Quantum Theory
Fall 2009

Problem Set 1: *Operator calculus and Spin-1/2*

Problem 1

Consider a physical property (given by an operator) A and two different matrix representations A' and A''. Show that the eigenvalues of A obtained by diagonalizing either A' or A'' are the same.

Problem 2

Consider the ammonia molecule: NH_3. Through quantummechanical tunneling of the nitrogen atom N it can be in two low-energy states $|1\rangle$ and $|2\rangle$ with energy E_0; the possibility of tunneling is described by an interaction energy $-A$. Because the molecule has an electric dipole moment D which has opposite directions for the two states, in an external electric field in this direction, with strength $\mathcal{E}$, the two states acquire dipole energies $\pm D\mathcal{E}$ $(= -\vec{D}\cdot\vec{\mathcal{E}})$. The Hamilton operator of the ammonia molecule in such an electric field then is:

$$\hat{H} = (E_0 + D\mathcal{E})\,|1\rangle\langle 1| + (E_0 - D\mathcal{E})\,|2\rangle\langle 2| - A\,(|1\rangle\langle 2| + |2\rangle\langle 1|)\,,$$

where E_0, A are numbers with the dimension of energy and $\{|1\rangle\,,|2\rangle\}$ is a complete, orthonormal basis set. E_0 can be considered to be large compared to A and $D\mathcal{E}$

(a) Find the exact energy eigenvalues and the corresponding energy eigenkets (as linear combinations of $|1\rangle$ and $|2\rangle$) of the ammonia molecule in an electric field.

(b) Compute in second order perturbation theory the energies for the case of a strong field, $D\mathcal{E} \gg A$.

(c) Compute in second order perturbation theory the energies for the case of a weak field, $D\mathcal{E} \ll A$.

(d) Compare the results of (b) and (c) with the exact results of (a).

Problem 3

Exercise 2.4.3 LB [1]

[1] LB stands for: *Quantum Physics* - M. Le Bellac (Cambridge Univ. Press, 2006); the textbook for the Quantum Theory course.

Problem 4

Exercise 2.4.11 LB

Problem 5

Exercise 3.3.3 LB

Problem 6

Exercise 3.3.4 LB
Note the printing errors on p. 91 of LB: the lower-left matrix elements of σ_x and σ_y under 3. should be $e^{i\alpha_x}$ and $i\,e^{i\alpha_x}$, respectively (these matrices should be hermitian).

Problem 7

A physical property A has only two (nondegenerate) eigenvalues a_1 and a_2 with eigenvectors $|a_1\rangle$ and $|a_2\rangle$, respectively. A general ket $|\psi\rangle$ can be written in the form:

$$|\psi\rangle = \cos\theta\,|a_1\rangle + \sin\theta\,\mathrm{e}^{i\phi}\,|a_2\rangle\,.$$

(a) How can you determine θ from the number of times n_1 and n_2 that the measurement of A gives results a_1 and a_2, respectively? (Assume $n_1 + n_2$ very large).

(b) Can any measurement of quantities which are functions of the operator A (*e.g.* measurement of any matrix element of any function $F(A)$) determine the phase ϕ? Explain. **Hint**: Start by computing $\langle\psi|\,F(A)\,|\psi\rangle$.

(c) Let B be a physical property *incompatible* with A. Show how to determine ϕ from a measurement of $|\langle b_1|\,\psi\rangle|^2$ or $|\langle b_2|\,\psi\rangle|^2$, where the $|b_m\rangle$ are eigenkets of B. Simplify the notation and calculation by using $\langle b_n|\,a_m\rangle = \alpha_{nm}\mathrm{e}^{i\gamma_{nm}}$ (with α_{nm} and γ_{nm} real).