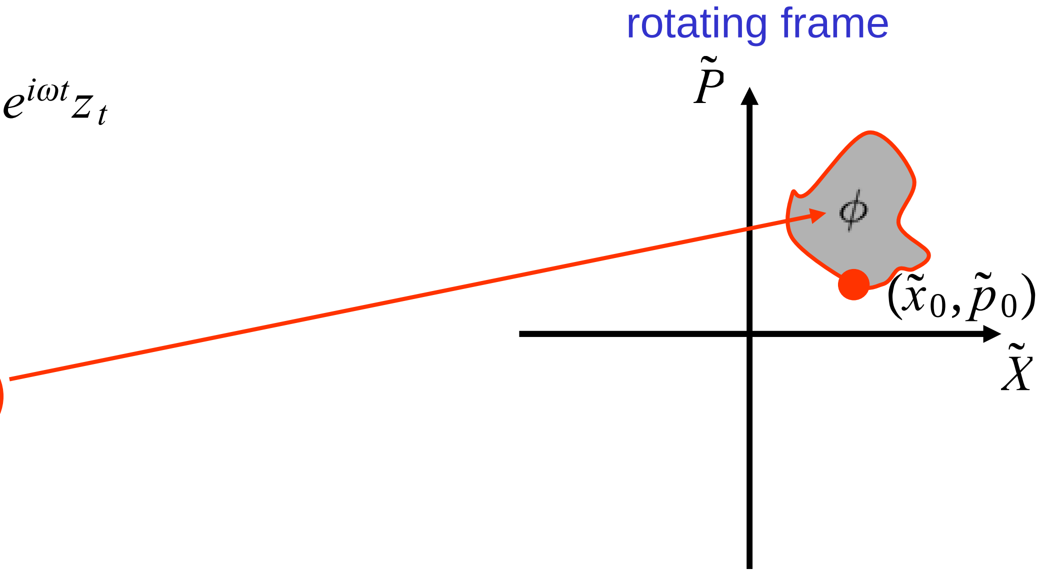

phase space



- Rotating frame $\tilde{z}_t \equiv \tilde{x}_t + i\tilde{p}_t = e^{i\omega t} z_t$

$$\frac{d\tilde{z}}{dt} = i e^{i\omega t}\frac{1}{\sqrt{2}} f(t)$$

$$\frac{d\phi}{dt} = \frac{d\tilde{p}}{dt}\tilde{x} - \frac{d\tilde{x}}{dt}\tilde{p} = 2\frac{dA}{dt}$$

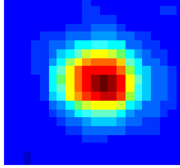


- Phase

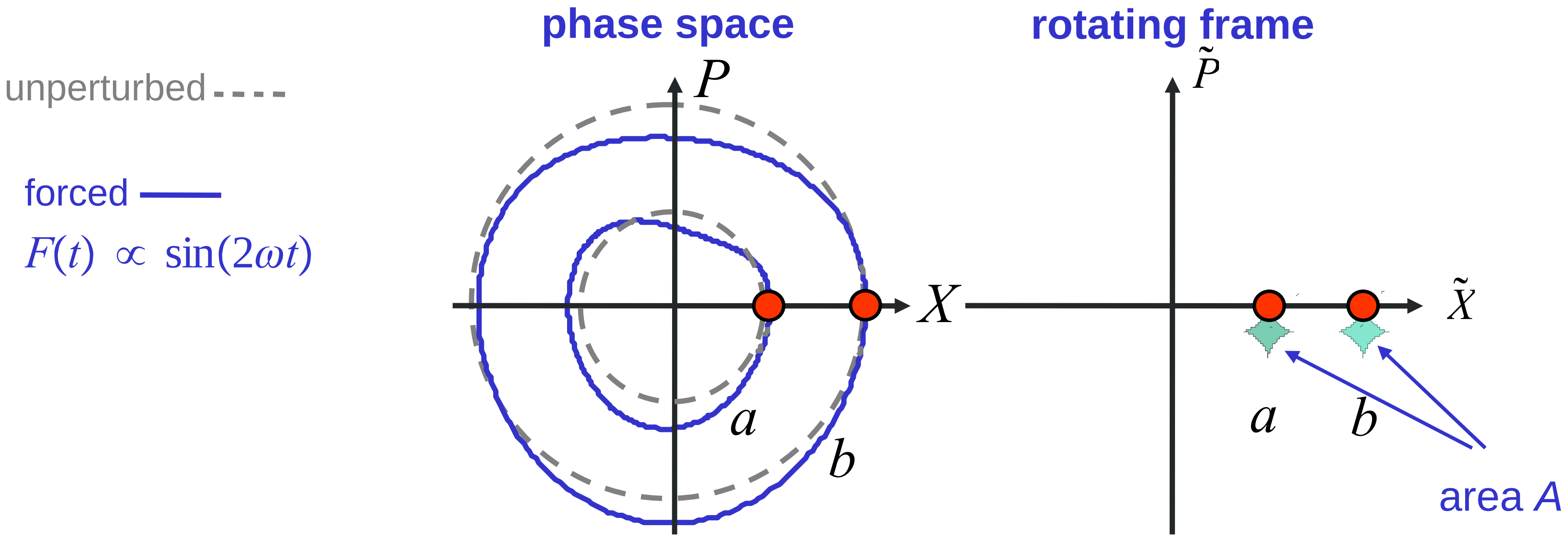
$$\phi(T) = \mathrm{Im}\frac{i}{\sqrt{2}}\int_0^T d\tau\, e^{i\omega\tau} f(\tau)\,\tilde{z}^*_\tau$$

