ON THE DYNAMICS OF SPIN GLASSES

A Theory of Aging

Ger Koper
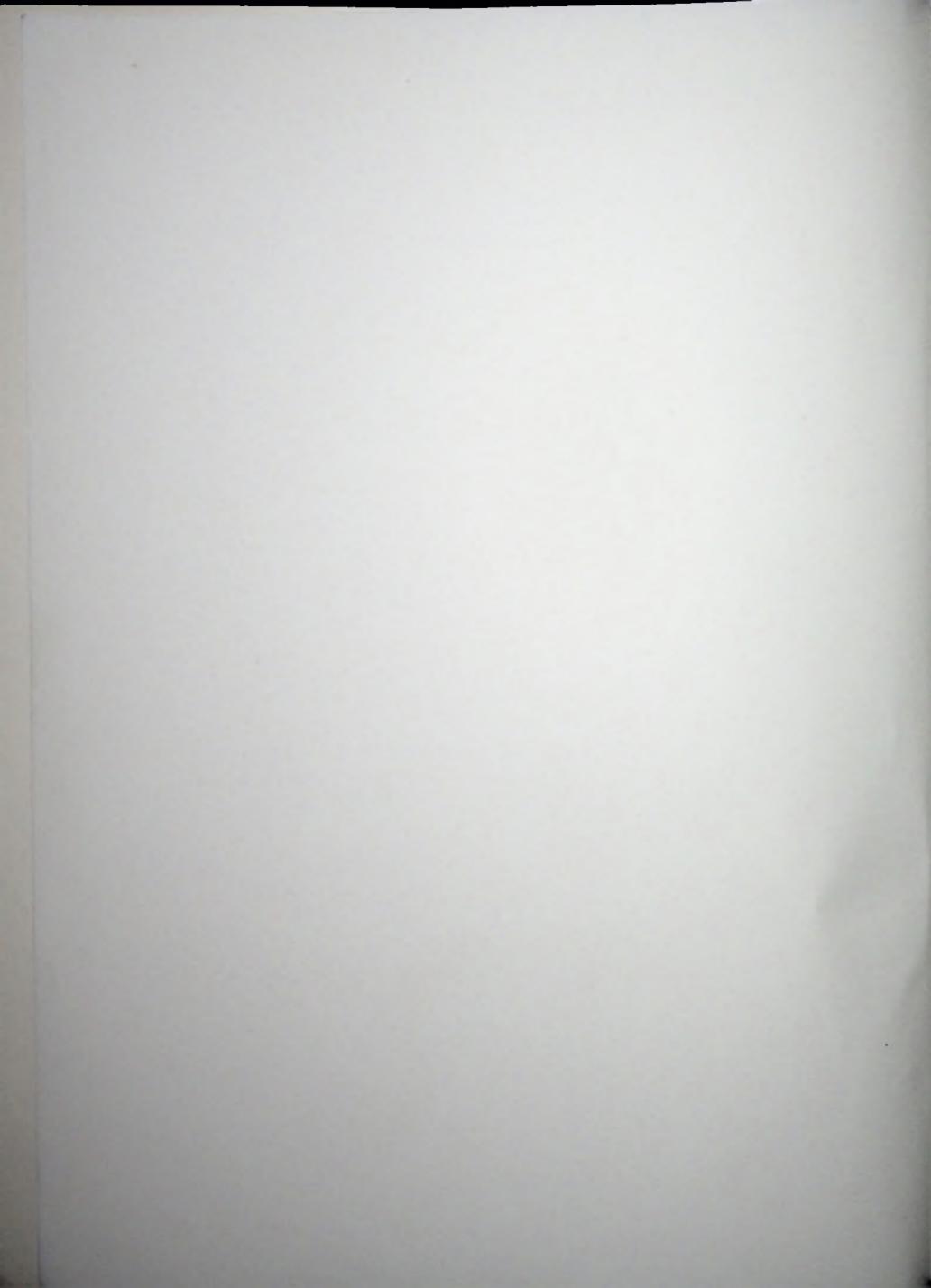
ON THE DYNAMICS OF SPIN GLASSES

A Theory of Aging

ON THE DYNAMICS OF SPIN GLASSES

A Theory of Aging

ON THE DYNAMICS
OF SPIN GLASSES

A Theory of Aging

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR AAN
DE RIJKSUNIVERSITEIT TE LEIDEN, OP GEZAG VAN DE
RECTOR MAGNIFICUS DR. J.J.M. BEENAKKER, HOOG-
LERAAR IN DE FACULTEIT DER WISKUNDE EN NATUUR-
WETENSCHAPPEN, VOLGENS BESLUIT VAN HET COL-
LEGE VAN DEKANEN TE VERDEDIGEN OP WOENSDAG
17 JANUARI 1990 TE KLOKKE 16.15 UUR

DOOR

GERARDUS JOSEPH MARIA KOPER

GEBOREN TE AMSTERDAM IN 1954

Contents

To Marvin VanDilla and Joe Gray
who talked me into this.

Contents

Chapter I

Spin glass theory:
Introduction and Survey

The purpose of this chapter is twofold. On the one hand it serves as an introduction to spin glasses, in particular to the more recent theoretical developments. On the other hand it surveys the subjects as treated — in more detail — in the subsequent chapters.

1 Characterization of spin glasses

The archetypal spin glasses are dilute magnetic alloys consisting of a noble metal, such as Cu, weakly diluted with transition metal ions such as Mn. The impurity concentrations range from 1 to 10%. For much lower concentrations, $c \ll 1\%$, the impurities can be considered as isolated and this relates to the Kondo problem. At higher concentrations these systems become magnetically ordered at low temperatures, e.g. CuMn orders antiferromagnetically.

The dominant contribution to the interaction between the magnetic ions is due to the scattering of the conduction electrons at the spins of the impurities. This indirect exchange interaction $J(r)$ oscillates strongly with distance r and is given by

$$J(r) \simeq J_0 \frac{\cos(k_F r + \phi_0)}{(k_F r)^3} \qquad (k_F r \gg 1), \tag{1.1}$$

where J_0 and ϕ_0 are constants and where k_F is the Fermi wavenumber of the host metal [1,2,3].

The r^{-3}-dependence of this Rudermann-Kittel-Kasuya-Yosida-interaction (RKKY) has the interesting consequence that the free energy of the impurity system obeys the scaling relation [4]

$$F \sim c^2 \mathcal{F}\left(\frac{T}{c}, \frac{H}{c}\right), \tag{1.2}$$

with temperature T, magnetic field H, and concentration c. For the scaling function $\mathcal{F}$ there is no microscopic theory but it follows, in principle, from a determination at one particular concentration. From the scaling relation (1.2) universal curves can be derived for e.g. the magnetic susceptibility and the specific heat as a function of temperature [5].

Another aspect of the RKKY-interaction is that it changes sign over distances of the order of the lattice spacing. Since the distance between the impurities is a random quantity the sign of the interaction between impurities is also random: either ferromagnetic or antiferromagnetic. This, in turn, leads to a high probability for frustrated loops of connected spins in the system. A loop of connected spins is frustrated if the product of the signs of the connecting bonds is negative: there is no spin configuration that satisfies all these bonds [6].

On the experimental side there is no single criterion that defines a spin glass, but there are some general properties that more or less identify systems as such:

(i) The dynamic susceptibility $\bar{\chi}(\omega; T)$ for small oscillating magnetic fields cusps at a temperature T_f which only weakly depends on frequency ω. The "sharpness" of the cusp may increase when the magnetic field amplitude is reduced [7]. An additional constant magnetic field also affects the shape of the cusp.

(ii) There are no Bragg reflections in neutron diffraction patterns; this indicates the absence of long range ferromagnetic or antiferromagnetic order [8].

