Prof. Dr. J.A. Schouten
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Delft, 29 June 1922

Dear colleague,

By registered mail I return to you the manuscript and the letter from Mr. A. Friedmann, with many thanks for allowing me to see them. The work is certainly of interest, but I think that a link is missing in the proof of the central theorem. It may also just be due to the limited knowledge of Russian possessed by my translator. I would therefore be much obliged if you could look into this point. I am referring to page 14.

I agree with the result

$$\alpha) \qquad (G^i_{rs} - G^i_{sr})\xi^s f_i = \omega f_r \ \text{ if } \ \xi^i f_i = 0,$$

from which it follows that

$$\beta) \qquad (G^i_{rs} - G^i_{sr})\xi^s = 0 \ \text{ for } \ i \neq r \ \underline{\text{and}} \ i \neq s.$$

However, Friedmann writes

$$\gamma) \qquad (G^i_{rs} - G^i_{sr})\xi^s = 0, \ \text{ for } \ i \neq r,$$

which is wrong in my opinion and the source of further errors.

A counter proof is easily provided. Take

$$G^i_{rs} - G^i_{sr} = P_r\delta^i_s - P_s\delta^i_r$$

with P_r an arbitrary vector. Then indeed α) and β) are satified, but not γ, and moreover for this G^i_{rs} the further equations of Fr. do not hold, in particular not the symmetry of G^i_{rs} in r and s.

Also the letter from Fr. to Weyl contains an error. The necessary and sufficient condition for the existence of "flat" hypersurfaces through every point and in every direction is <u>not</u> that the curvature tensor of G^α_{is} should vanish. Counter proof: In every V_n of constant curvature one has $G^\alpha_{is} = \Gamma^\alpha_{is} = \left\{ {is \atop \alpha} \right\}$, so G^α_{is} is then symmetric and the curvature tensor of G^α_{is} is certainly <u>not</u> zero, while still the required flat hypersurfaces exist. One can demonstrate that the necessary and sufficient condition is that all sets of 4 <u>distinct</u> indices of the curvature tensor with respect to one basis (and thus with respect to any basis) vanish.

If you do not object then I would like to write to Fr. about these issues. I hope to speak with you about this on Monday July 3 in Leiden.

With friendly wishes, sincerely yours,

J.A. Schouten