$$= \mathrm{Im}\frac{i}{\sqrt{2}}\underbrace{\left[\int_0^T d\tau\, e^{i\omega\tau} f(\tau_1)\right]}_{\substack{=0\\ \text{return condition}}}\tilde{z}^*_0 + \frac{1}{2}\underbrace{\mathrm{Im}\int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2\, e^{i\omega(\tau_1-\tau_2)} f(\tau_1) f(\tau_2)}$$

The phase does *not* depend on the initial state, (x_0,p_0)

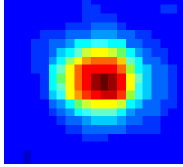


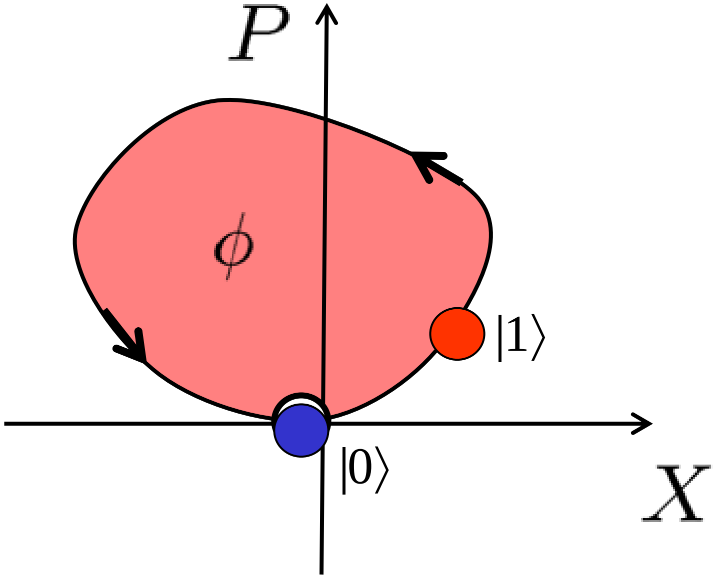
- Example



- The phase does not depend on the initial state, (x_0,p_0) ☺ (temperature independent)

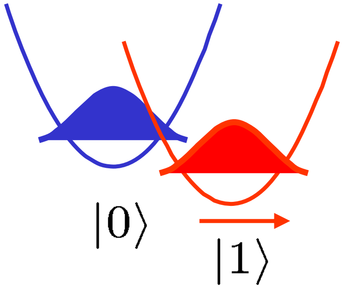
Geometric Phase Gate: Single Ion





- Hamiltonian

$$H = \tfrac{1}{2}\hbar\omega(\hat{p}^2 + \hat{x}^2) - |1\rangle\langle 1| f(t)\hat{x}$$



- Time evolution operator

$$U(T) = e^{i\phi|1\rangle\langle 1|}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |z_0\rangle$$

$$\xrightarrow{U(T)} (\alpha|0\rangle + \beta e^{i\phi}|1\rangle) \otimes |z_T\rangle$$

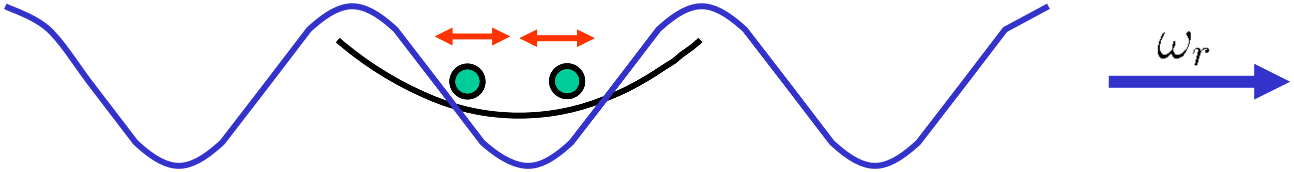
single ion phase gate

motion factors out

© NIST D. Leibfried et al.