(iii) If feasible, as for instance in AuFe, Mössbauer spectra show a splitting of the hyperfine line at the cusp temperature T_f. This is a clear proof of the formation of static or quasistatic internal fields at temperatures below T_f and indicates the freezing-in of the spins [9].

(iv) The magnetic contribution to the specific heat shows, for low temperatures, linear temperature dependence with only a broad maximum at a temperature well above T_f [10].

(v) Around and below T_f magnetization and susceptibility depend strongly on "history" and relax on macroscopic time scales [11].

The linear specific heat dependence on temperature is characteristic of many "glassy" systems (see section 4) and can be explained by considering the system as an ensemble of subsystems with only two accessible energy levels [12]. Such a theory, however, cannot account for the cusp in the dynamic susceptibility. On the other hand this same cusp, and the formation of local moments, are arguments in favour of a phase transition at T_f. But if there is a phase transition then it is an unfamiliar one without long range ferromagnetic or antiferromagnetic order and with no anomaly in the specific heat.

2 Mean field theory

2.1 The Edwards-Anderson model

The first milestone in the development of spin glass theory is the work of Edwards and Anderson (EA) [13]. They considered the Heisenberg Hamiltonian

$$\mathcal{H} = - \sum_{i,j} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j - \mathbf{H} \cdot \sum_i \mathbf{s}_i, \qquad (2.1)$$

where i and j are sites of a regular d-dimensional lattice and where the spins are considered as classical vectors pointing in directions $\mathbf{s}_i$. In a spin glass with substitutional disorder (such as CuMn) the coupling constants should be written as $J_{ij} = \varepsilon_{ij} J(r_{ij})$ with $\varepsilon_{ij} = 1$ if both sites i and j are occupied and $\varepsilon_{ij} = 0$ otherwise. The coupling strength $J(r_{ij})$ is, at least for the case of CuMn, given by the RKKY-interaction (1.1) that varies with spin spacing r_{ij}. EA chose a simpler model where the coupling constants J_{ij} are independent random variables drawn from the Gaussian probability distribution

$$P(J_{ij}) = \frac{1}{cJ\sqrt{2\pi}} \exp\left(-\frac{J_{ij}^2}{2c^2 J^2}\right), \qquad (2.2)$$

where J is the typical coupling strength and c is the impurity concentration.

The experimental evidence that below a well-defined temperature T_f the spins freeze in random directions [9] led EA to consider the order parameter

$$q = \langle \mathbf{s}_i^{(1)} \cdot \mathbf{s}_i^{(2)} \rangle, \qquad (2.3)$$

3

where $s_i^{(1)}$ and $s_i^{(2)}$ are two observations of the same spin at widely seperated instants of time. The angular brackets, $\langle \ldots \rangle$, denote an average over thermal fluctuations. One expects $q = 0$ for $T \geq T_f$ and $q \neq 0$ voor $T < T_f$, even $q = 1$ for $T = 0$.

The thermodynamic properties of systems described by a Hamiltonian $\mathcal{H}$ as a function of temperature T follow from of the free energy

$$F = -T \log Z \tag{2.4}$$

(Boltzmann's constant is taken to be unity, $k_B = 1$) where Z is the partition function

$$Z = \text{Tr} \, e^{-\beta \mathcal{H}} \tag{2.5}$$

with $\beta = T^{-1}$. A problem that arises here is that the Hamiltonian of Eq. (2.1) is a random quantity due to the randomness of the bonds that obey the probability distribution (2.2). One seems to have the choice of either averaging the partition function Z over disorder (which is easy), or the free energy F (which is hard). The coupling constants are to be considered as "quenched" random variables with their values fixed for a particular sample. One should therefore take the average of the free energy, which represents the behaviour of a *typical* system.

EA introduced the replica method to calculate the free energy. The idea is the following. One considers n independent copies (replicas) of the same realization of the system and calculates the total partition function

$$Z^n = \int \prod_i ds_i^{(1)} \int \prod_i ds_i^{(2)} \cdots \int \prod_i ds_i^{(n)}$$

$$\times \quad \exp \left[\beta \sum_{a=1}^n \left\{ \sum_{ij} J_{ij} s_i^{(a)} \cdot s_j^{(a)} + \mathbf{H} \cdot \sum_i s_i^{(a)} \right\} \right]. \tag{2.6}$$

The average of the free energy can be expressed as

$$\overline{F} = -T \, \overline{\log Z} = T \lim_{n \to 0} \frac{\overline{Z^n} - 1}{n}, \tag{2.7}$$

where the overbar $\overline{\ldots}$ denotes the disorder average. In principle it is not rigorous to take the limit of $n \to 0$ as Eq. (2.6) allows one to calculate Z^n only for integral values of n (in practice it is easy since one only keeps terms linear in n). Nevertheless, that is what EA did and they found a transition temperature T_c where the susceptibility has a cusp. The cusp is smoothened when a magnetic field is applied. Unlike in real spin glasses, the specific heat also cusps at T_c.

The order parameter (2.3), but now with $s_i^{(1)}$ and $s_i^{(2)}$ (as suggested by the notation) taken from different replicas in (2.6), plays a particular role in their calculation of the free energy. The transition temperature T_c is defined as the temperature where q vanishes. The low temperature ("spin glass") phase is characterized by a nonzero order parameter q, whereas the total magnetization vanishes in the absence of a magnetic field.

It was Blandin [14] who established the equivalence of the order parameter (2.3) and the expression

4

$$q_{EA} = \frac{1}{N} \sum_{i=1}^{N} \overline{\langle s_i \rangle^2}, \qquad (2.8)$$

which is usually referred to as the EA order parameter.

2.2 The Sherrington-Kirkpatrick model

The work of EA is not fully exact in the sense that approximations are made that cannot be rigorously justified. The development of a full-fledged mean field theory was initiated by Sherrington and Kirkpatrick (SK) [15,16]. They considered a system consisting of N Ising spins, $s_i = \pm 1$, with Hamiltonian

$$\mathcal{H} = - \sum_{1 \le i < j \le N} J_{ij} s_i s_j - H \sum_{i=1}^{N} s_i, \qquad (2.9)$$

where the coupling constants J_{ij} are independent, identically distributed random variables drawn from a Gaussian probability distribution with zero mean and variance N^{-1}. The scaling of the typical coupling strength (i.e. the variance of the Gaussian distribution) with the system size N guarantees the existence of the thermodynamic limit.

SK used the replica method introduced by EA to calculate the free energy of the model. After the disorder average of the n-fold partition function (2.6) one has

$$\overline{Z^n} = \sum_{\{s^1\}} \sum_{\{s^2\}} \cdots \sum_{\{s^n\}} e^{-\beta \mathcal{H}_n} \qquad (2.10)$$

with

$$\mathcal{H}_n \equiv -\frac{\beta}{2N} \sum_{a,b=1}^{n} \sum_{i,j=1}^{N} s_i^a s_i^b s_j^a s_j^b - H \sum_{a=1}^{n} \sum_{i=1}^{N} s_i^a, \qquad (2.11)$$

and where $\{s^a\}$ denotes all possible configurations of the spins $s_i^a, i = 1, 2, \ldots, N$.

