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Editor's Note

The Field of a Single Centre in Einstein 's Theory of Gravitation, and the Motion of a Particle in that Field

By J. Droste. (Communicated by Prof. H. A. Lorentz).
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(*Koninklijke Nederlandsche Akademie van Wetenschappen Proceedings* **19**, 197 (1917)).

If one harbors any doubt that luck, good and bad, plays a significant role in scientific success, that doubt should be dispelled by the republication of Johannes Droste's "Field of a single center in Einstein's theory of gravitation, and the motion of a particle in that field." It is a remarkable paper, arguably one of the most remarkable in the annals of general relativity and yet, although the paper is known to historians of science, practitioners of relativity themselves have been almost universally unaware of its existence for nearly a century, and no mention of it appears in any standard text. Why should this obscure paper by Droste command our attention? If it contained nothing else, one would nevertheless remain startled to find on its opening pages a derivation of the famous Schwarzschild metric, a derivation more transparent than Schwarzschild's own, and announced independently just four months after the presentation of Schwarzschild's solution to the Prussian Academy [1]. That is only the beginning.

We know little about Johannes Droste. From what we do know (see the biographical note), Einstein's theory of gravitation was the subject of his Ph.D. thesis. As he tells us in the introduction to the current paper, he had been working on the equations of motion in general relativity as early as 1913 after Einstein published a preliminary version of the field equations. Droste must have thus been well positioned to take on the final theory once it appeared. From a variational

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principle he first derives the exterior line element for a point mass (Eq. 7), though the derivation is much cleaner than Schwarzschild's. (With the benefit of hindsight we can say that both Schwarzschild and Droste were fortunate to obtain the correct result, since it is well known that applying variational principles after symmetry is imposed frequently results in wrong answers.) In his own paper Schwarzschild then recovers the perihelion shift of Mercury but otherwise abandons orbital analysis in favor of the interior solution, which appeared in his next paper [2]. Droste, on the other hand, provides a complete analysis of orbital motion in what today would be termed a black-hole background. Indeed, his first step (Eq. 8) is to introduce the "Regge-Wheeler" tortoise coordinate, which he then exploits in several instances further on. Some of Droste's conclusions strike us as startling, anticipations of results that were apparently well understood only decades later. For example, in examining radial orbits he states (below Eq. 16) that the acceleration of a test particle is greatest at $r = 3M$, which corresponds to the peak of the effective potential for a Schwarzschild black hole. By examining the derivatives of the tortoise coordinate he arrives at the conclusion (above Eq. 21 and Eq. 27) that it takes in infinite amount of "Schwarzschild" time to reach $r = 2M$. He has even managed to do this for general orbits, whereas today texts only present the radial case. (Judging from his remarks below Eq. 8, however, he did not understand that the infinite time to reach $r = 2M$ is merely a coordinate effect.) After an exhaustive analysis of the general orbital equation (Eq. 27) Droste finally turns to the perihelion shift of Mercury and retrieves the result found earlier by Einstein and Schwarzschild.

It is not difficult to guess why Droste's paper never received the attention it deserved. Schwarzschild's paper was announced to the Prussian Academy of Sciences on January 13th, 1916 by Einstein himself; Droste's paper was communicated to the Royal Dutch Academy on May 27 by Lorentz. Schwarzschild had been director of the Astrophysical Observatory in Potsdam; Droste was an unknown Ph.D. student. Furthermore, the implications of some of Droste's conclusions, such as the infinite time required to reach $r = 2M$, could not have been clear at the time. We can grant Schwarzschild priority, but given that Nordström rediscovered the charged black-hole solution 2 years after Reissner, and that the successive rediscoveries of the standard model by Lemaître, Robertson and Walker were spread out over a period of 13 years after Friedmann, it would seem appropriate to begin referring to the Schwarzschild-Droste black hole, even 100 years too late.

REFERENCES

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Short Biography

Johannes Droste. Born in Grave (The Netherlands) on 28 May 1886. He studied with Professor H.A. Lorentz in Leiden and received a Ph.D. in 1916, on the thesis "The gravitational field of one or more bodies according to the theory of Einstein". (The thesis is in Dutch.) As far as I am aware, this is the only Ph.D. research supervised by Lorentz on the topic of general relativity. From 1914 to 1919 Droste taught mathematics at a Gymnasium in Gorkum. He then joined the faculty of Leiden University, becoming professor of mathematics in 1930. (The route from high school teacher to university professor was not uncommon in those days; the career of Lorentz had followed the same path.)

Droste's activities as a professor were devoted to the teaching of mathematical analysis. He did not continue his research in physics, beyond a few contributions on mathematical points in the theory of elasticity and thermodynamics (published in the proceedings of the Dutch Royal Academy). He died in Leiden on 16 September 1963.

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