NIST Gate: Leibfried *et al* Nature 2003

- 2 ions in a running standing wave tuned to ω_r



$$H = \omega_r a^\dagger a - F(t)(\sigma_z^1 + \sigma_z^2)(a_r + a_r^\dagger)$$

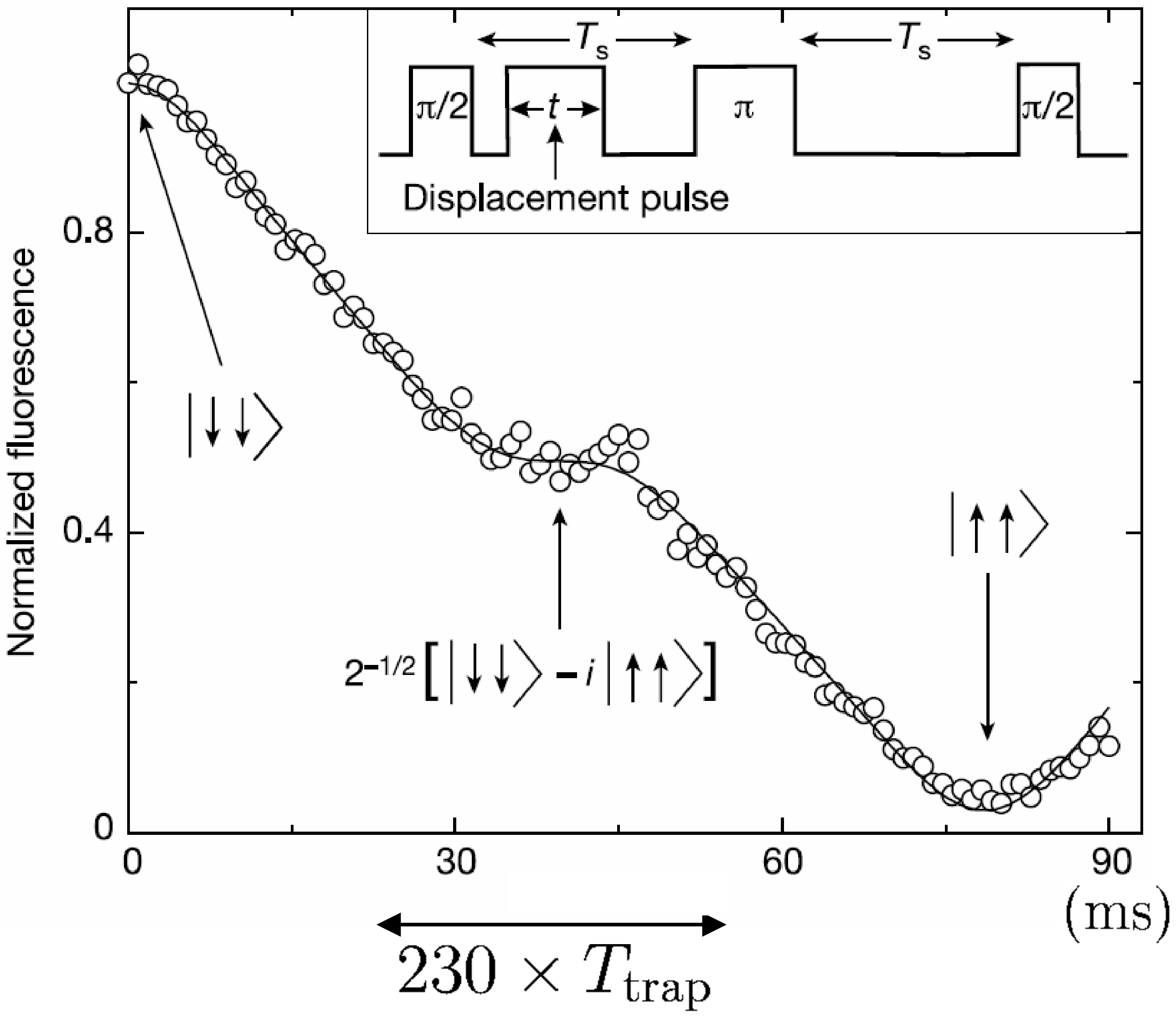
- If F(t) is periodic with a period multiple of ω_r, after some time the motional state is restored, but now the total phase is

$$\phi = A\sigma_z^1\sigma_z^2 \qquad U(T) = \exp(i\phi\sigma_1^z\sigma_2^z)$$

- To address one mode, the gate must be slow ☹

$$T \gg 2\pi/\omega_r$$

NIST Gate: Leibfried *et al.* Nature 2003



Best gate?

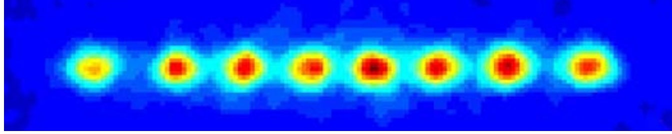
- What is the best possible gate?

  requirements: ...