The natural variational parameters that appear in the calculations are the elements of the matrix

$$Q^{ab} \equiv \langle s_i^a s_i^b \rangle_n \quad (a \ne b) \qquad (2.12)$$

where

$$\langle X \rangle_n \equiv \sum_{\{s^1\}} \sum_{\{s^2\}} \cdots \sum_{\{s^n\}} X e^{-\beta \mathcal{H}_n} / \overline{Z^n}. \qquad (2.13)$$

A priori, the replicas are identical so SK postulated that the matrix Q^{ab} should take the form

$$Q^{ab} = Q \quad (a \ne b). \qquad (2.14)$$

With this choice the variational free energy per replica becomes, in the limit $n \to 0$,

$$-\beta F_Q(T, H) = \frac{\beta^2(1-Q)^2}{4} + \int\limits_{-\infty}^{\infty} \frac{dz}{\sqrt{2\pi}}\, e^{-z^2/2} \log\left[2\cosh\left(\beta z\sqrt{Q} + \beta H\right)\right] \quad (2.15)$$

and Q is determined by the requirement $\partial F_Q/\partial Q = 0$. In zero magnetic field there is the solution $Q(T, 0) = 0$ for $T > T_c = 1$, but for $T < T_c$ there is another solution where $Q(T, 0) \neq 0$. For nonzero magnetic field there is no transition. From the free energy SK found cusps at T_c for the susceptibility, as occurs in reality, and for the specific heat, which does not occur in real spin glasses. The magnetization is zero in the absence of a magnetic field for all temperatures.

There are problems with this solution for $T < T_c$ and the most important one is that the zero temperature entropy per spin $S(0) = -1/2\pi$ whereas entropy, being the logarithm of an integer number, should be nonnegative.

A more careful analysis performed by De Almeida and Thouless (AT) [17] showed that the replica symmetric solution is only stable beyond the "Almeida-Thouless-line", i.e. for temperatures $T > T_c(H)$. One has $T_c(0) = 1$ and $T_c(H) \to 0$ for $|H| \to \infty$. Where it is stable the replica symmetric solution is exact and has been verified by other methods. For temperatures $T \leq T_c(H)$ the replica symmetric solution is invalid and in order to find a solution replica symmetry has to be broken [17].

In 1979 Parisi suggested a scheme for replica symmetry breaking that led to a stable solution for temperatures below the AT-line [18]. The structure of Parisi's matrix, for the example of $n = 8$, is sketched in Fig. 1. It involves a sequence of

$$(Q^{ab}) = \begin{pmatrix} 0 & Q_0 & Q_1 & Q_1 & Q_2 & Q_2 & Q_2 & Q_2 \\ Q_0 & 0 & Q_1 & Q_1 & Q_2 & Q_2 & Q_2 & Q_2 \\ Q_1 & Q_1 & 0 & Q_0 & Q_2 & Q_2 & Q_2 & Q_2 \\ Q_1 & Q_1 & Q_1 & 0 & Q_2 & Q_2 & Q_2 & Q_2 \\ Q_2 & Q_2 & Q_2 & Q_2 & 0 & Q_0 & Q_1 & Q_1 \\ Q_2 & Q_2 & Q_2 & Q_2 & Q_0 & 0 & Q_1 & Q_1 \\ Q_2 & Q_2 & Q_2 & Q_2 & Q_1 & Q_1 & 0 & Q_0 \\ Q_2 & Q_2 & Q_2 & Q_2 & Q_1 & Q_1 & Q_0 & 0 \end{pmatrix}$$

Fig. 1. Structure of Parisi's matrix for $n = 8$.

decreasing coefficients $Q_0, Q_1, \ldots$ that, in the limit $n \to 0$, changes into a monotonic "order parameter function" $Q(x)$ with $0 \leq x \leq 1$. For temperatures $T > T_c(H)$ the order parameter function $Q(x) = 0$ identically. This leads again to the replica symmetric solution. For $T < T_c(H)$ one has $Q(x) \neq 0$ with a maximum value $Q_M = Q(1)$ (which is identical to the EA order parameter (2.8)) that tends to 1 for $T \to 0$.

It was also Parisi who in 1983 gave a physical interpretation of the order parameter function $Q(x)$ [19]. This picture, which now is widely accepted, is that below

the AT-line the spin glass has an infinity of phases. Each phase α is identified by a set of local magnetizations $m_i^\alpha = \langle s_i \rangle^\alpha$, where $\langle \ldots \rangle^\alpha$ denotes a thermodynamic average in the phase α. The free energy of a phase α is given by

$$F_\alpha = N\bar{f} + f_\alpha, \tag{2.16}$$

where $\bar{f}$ and f_α are of order N^0. Eq. (2.16) expresses the fact that all the phases have the same thermodynamic properties as determined by $\bar{f}$. Mézard et al. showed that the phase free energies f_α are to be considered as independent random objects [20] with a distribution that depends on temperature. From Eq. (2.16) one has for the probability that the system is in the phase α

$$P_\alpha = \frac{e^{-\beta f_\alpha}}{\sum_\gamma e^{-\beta f_\gamma}} \tag{2.17}$$

so that the thermal average of an observable O can be expressed as

$$\langle O \rangle = \sum_\alpha P_\alpha \langle O \rangle^\alpha. \tag{2.18}$$

It is instructive at this stage to point out an analogy with the low temperature ferromagnet where there are only two phases, labelled $+$ and $-$. One would write for the average magnetization

$$\langle s_i \rangle = \tfrac{1}{2}\langle s_i \rangle^+ + \tfrac{1}{2}\langle s_i \rangle^- = \tfrac{1}{2}(m - m) = 0. \tag{2.19}$$

Mézard et al. [21] showed that the set of the phases α is ultrametric. To demonstrate this one defines the *overlap* $q^{\alpha\beta}$ of two phases α and β by

$$q^{\alpha\beta} = \frac{1}{N} \sum_{i=1}^{N} m_i^\alpha m_i^\beta. \tag{2.20}$$

The self overlap $q^{\alpha\alpha}$ is the previously defined EA order parameter, see Eq. (2.8). For three different phases α, β, and γ one either has $q^{\alpha\beta} = q^{\beta\gamma} = q^{\gamma\alpha}$ or two of them are equal and one is larger. This is the ultrametric property (see e.g. [22]). The relation between the order parameter function $Q(x)$ and the overlap $q^{\alpha\beta}$ is an intricate one which will not be discussed here, but see the references of Section 5.

The question whether the Parisi solution is unique has not yet been answered. Also the lower critical dimension, below which there is no longer a phase transition, is unknown. The upper critical dimension, below which the mean field picture is no longer valid, is generally believed to be six [23].

2.3 Dynamics in the mean field model

The SK-model deals with Ising spins that have no natural intrinsic dynamics. One way of introducing a time-dependence in this model is by means of Glauber dynamics [24]. An other one is to introduce a "soft spin" version of the model in which the spins s_i are continuous variables submitted to a double well symmetric potential which strongly favors $s_i = \pm 1$, the Ising model being recovered in a suitable

limit. Relaxational dynamics is then introduced through a Langevin equation with a Gaussian noise term [25,26].

The results can be summarized as follows: Above the AT-line ($T > T_c(H)$) the relaxation is exponential with a relaxation time that diverges as $T \to T_c(H)$. Below the AT-line one distinguishes relaxation within one phase, which is nonexponential, and the relaxation associated with the "hopping" of the system between the phases. The latter appears to occur on timescales that diverge with system size. For instance Mackenzie and Young [27,28] estimate for this timescale τ

$$\log \tau \sim N^{\frac{1}{4}}. \tag{2.21}$$

One often assumes that, since the dynamics of real spin glasses appears to occur on macroscopic timescales, the dominant process in the relaxation can be identified as the hopping between the phases. This led to the studies of models with a hierarchical distribution of timescales [29] and hopping dynamics in ultrametric spaces [30,31,32,33].

Another approach is to study a system of states that have random free energies. The simplest possible model of this kind has been introduced by Derrida [34,35] and it consists of a set of independent, Gaussian distributed energy levels. It has several important features in common with the SK model below the AT-line. In particular it is known (see Eq. (2.16)) that in both models the (free) energies of the lowest lying states are random variables with an exponential distribution.