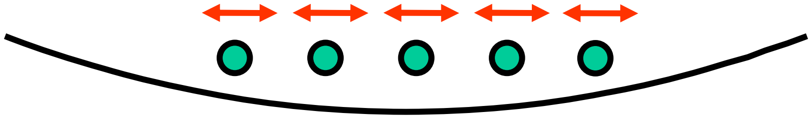
  constraints: …

- … an optimal control problem

N Ions



- We will consider N trapped ions (linear traps, microtraps…), subject to state-dependent forces:



$$H = \sum_{i=1}^{N}\left[\frac{1}{2m}p_i^2 + V_{e,i}(x_i) - F_i(t)\sigma_z^i x_i\right] + \sum_{i<j}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{|x_i - x_j|}$$

- normal modes

$$H = \sum_i\left[\frac{1}{2m}P_i^2 + \frac{1}{2}m\nu_k^2 Q_k^2\right] - \sum_k F_i(t)\sigma_z^i M_{ik} Q_k$$

integrable

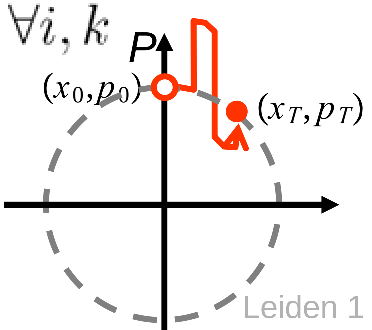
- unitary evolution operator

$$U(T) = \exp\left(i\sum_{ij} J_{ij}\sigma_z^i\sigma_z^j\right)$$

general Ising interaction

- constraints on forces

$$\int_0^T d\tau\, e^{i\omega_k\tau}F_i(\tau) = 0, \quad \forall i,k$$



Quantum Control Problem

- Target: the Ising interaction, is a function of the forces

$$J_{ij} = \frac{1}{2m\hbar}\int_0^T\int_0^T d\tau_1 d\tau_2\, F_i(\tau_1)F_j(\tau_2)\mathcal{G}_{ij}(\tau_1-\tau_2).$$

given

determine

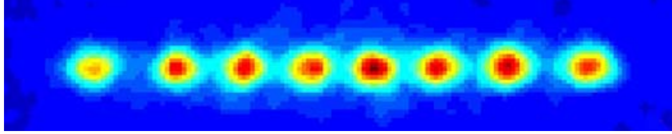
The kernel G depends only on the trapping potential.

- Constraints: displacements, z_k, depend both on the forces and on the internal states. To cancel them, we must impose

$$\int_0^T d\tau\, e^{i\omega_k\tau}F_i(\tau)=0, \quad \forall i,k$$

- Additional constraints: the total time, T; smoothness & intensity of the forces, no local addressing of ions …

fastest gate?



More results

- **Theorem:** For N ions and a given Ising interaction J_{ij} , it is always possible to find a set of forces that realize the gate

simulate spin models

$$\exp\left(-iT\sum_{ij} J_{ij}\sigma_z^i\sigma_z^j\right),$$

$$\exp\left(-iT(\sum_{ij} J_{ij}\sigma_z^i\sigma_z^j + \sum_i h_i\sigma_z^i)\right),$$

although now the solution has to be found numerically.

- **Applications:** Generation of cluster states, of GHZ states, stroboscopic simulation of Hamiltonians, adiabatic quantum computing,…

The time, T, is arbitrary!

---

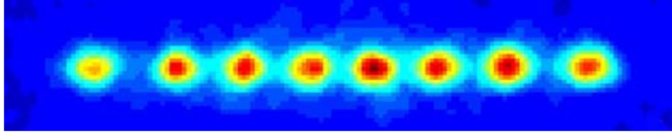
cluster state

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_z + |1\rangle_z)$$

$$|\phi\rangle_c = \exp(i\int_0^t \frac{1}{4}\hbar g(t)dt \sum\nolimits_{<a,b>} \sigma_z^{(a)} \otimes \sigma_z^{(b)} dt)\ \ (\otimes_{a\in C}\, |+\rangle_a)$$

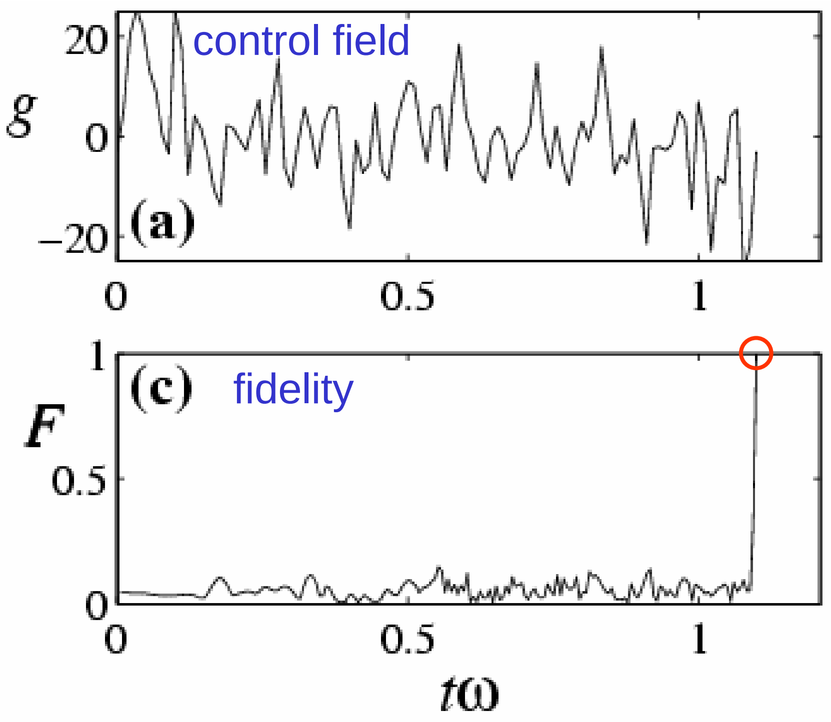
GHZ state

$$|\phi\rangle_{\mathrm{GHZ}} \sim e^{-iJ_z^2 t}|+\rangle \equiv e^{-i(\sum_i \frac{1}{2}\sigma_z^i)^2 t} \sim |00\ldots\rangle + |11\ldots\rangle$$

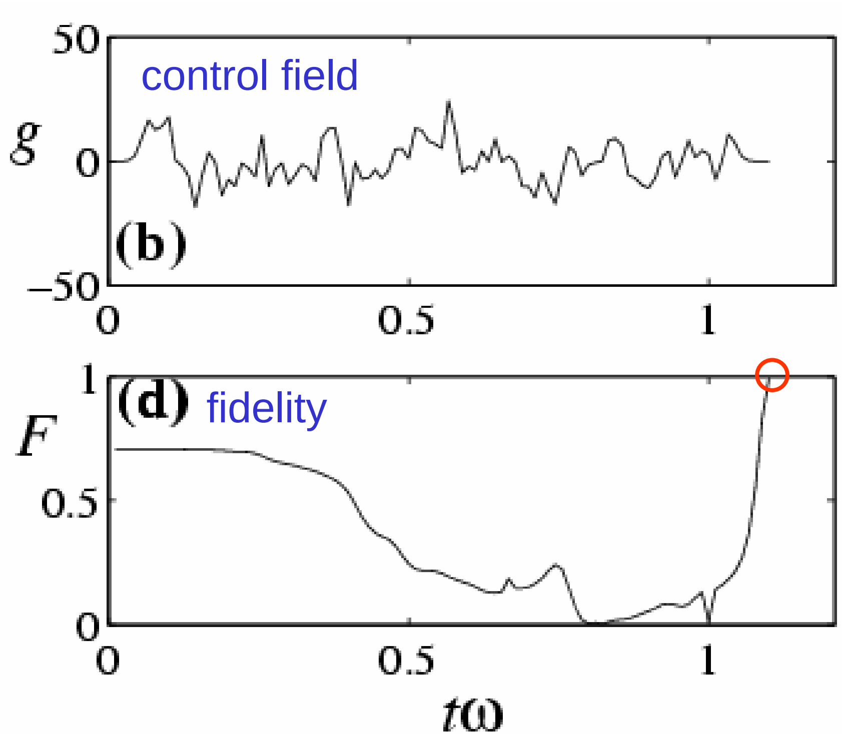


Engineering cluster and GHZ states

Cluster state N=10



GHZ state N=20



These examples use a common force: $F_i(t) = x_i\ g(t)$

Juanjo Garcia-Ripoll has calculated this up to N=30 ions

A final remark:
Analogies with Condensed Matter Hamiltonians

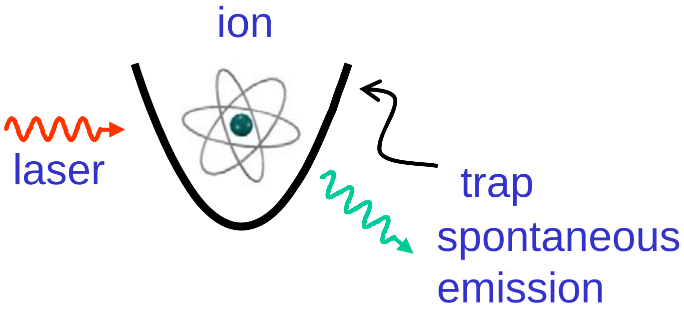
- Cavity QED: optical / microwave CQED / ion trap *vs.* JJ + transmission line