De Dominicis et al. [36] were the first to consider the kinetics of a system of exponentially distributed random energy levels. They postulated a master equation for the occupation probabilities of the energy levels and for one particular choice of the transition rates (for going from one level to another) they found stretched exponential relaxation. Some time later Koper and Hilhorst [37] showed that this same model, with slightly modified transition rates, also exhibits power law relaxation. In Chapter II a study is presented that deals with a more general class of transition rates for which it is possible to obtain analytic results. In all cases considered either power law or stretched exponential decay is found for the asymptotic behaviour of the equilibrium autocorrelation function. All exponents are temperature dependent. From this study it can be concluded that in order to obtain nonexponential relaxation no hierarchical tree structure is needed, let alone ultrametricity; the fact that the level energies are random suffices.

3 Finite-dimensional spin glasses

The solution of the SK-model reveals a rich structure: below the critical temperature $T_c(H)$ the system condenses into a "non-ergodic" state characterized by an infinite number of pure phases unrelated by symmetry. It is important to know whether this complex ordered phase persists when one goes from a mean field ($d = \infty$) model to real (three-dimensional) spin glasses and indeed whether real spin glasses exhibit an equilibrium phase transition at all. One possibility to answer these questions is to take the solution of the SK-model as a starting point,

and to obtain the properties of finite-dimensional systems by systematic expansion around the SK-model. Such a program is technically rather difficult but nevertheless De Dominicis and Kondor have made considerable progress, the latest result being a continuation of spin glass field theory below the upper critical dimension $d = 6$ [38]. An alternative approach to the study of finite-dimensional spin glasses is referred to as "zero-temperature scaling": a self-consistent, phenomenological, theory of the ordered phase.

3.1   Zero-temperature scaling

Central to this theory is the idea that one can understand the ordered phase of a spin glass, which at least should exist at zero temperature, from its ground state properties. A key role is played by an exponent y which is related to the sensitivity of the ground state energy to changes in the boundary conditions and which can be determined in the following way: consider a hypercubic spin system (in d-dimensions) of linear size L with periodic boundary conditions in all directions. Its ground state energy is denoted by E_p. Next impose anti-periodic boundary conditions in one direction; the new ground state energy is denoted by E_a. For the case of an Ising ferromagnet it is immediately clear that $E_L = (E_a - E_p)/2$ is the energy that is associated with the interface ("domain wall") introduced by the anti-periodic boundary condition so that it scales with linear dimension L as $E_L \sim L^{d-1}$. For spin glasses one also finds interface energies scaling as $E_L \sim L^y$ and obviously $y \leq d - 1$ as a consequence of the frustration inherent to these systems. When the exponent y is negative there can be no ordered phase for any positive temperature T since arbitrarily large connected clusters ("domains"), which cost arbitrarily small energy, will be thermally excited. So the sign of y determines whether the system is ordered for nonzero temperature or not.

Both McMillan [39,40] and Bray and Moore [41,42] have numerically determined the exponent y for Ising spin glass models with a Gaussian bond distribution centered around zero. For $d = 2$ they found $y \approx -0.3$ and for three dimensions $y \approx 0.2$. Thus there should be no phase transition (except for $T = 0$) in two-dimensional Ising spin glasses, whereas three-dimensional systems should exhibit a phase transition. These predictions are supported by Monte-Carlo studies by Bhatt and Young [43] and by Ogielski [44].

In the framework of the renormalization group (RG) theory (see for instance [45]) the energy of the interface is regarded as a scale-dependent width of the distribution of effective coupling constants at length scale L: $J_L \sim J L^y$ where J is the typical, microscopic, bond strength. The scale-dependent coupling constant J_L scales to infinity ("strong coupling") with increasing L if $y > 0$ and to zero ("weak coupling") if $y < 0$. This is consistent with the requirement $y > 0$ for an ordered phase at low temperatures.

Physical properties at nonzero temperature depend on dimensionless ratios like J/T so it makes sense to reinterpret the increase (decrease) of the characteristic coupling with increasing length scale as a decrease (increase) of the temperature
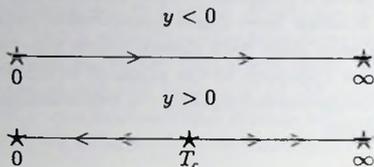
Fig. 2. Schematic RG flows. For $y > 0$ the zero temperature fixed point is stable whereas for $y < 0$ it is unstable. This implies an ordered phase only for $y > 0$.

at fixed coupling strength. From this point of view an initially small temperature decreases under coarse graining for $y > 0$, and increases for $y < 0$: there is a stable zero temperature fixed point for $y > 0$. Since for $T \to \infty$ the system loses order at longer length scales there must, for $y > 0$, be a critical temperature T_c: an unstable fixed point of the RG. Schematic flows are depicted in Fig. 2.

The exponent y is associated with the physical properties at or near zero temperature — hence the name "zero-temperature scaling". Indeed all quantities that are dominated by large scale excitations can be described by exponents related to y. Examples are the decay of the correlations in the spin glass phase [46]:

$$\overline{(\langle s_i s_{i+r} \rangle - \langle s_i \rangle \langle s_{i+r} \rangle)^2} \sim r^{-y} \qquad (r \to \infty), \tag{3.1}$$

and the dependence of the singular part of the magnetization on the magnetic field [42]

$$M_{sing} \sim H^{\frac{d}{d-2y}} \qquad (H \to 0). \tag{3.2}$$

3.2 The droplet model

Based on the results of the domain wall studies by MacMillan [39,40] and by Bray and Moore [41,42] Fisher and Huse (FH) have developed a "droplet"-picture of the ordered spin glass phase [46,47,48]. This discussion will be limited to Ising systems.

A definition of a *droplet* in a ground state can be given as follows:
One picks a particular site, say i, one chooses a length scale L, and considers all the connected clusters of L^d spins that contain i. With each such cluster there is associated an energy that is the difference between the energy of the ground state with and without the spins of the cluster flipped. Those clusters with lowest energy are called droplets and FH assume that their typical energy E_L scales as $E_L \sim L^y$ as described above.

The basic hypothesis of FH is that the droplets are the low-lying excitations from the ground state. For small positive temperatures T the behaviour of the

10

ordered phase (which only exists for $y > 0$) is dominated by the "thermally active" droplets, i.e. those droplets with energies not greater than T. For general positive temperatures FH argue that the properties of the spin glass phase can still be understood in terms of such droplets that now have a typical *free* energy F_L which also scales as $F_L \sim L^y$.

The static properties of the spin glass phase as derived from the droplet picture are none other than those given by the zero-temperature scaling theory discussed above (see Eqs. (3.1) and (3.2)).

The two main features of the ordered spin glass phase, as emerge from FH's droplet picture, are

- At any given temperature below the ordering temperature T_c (which FH assume positive for $d = 3$) the system has only two pure equilibrium states which are simply related by global spin reversal. In such a state each spin s_i has a nonzero expectation value $\langle s_i \rangle$ and thus the EA order parameter (2.8) is nonzero.

- In a nonzero magnetic field, which breaks the global spin flip symmetry, the spin glass phase is destroyed.

This contrasts with the mean field theory, which predicts an infinity of states in the spin glass phase where the latter exists for all temperatures below the AT-line $T_c(H)$.

In order to deal with the dynamical behaviour of the ordered phase a new hypothesis is required. FH follow MacMillan [39] who assumed that the relaxation takes place via *thermally activated processes* over free energy barriers. They conjecture that the typical free energy barriers B_L associated with the reversal of droplets of linear size L scales as

$$B_L \sim L^\psi \tag{3.3}$$

with $y \leq \psi \leq d - 1$ a new exponent. From this assumption they derive logarithmic decay for the time correlations

$$\overline{\langle s_i(0)s_i(t)\rangle - \langle s_i(0)\rangle^2} \sim (\log t)^{-y/\psi} \quad (t \to \infty) \tag{3.4}$$

and a magnetic noise spectrum $S(\omega)$ that behaves as

$$S(\omega) \sim 1/\omega |\log \omega|^{(1+y/\psi)} \quad (\omega \to 0) \tag{3.5}$$

yielding $1/f$-noise up to logarithmic corrections.