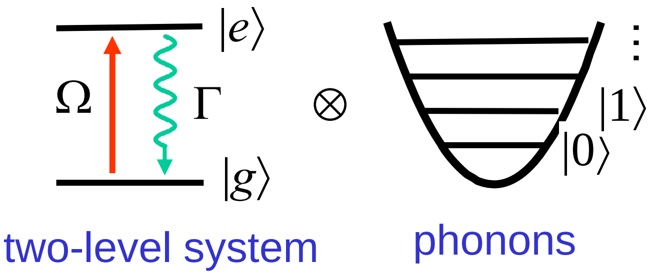
  see Yale & Delft

- ➡ Trapped Ion *vs.* Nanomechanical Systems + Quantum Dot / Cooper pair box

Trapped ion

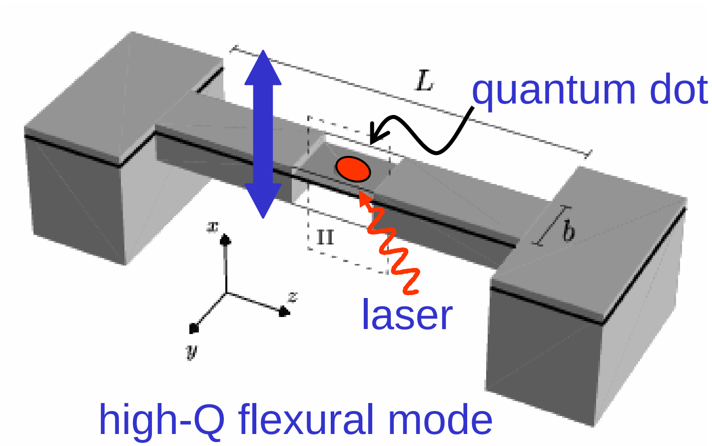
- trapped ion driven by laser





Nano-mechanical system

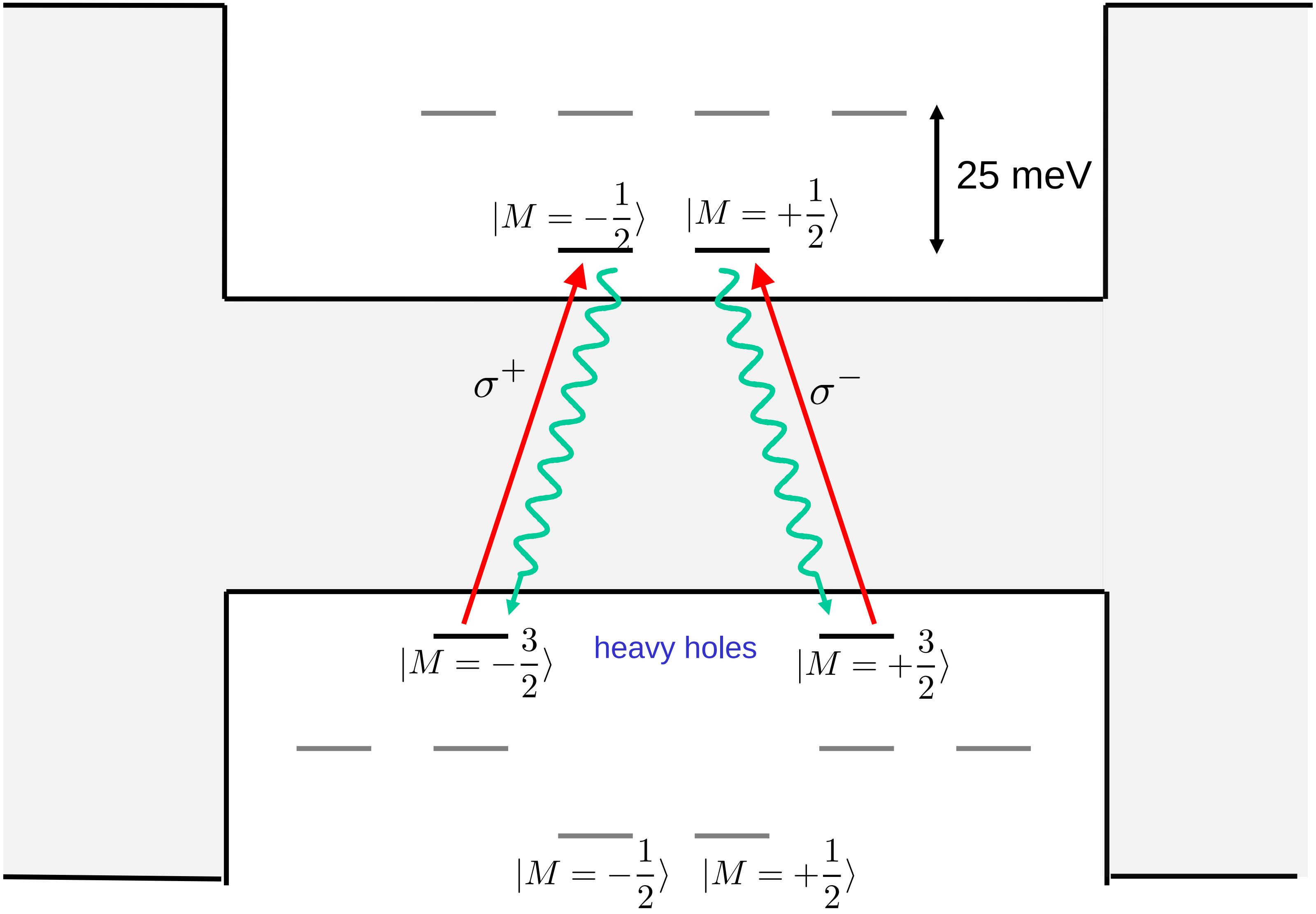
- quantum dot in a phonon cavity



I. Wilson-Rae, PZ, A. Imamoglu,
PRL 2004

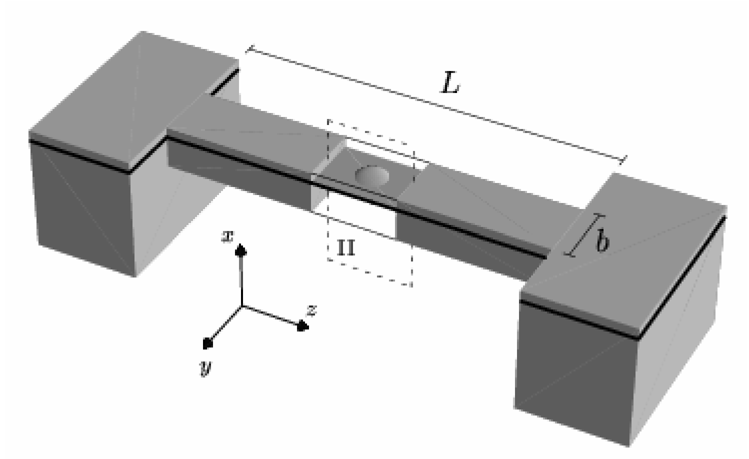
see A. Imamoglu's
lecture

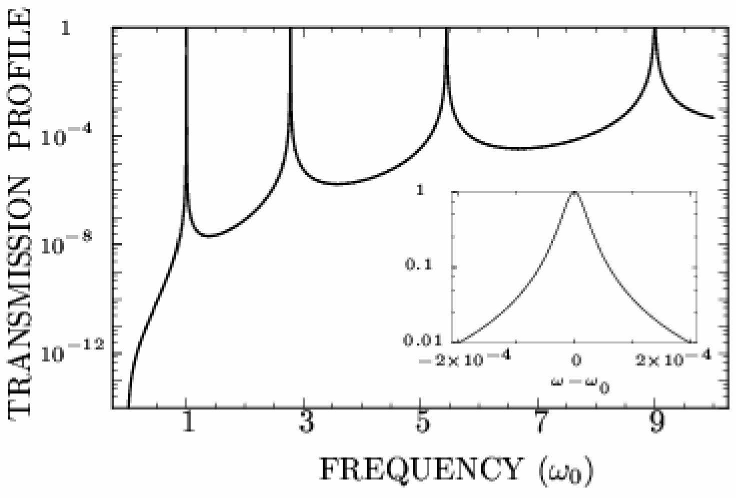
Spectroscopy of Quantum Dots



Quantum dot in a phonon cavity

- system





Q = 25,000 has been measured for modes with $\omega = 2\pi$ x 200 MHz.

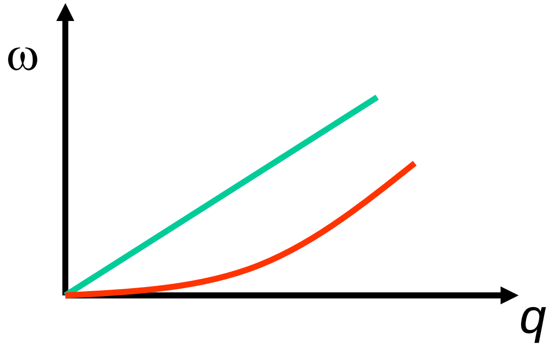
- Thin rod elasticity: $\lambda_p \sim L >> b,d$

four branches with no infrared cutoff:

✓ flexural & in-plane bending $\omega \sim q^2$

$$\frac{\partial^2 u}{\partial t^2} + \frac{EI_2}{\rho}\frac{\partial^4 u}{\partial y^4} = 0$$