3.3 The chaotic nature of the ordered phase

If one had to estimate the energy associated with the reversal of a droplet of L^d spins one would guess that it should scale as $L^{d_s/2}$ being the sum of order L^{d_s} random bond contributions of uncorrelated sign, with d_s the fractal dimension of the surface area of the cluster. Ofcourse $d - 1 \leq d_s < d$. This estimate is much larger that the

actual free energy of order L^y which implies that large cancellations from different parts of the interface take place. Such cancellations should be sensitive to changes in temperature T implying that the relative orientations of spins sufficiently far apart can change with arbitrary small changes in temperature [46].

A simple Imry-Ma-style argument [49] will be given not for temperature changes but for small changes in bond strength: Consider an Ising spin glass with a continuous bond strength distribution of width J. A low energy excitation from the ground state, involving an overturned droplet of linear dimension L, costs an energy of order L^y. Now a small random perturbation of typical size ΔJ is added to each bond. If the ground state remains unchanged, the contribution to the droplet energy is of order $\Delta J L^{d_s/2}$. Hence, provided $\zeta \equiv \frac{1}{2}d_s - y$ is positive, the ground state will be unstable to the perturbation on length scales $L \gtrsim \ell_{\Delta J}$ with

$$\ell_{\Delta J} \sim \left(\frac{J}{\Delta J}\right)^{\frac{1}{\zeta}} \tag{3.6}$$

The relative orientations of spins separated by more than $\ell_{\Delta J}$ will be strongly affected. This sensitivity is a fundamental property of spin glasses as one has $\zeta > 0$ due to $d_s \geq d - 1$ and $y \approx 0.3$ for $d = 3$ and $y \approx -0.2$ for $d = 2$. The behaviour (3.6) has been numerically confirmed for $d = 2$.

For temperature changes ΔT Bray and Moore constructed a similar argument [50] which gives

$$\ell_{\Delta T} \sim |\Delta T|^{-1/\zeta}. \tag{3.7}$$

Hence, for any change of temperature ΔT, however small, there exists a length scale $\ell_{\Delta T}$ beyond which the equilibrium state becomes unstable.

It is interesting to point out a correspondence between this result and Parisi's solution of the SK-model: both show a chaotic dependence of the spin correlations on temperature. This correspondence has recently been discussed by Kondor [51].

4 Aging

A system is said to be in thermodynamic equilibrium when all of its physical observables are stationary with respect to time. The properties of a thermodynamic state depend on a set of thermodynamic variables, e.g. temperature and magnetic field for a magnetic system. After a change of these thermodynamic variables, such as a sudden reduction of temperature or magnetic field, it takes some time before the new thermodynamic state is reached. For many systems in nature this equilibration time is very short, of the order of seconds or less. But there are marked exceptions where the equilibration time can (depending on the circumstances) be much longer than experimentally accessible observation times. When, during the equilibration process, the system appears to be in equilibrium on experimental time scales (i.e. when many observables appear to be stationary) the system is said to undergo *aging*.

12

The best known systems that exhibit aging are the glasses of which vitreous silica (window glass) is a familiar example. Glasses are to be regarded as solidified supercooled liquids that undergo extremely slow processes which attempt to establish equilibrium. It was the *glassy* behaviour that led Coles, at the end of the 60's, to coin the name "spin glass" for the random spin systems such as CuMn [52].

Detailed studies of systems that undergo aging, in particular performed by Struik on amorphous polymers [53] and by Lundgren et al. on spin glasses [54], have firmly established that, during the aging process, linear response relations hold provided that the perturbations are sufficiently small. For spin glasses this means that for instance the magnetization per spin $m(t)$ responds linearly to the applied magnetic field $H(t)$:

$$m(t) = m(0) + \int_0^t dt' \, \chi(t, t - t') H(t - t') \quad (t > 0). \tag{4.1}$$

The *magnetic response function* $\chi(t, t - t')$ gives the response at time t to a unit magnetic field impulse at time $t - t'$. Causality implies that $\chi(t, t - t')$ vanishes for $t' < 0$. The time instant $t = 0$ *cannot* be chosen arbitrarily (the system is not stationary with respect to time): it is the instant of time on which the system was "born", i.e. for spin glasses it is the moment on which the quench into the spin glass phase took place. In thermodynamic equilibrium the response function depends only on the time difference t' and not on the absolute time t.

A system that undergoes aging exhibits *waiting time effects* or *memory effects*: the behaviour of such a system depends on its *history*, i.e. on what happened to it from the moment that it was born. The dependence of the response functions of an aging system on absolute time accounts for such phenomena. The simplest demonstration of the waiting time effect in spin glasses is done with magnetization relaxation: At time $t = 0$ one quenches the system into the spin glass phase while keeping the magnetic field at a constant small value (e.g. 10 Gauss). One then waits for some time t_w and at the instant $t = t_w$ one cuts the magnetic field and from then on one records the decay of the magnetization. If one repeats the experiment for different values of the waiting time (experimental values range from 1 second to several days) one finds that the observed magnetization profiles differ with waiting time: the decay becomes *slower* as the waiting time t_w gets larger. Moreover, at the absolute time $t \approx 2t_w$ a change in the decay rate of the magnetization takes place which shows up as an inflection point in log-log plots of magnetization versus time. It is because of the latter effect that sometimes the term "memory effect" is used.

Linear magnetic response of a spin glass can only be observed for very small magnetic field changes (10 Gauss or smaller). For larger values nonlinearities come into play. These nonlinearities have nothing to do with those of the static susceptibility but rather manifest themselves in the dynamics. The studies of Alba et al. [55], Nordblad et al. [56] and Refregier et al. [57] are concerned with these dynamical nonlinearities and their discovery made clear that there was a need for

a theory of, in particular, nonlinear aging phenomena. In 1988 Koper and Hilhorst presented a phenomenological theory for aging in spin glasses [58]. Not long after, and independently, Fisher and Huse presented a similar theory [59] that puts more emphasis on the activated dynamics of the droplets (see section 3.2). The theory, in the language of Koper and Hilhorst, is presented in full detail in Chapter III. A concise description follows in the next subsection.

4.1 Domain theory

For definiteness the discussion will be focussed on a spin glass model which consists of Ising spins on a simple d-dimensional lattice. The Hamiltonian is given by

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j - H \sum_i s_i, \tag{4.2}$$

where the first sum runs over all neighbouring sites on the lattice. The coupling constants J_{ij} are independent random variables drawn from a probability distribution with zero mean (no residual ferromagnetic or antiferromagnetic component) and variance J.