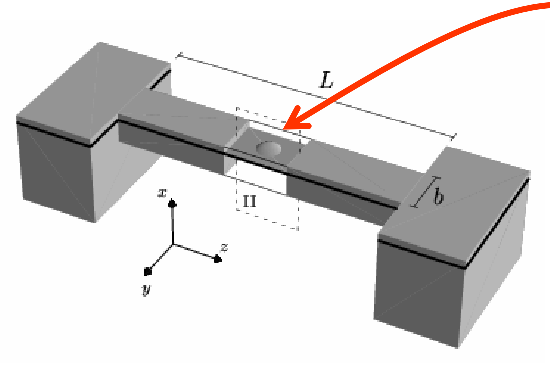
✓ torsional & compression modes $\omega \sim q$

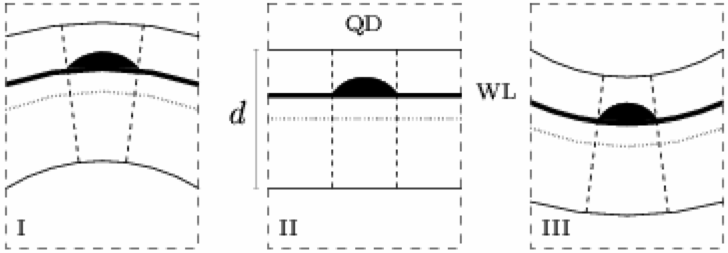


Hamiltonian

deformation coupling

$$H_{\mathrm{DP}} = \int \mathrm{d}\bar{r}^3 [D_c \hat{\rho}_{\mathrm{el}}(\bar{r}) - D_v \hat{\rho}_{\mathrm{h}}(\bar{r})] \nabla \cdot \hat{\bar{u}}(\bar{r})$$





- Hamiltonian: single mode coupled to a QD via deformation coupling

exciton

quantum dot

phonons

$$H = \hbar\omega_0 b_0^\dagger b_0 + \hbar[-\Delta \boxed{+\omega_0\eta(b_0 + b_0^\dagger)}]|e\rangle\langle e| + \hbar\tfrac{1}{2}\Omega(|e\rangle\langle g| + \mathrm{h.c.})$$

mode

laser driven quantum dot

deformation potential coupling: spin-boson model

- unitary transformation to polaron representation: $B = e^{\eta(b_0 - b_0^\dagger)}$

NMS + QD

$$H = \hbar\omega_0 b_0^\dagger b_0 - \hbar\Delta|e\rangle\langle e| + \hbar\tfrac{1}{2}\Omega\left(e^{\eta(b_0 - b_0^\dagger)}|e\rangle\langle g| + \text{h.c.}\right)$$

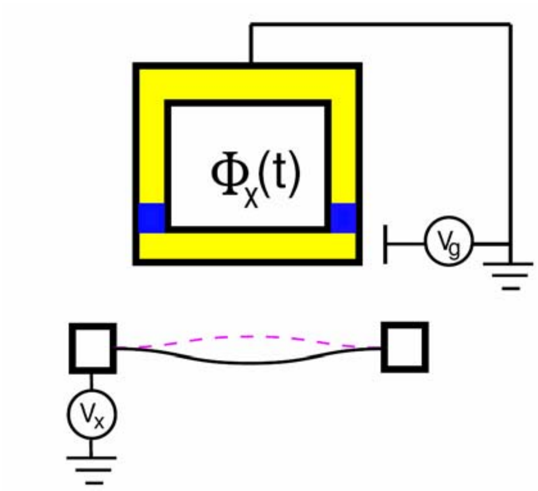
looks like ion trap Hamiltonian with effective Lamb-Dicke parameter (replacing the recoil) : η ~ 0.1

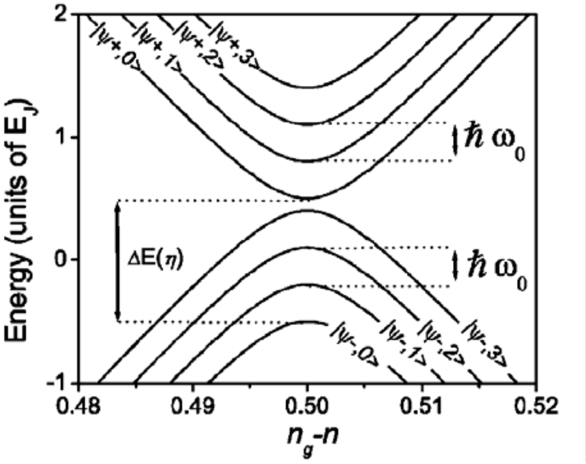
ion trap

$$H = \frac{P^2}{2M} + \frac{1}{2}M\nu^2X^2 - \Delta|e\rangle\langle e| - \frac{1}{2}\Omega(e^{ik_LX}|e\rangle\langle g| + \text{h.c.})$$

$$\equiv e^{i\eta(a+a^\dagger)}$$

- another example: Cooper pair box





cooling: I.Martin, S.Shnirman; L. Tian, …

"cavity QED": K. Schwab et al.