A fundamental role in the theory is played by the *overlap* between two states of the system which is, loosely speaking, the degree of similarity of the correlations of the two states. (This overlap of *correlations* is not to be confused with the overlap of *magnetizations* defined by Eq. (2.20)). For two thermodynamic equilibrium states of the same system at temperatures T and T' and in magnetic fields H and H', the similarity between the correlations can be expressed by means of an *overlap function* that essentially is the *projection* of one of the states onto the other

$$C^{eq}_{X,X'}(r) \equiv \overline{(\langle s_i s_{i+r} \rangle^{eq}_X - \langle s_i \rangle^{eq}_X \langle s_{i+r} \rangle^{eq}_X)(\langle s_i s_{i+r} \rangle^{eq}_{X'} - \langle s_i \rangle^{eq}_{X'} \langle s_{i+r} \rangle^{eq}_{X'})}, \tag{4.3}$$

where $X \equiv (T, H)$ and where $\langle \ldots \rangle^{eq}_X$ denotes a thermal average at X. The overbar denotes, as usual, an average over disorder. The self-overlap function

$$C^{eq}_{X,X}(r) = \overline{(\langle s_i s_{i+r} \rangle^{eq}_X - \langle s_i \rangle^{eq}_X \langle s_{i+r} \rangle^{eq}_X)^2} \tag{4.4}$$

is nothing but the correlation function associated with the EA order parameter (2.8). In the paramagnetic phase this correlation function decays exponentially and one may define a spin glass correlation length $\xi_{eq}(X)$ by

$$C^{eq}_{X,X}(r) \sim e^{-2r/\xi_{eq}(X)} \qquad \text{for} \qquad r \to \infty. \tag{4.5}$$

The correlation length ξ_{eq} diverges when X enters the spin glass region, in which it is infinite. The question whether there exists an AT-line $T_c(H)$, which delimits a whole spin glass region, or whether there is only a spin glass phase for zero magnetic field (as FH argue) is not a relevant one as far as the domain theory is concerned. If the correlation length ξ_{eq} is finite for all nonzero magnetic fields then the domain theory is still applicable provided that the magnetic fields are small: as long as the correlation length is the largest length in the problem the system can be considered to be *effectively* in the spin glass phase.

A consequence of the chaotic nature of the spin glass phase (section 3.3) is that there exists an *overlap length* $\ell_{eq}(X, X')$ such that

$$C_{X,X'}^{eq}(r) \sim e^{-2r/\ell_{eq}(X,X')} \qquad \text{for} \qquad r \to \infty. \tag{4.6}$$

In the limit where the two thermodynamic states become identical, i.e. $X = X'$, the overlap length diverges up to the correlation length ξ_{eq}. For a finite difference between X and X' the correlations of the two thermodynamic states are nearly identical on length scales less than $\ell_{eq}(X, X')$ and different on larger length scales. This overlap length is a generalization to nonzero magnetic fields of the length $\ell_{\Delta T}$, discussed in section 3.3. It is a key concept in this thesis as it allows for the description of, in particular, nonlinear aging effects.

Let X be an arbitrary reference temperature and magnetic field. Then, within this framework, a definition can be given of X-*domains* in a thermodynamic state at X': these are regions where the correlations are identical to the correlations of a copy of the same system in equilibrium at the reference X. According to the above definitions these domains have a characteristic size $\ell_{eq}(X, X')$. It should be emphasized that for a particular thermodynamic state at X' there is an infinite set of domain structures each of which belongs to a different reference temperature and magnetic field X.

In a nonequilibrium state one can again distinguish X-domains, whose linear size then depends on time. The overlap function (4.3) can be generalized to the nonequilibrium case by

$$C_X(r,t) \equiv \overline{(\langle s_i s_{i+r}\rangle_t - \langle s_i\rangle_t\langle s_{i+r}\rangle_t)(\langle s_i s_{i+r}\rangle_X^{eq} - \langle s_i\rangle_X^{eq}\langle s_{i+r}\rangle_X^{eq})}, \tag{4.7}$$

where $\langle \ldots \rangle_t$ denotes an average over a time-dependent ensemble. The time-dependent characteristic length of the overlap function (4.7) is referred to as the time-dependent characteristic size $\ell_X(t)$ of the X-domains:

$$C_X(r,t) \sim e^{-2r/\ell_X(t)} \qquad \text{for} \qquad r \to \infty. \tag{4.8}$$

It describes up until what length scale correlations in the time-dependent state are identical to those of the same system in equilibrium at X.

A natural assumption of the theory is that once the system (at $t = 0$) is quenched into the spin glass phase at X it equilibrates by developing X-domains, i.e. the characteristic X-domain size grows with time e.g. according to a power law

$$\ell_X(t) \sim t^p \tag{4.9}$$

with p a positive constant (but this particular form of the growth law is not crucial for qualitative conclusions). The characteristic size of the X'-domains also grows with time t until it finally reaches the overlap length $\ell_{eq}(X, X')$. For the special choice $X = X'$ this upper limit is the correlation length $\xi_{eq}(X)$ and so in a system that for X exhibits spin glass order the characteristic X-domain size grows without limit (in principle until one single mono-domain is formed).

The necessity to consider simultaneously and within one and the same system X'-domains for more than one value of X' (or even for a continuum of such values) becomes clear when there are time-dependent magnetic fields or temperatures. Such a description is essential for a correct qualitative understanding of field and

temperature jump experiments. But it is also logically unavoidable: if e.g. the temperature and magnetic field keep alternating between two values X' and X'', then neither the X'-domain structure nor the X''-domain structure should be priviliged.

The relaxation of magnetic fluctuations of the system in equilibrium at X, where it consists of one "mono-domain", will be referred to as equilibrium relaxation at X. According to experiments [55,60] and to theoretical predictions (see section 2.3) this relaxation is very slow, e.g. power law (like $\sim t^{-\alpha}$ with $\alpha \approx 0.1$) or even logarithmic (like $\sim 1/\log t$). In a system equilibrating at a temperature and magnetic field X, where the range of the X-correlations is the time-dependent X-domain size $\ell_X(t)$, it is natural to conjecture that the magnetic relaxation spectrum is characterized by a maximum relaxation time τ_{max} related to the X-domain size. In this thesis it is postulated that at fixed X

$$\tau_{max}(t) \sim \ell_X^z(t) \tag{4.10}$$

with z a positive constant. (Only the fact that τ_{max} increases with the domain size matters, the actual analytic form has been chosen to fit experimental data). The important consequence is that in a time-dependent temperature and magnetic field X_t the instantaneous maximum relaxation time is related to the range of the X_t-correlations:

$$\tau_{max}(t) \sim \ell_{X_t}^z(t). \tag{4.11}$$

It is easy to see how the above postulates can be used to explain linear aging effects. Consider for instance the field jump experiment described in the beginning of this section: At time $t = 0$ the system is quenched into the spin glass phase while a small magnetic field is applied. At time $t = t_w$ the magnetic field is cut. The longer one waits, i.e. the larger t_w, the larger the $(T, 0)$-domain size (the magnetic field is so small that it has no influence). Consequently, the relaxation for larger waiting times is slower than for shorter waiting times, because through the domain size the maximum relaxation time gets larger with increasing waiting time. In Chapter III a more detailed explanation of aging effects is given. In particular the interplay between the time-dependent domain size and the overlap length ℓ_{eq} is discussed as it gives rise to the more interesting nonlinear aging effects.

At this point it is of interest to remark that Rossel et al. have observed aging effects in a single crystal of the high T_c superconductor $Y_1Ba_2Cu_3O_{7-x}$ [61]. Much of the theory sketched above appears also to be applicable to this system.

4.2 A one-dimensional model

The domain theory of aging is built upon quite some postulates. One would like to construct however simple a model that demonstrates some of the ideas of the theory.

The model of choice is a one-dimensional Ising spin glass chain with randomly temperature-dependent nearest-neighbour couplings. The coupling constants are taken to be independent random variables such that

(i) for $T = 0$ a bond is either $+J$ or $-J$ with equal probability,

(ii) for a temperature increment ΔT the bond changes sign with probability $\Gamma \Delta T/2$,

where J and Γ are constants. (Only infinitesimally small magnetic fields are considered and hence the magnetic field dependence of the bonds is not taken into account.) As a consequence the equilibrium correlations of the model vary randomly with temperature (as they should according to section 3.3) in such a way that the overlap function (4.3) indeed behaves as (4.6) with an overlap length $\ell_{eq}(T, T')$ that scales as

$$\ell_{eq}(T, T') \sim |T - T'|^{-\frac{1}{\zeta}} \tag{4.12}$$

up to length scales of the order of the equilibrium correlation length ξ_{eq}. The above described model may arise if one renormalizes a d-dimensional spin glass bar of size $L^{d-1} \times \infty$.

The model is endowed with Glauber dynamics [24]: a master equation, with specific Glauber transition rates, is postulated for the evolution of the probability of finding a particular spin configuration of the chain at a particular instant of time.

The full analysis of the model is given in Chapter IV where it is indeed shown that aging effects appear on length scales less than the correlation length ξ_{eq} and on a time scale less than the maximum relaxation time. The growth law for the characteristic domain size (4.9) and the dependence of the relaxation time on this domain size (4.10) are demonstrated. Within this same model one can also calculate *(i)* the time-dependent dynamic susceptibility $\bar{\chi}_{\tau_o}(\omega; t)$, which is roughly speaking the proportionality between the amplitudes of an oscillating magnetization and magnetic field *at time* t, and *(ii)* the time-dependent spectrum of magnetization fluctuations $S_{\tau_o}(\omega; t)$. Both these quantities depend on the observation time τ_o which is the time window around the absolute time t within which the magnetization and its fluctuations are analysed. The observation time is chosen much smaller than the age t of the system. In equilibrium the two quantities *(i)* and *(ii)* are related by the fluctuation dissipation theorem (FDT). Experimentally the validity of the FDT for the nonequilibrium spin glass was reported in 1987 by Refregier [62]. The calculations of Chapter IV also demonstrate the validity of the FDT for this model in nonequilibrium, however with a proportionality factor that depends weakly on the observation time.

The interesting question is: How can one explain this result? Refregier [62] argues that, during the observation time, the system is to be considered as in "local equilibrium", i.e. the system visits only part of its phase space but each phase point is visited with a probability that is proportional to the equilibrium probability. From the analysis in Chapter IV it appears that this may very well be true for the one-dimensional model. It should be remarked, however, that every almost-stationary distribution is sufficient for the FDT to hold because it approximately satisfies Glauber's master equation for the system.

4.3 Suppression of aging effects

Up till here no predictions were made from the domain theory. Even the validity of the FDT in a nonequilibrium spin glass [62] was demonstrated experimentally before it was derived theoretically.

But recently it has been found from calculations within the one-dimensional model that one can accelerate the dynamics of the spin glass drastically if one applies a linearly time-dependent temperature instead of a constant temperature. Theory predicts that under these circumstances the system quickly reaches a quasistationary state (which is unlike any equilibrium state) in which no aging effects are present. The details of this analysis are presented in Chapter V. Here it is shown how, within the framework of the domain theory, one can arrive qualitatively at these conclusions.

The basic assumption of the theory is that the spin glass equilibrates through the growth of domains. When the temperature is continually changed at no matter how small a rate the growing domain structure is continually disrupted. The phenomenon of interest here is that a situation arises where the growing and the breaking of the domains balance each other such that the characteristic domain size is stationary. In this situation there is no longer a growing length scale in the system and hence there is no aging. The characteristic domain size remains finite (and dependent on the temperature rate of change) so that there is also a finite and time-independent maximum relaxation time. This maximum relaxation time depends, through the characteristic domain size, on the temperature rate of change and can presumably be made small compared to experimental time scales.

It is certainly interesting to know whether the predicted behaviour occurs in nature. In the experiment one would have to look for the value of the temperature rate of change that is required to provoke the effect. Theoretically this question is answered in this thesis only within the framework of the one-dimensional model. There one distinguishes two types of temperature dependencies: a "weak" temperature dependence of the "equilibrium" relaxation (within the domains) of the system and a "strong" and chaotic temperature dependence of the correlations. In the calculations the temperature rate of change is taken so small that during for instance a magnetic relaxation experiment the equilibrium relaxation is not affected significantly. The calculations show that, subject to a temperature rate of change $\dot{T}$, the magnetic relaxation takes place on a time scale $\tau_{qs} \sim \dot{T}^{-2/3}$. This timescale is much smaller than the corresponding time scale of the system at constant temperature.

5 Further reading

In this introductory chapter a concise review has been given of the developments in spin glass theory with an emphasis on aging. The aim has been to place the subsequent chapters in the proper context. More elaborate reviews are given by Binder and Young [63], Fischer [64,65] and by Chowdhury [66]. A fairly complete treatment of mean field theory is given by Mézard et al. [67]. Through these reviews

the interested reader has access to the bulk of the spin glass literature.

References

[1] Ruderman, M. A. and Kittel, C., Phys. Rev. **96** (1954) 99.

[2] Kasuya, T., Prog. Theor. Phys. **16** (1956) 45.

[3] Yosida, K., Phys. Rev. **106** (1957) 893.

[4] Blandin, A., PhD thesis University of Paris (1961).

[5] Souletie, J. and Tournier, R., J. Low Temp. Phys. **1** (1969) 95.

[6] Toulouse, G., Comm. Phys. **2** (1977) 115.

[7] Canella, V. and Mydosh, J. A., Phys. Rev. **B 6** (1972) 4220.

[8] Arrot, A., J. Appl. Phys. **36** (1965) 1093.

[9] Violet, C. and Borg, R. J., Phys. Rev. **149** (1966) 540.

[10] De Nobel, J. and Du Chantenier, J. J., Physica **25** (1959) 969.

[11] Tholence, J. L. and Tournier, R., J. Physique **35** (1974) C4-229.

[12] Anderson, P. W., Halperin, B. I., and Varma, C. M., Phil. Mag. **25** (1972) 1.

[13] Edwards, S. F. and Anderson, P. W., J. Phys. **F 5** (1975) 89.

[14] Blandin, A., J. Physique **C6-39** (1978) 1499.

[15] Sherrington, D. and Kirkpatrick, S., Phys. Rev. Lett. **35** (1975) 1792.

[16] Kirkpatrick, S. and Sherrington, D., Phys. Rev. **B 17** (1978) 4384.

[17] De Almeida, J. R. L. and Thouless, D. J., J. Phys. **A 11** (1978) 983.

[18] Parisi, G., J. Phys. **A 13** (1980) 1101.

[19] Parisi, G., Phys. Rev. Lett. **50** (1983) 1946.

[20] Mézard, M., Parisi, G., and Virasoro, M. A., J. Phys. (Paris) Lett. **46** (1985) L217.

[21] Mézard, M., Parisi, G., Sourlas, N., Toulouse, G., and Virasoro, M. A., J. Physique **45** (1984) 843.

[22] Rammal, R., Toulouse, G., and Virasoro, M. A., Rev. Mod. Phys. **58** (1986) 765.

[23] Harris, A. B., Lubensky, T. C., and Chen, J. H., Phys. Rev. Lett. **36** (1976) 415.

[24] Glauber, R. J., J. Math. Phys. **4** (1963) 294.

[25] Sompolinski, H., Phys. Rev. Lett. **47** (1981) 935.

[26] Sompolinski, H. and Zippelius, A., Phys. Rev. **B 25** (1982) 6860.

[27] Mackenzie, N. D. and Young, A. P., Phys. Rev. Lett. **49** (1982) 301.

[28] Mackenzie, N. D. and Young, A. P., J. Phys. **C 16** (1983) 5321.

[29] Palmer, R. G., Stein, D. L., Abrahams, E., and Anderson, P. W., Phys. Rev. Lett. **53** (1984) 958.

[30] Ogielski, A. T. and Stein, D. L., Phys. Rev. Lett. **55** (1985) 1634.

[31] Paladin, G., Mézard, M., and De Dominicis, C., J. Physique **46** (1985) L985.

[32] Schreckenberg, M., Z. Phys. **B 60** (1985) 483.

[33] Grossman, S., Wenger, F., and Hoffman, K. H., J. Phys. (Paris) Lett. **46** (1985) L575.

[34] Derrida, B., Phys. Rev. Lett. **45** (1980) 79.

[35] Derrida, B., Phys. Rev. **B 24** (1981) 2613.

[36] De Dominicis, C., Orland, H., and Lainée, F., J. Phys. (Paris) Lett. **46** (1985) L463.

[37] Koper, G. J. M. and Hilhorst, H. J., Europhys. Lett. **3** (1987) 1213.

[38] De Dominicis, C. and Kondor, I., J. Phys. **A 22** (1989) L743.

[39] McMillan, W. L., J. Phys. **C 17** (1984) 3179.

[40] McMillan, W. L., Phys. Rev. **B 30** (1984) 476.

[41] Bray, A. J. and Moore, M. A., J. Phys. **C 17** (1984) L463.

[42] Bray, A. J. and Moore, M. A., J. Phys. **C 17** (1984) L613.

[43] Bhatt, R. N. and Young, A. P., Phys. Rev. Lett. **54** (1985) 924.

[44] Ogielski, A. T. and Morgenstern, I., Phys. Rev. Lett. 54 (1985) 928.

[45] Fisher, M. E., Rev. Mod. Phys. **46** (1974) 597.

[46] Fisher, D. S. and Huse, D. A., Phys. Rev. Lett. **56** (1986) 1601.

[47] Fisher, D. S. and Huse, D. A., Phys. Rev. **B 38** (1988) 386.

[48] Bray, A. J. and Moore, M. A., in: *Heidelberg Colloquium on Glassy Dynamics*, eds. J. L. van Hemmen and I. Morgenstern (Springer, Heidelberg, 1987).

[49] Imry, Y. and Ma, S., Phys. Rev. Lett. **35** (1975) 1399.

[50] Bray, A. J. and Moore, M. A., Phys. Rev. Lett. **58** (1987) 57.

[51] Kondor, I., J. Phys. **A 22** (1989) L163.

[52] Toulouse, G., in: *Disordered systems and localization*, ed. J. A. Mydosh (Springer, Heidelberg, 1981).

[53] Struik, L. C. E., *Physical aging in amorphous polymers and other materials* (Elsevier Scient. Publ., Amsterdam, 1978).

[54] Lundgren, L., Nordblad, P., and Sandlund, L., Europhys. Lett. **1** (1986) 529.

[55] Alba, M., Vincent, E., Hammann, J., and Ocio, M., J. Appl. Phys. **61** (1987) 4092.

[56] Nordblad, P., Svedlindh, P., Sandlund, L., and Lundgren, L., Phys. Lett. **A 120** (1987) 475.

[57] Refregier, P., Vincent, E., Hammann, J., and Ocio, M., J. Physique **48** (1987) 1533.

[58] Koper, G. J. M. and Hilhorst, H. J., J. Physique **49** (1988) 429.

[59] Fisher, D. S. and Huse, D. A., Phys. Rev. **B 38** (1988) 373.

[60] Nordblad, P., Svedlindh, P., Lundgren, L., and Sandlund, L., Phys. Rev. **B 33** (1986) 645.

[61] Rossel, C., Maeno, Y., and Morgenstern, I., Phys. Rev. Lett. **62** (1989) 681.

[62] Refregier, P., PhD thesis Université de Paris-Sud, Centre d'Orsay (1987).

[63] Binder, K. and Young, A. P., Rev. Mod. Phys. **58** (1986) 801.

[64] Fischer, K. H., Phys. Stat. Sol. (b) **116** (1983) 357.

[65] Fischer, K. H., Phys. Stat. Sol. (b) **130** (1985) 13.

[66] Chowdhury, D., *Spin glasses and frustrated systems* (World Scientific, Singapore, 1986).

[67] Mézard, M., Parisi, G., and Virasoro, M. A., *Spin glass theory and beyond* (World Scientific, Singapore, 1987).

Chapter II

Nonexponential Relaxation
in the Random Energy Model *

In this chapter a general class of master equations for a system consisting of exponentially distributed random energy levels is solved. The equations differ in the choice of transition rates. The relaxation is found to be nonexponential in all cases.

1 Introduction

The simplest possible model of a random system is probably the model of independent Gaussian distributed random energy levels (REM) introduced by Derrida [1,2]. It has several important features in common with the Sherrington-Kirkpatrick mean field model [3,4] of a spin glass which has been extensively studied in recent years, notably by Parisi and coworkers [5,6]. In particular it is known (see Mézard et al. [7]) that in both models the (free) energies of the lowest-lying states are random variables with an exponential distribution. For the REM we rederive the connection between the Gaussian and the exponential distribution in a very simple way in appendix A.

It is therefore of great interest to study also the kinetics of a system of exponentially distributed random energy levels. This was first done by De Dominicis et al. [8], who considered a master equation for the occupation probabilities of the energy levels. For one particular choice of the transition rates De Dominicis et al. [8] found stretched exponential relaxation. Some time later we showed [9] that this model, with slightly modified transition rates, also exhibits power law relaxation.

This chapter deals with a more general class of transition rates for which it is possible to obtain analytic results. In sections 2 and 3 this class is defined and it is shown how the solution of the master equation can be found. In sections 4 and 5 the relaxation for an interpolating set of transition rates, parametrized by a variable q, that has the models of De Dominicis [8] and of Koper and Hilhorst [9] as limiting cases, is studied. The time dependent behaviour of an autocorrelation function of the occupation probabilities is determined. The special case of $q = 0$, that corresponds to De Dominicis' transition rates [8], is treated separately in section 4, the general case in section 5. The behaviour of the relaxation can be depicted in a "phase diagram" with temperature and q as parameters. In section 6 transition rates based on the existence of barriers of random height between the energy levels are considered. In all cases it is found that the relaxation is either of the stretched exponential or of the power law type.

2 The kinetic random energy model

Derrida's [1,2] random energy model (REM) is defined as

(i) a system of $2^{\mathcal{N}}$ energy levels E_i

(ii) that are independent random variables

(iii) drawn from a Gaussian probability distribution with zero mean and variance $\frac{1}{2}\mathcal{N}J^2$:

$$P(E) = \frac{e^{-E^2/\mathcal{N}J^2}}{\sqrt{\pi\mathcal{N}J^2}}. \tag{2.1}$$

The model exhibits a phase transition at the critical temperature $T_c = J/2\sqrt{\log 2}$. For temperatures T below T_c and in the thermodynamic limit the free energy becomes a constant $E_0(\mathcal{N}) \simeq \mathcal{N}J\sqrt{\log 2}$ and the entropy vanishes. This indicates a completely frozen low temperature phase.

The probability $\mathcal{P}_i^{eq}$ to find the equilibrated system in level i is given by

$$\mathcal{P}_i^{eq} \equiv Z^{-1} \exp(-\beta E_i), \tag{2.2}$$

where the partition function is given by

$$Z \equiv \sum_{j=1}^{\mathcal{N}} \exp(-\beta E_j) \tag{2.3}$$

and where β is the inverse temperature (Boltzmann's constant $k_B = 1$). Clearly the lowest lying energy levels dominate the low temperature phase. The energies E_i of the lowest N energy levels (where N is arbitrary) will be written as

$$E_i = E_0(\mathcal{N}) + \varepsilon_i \quad (i = 1, 2, \ldots, N), \tag{2.4}$$

where ε_i is a small and nonextensive random energy. By linearizing the Gaussian probability distribution of the full REM one can show that the ε_i are distributed according to the exponential probability distribution

$$p(\varepsilon) = \begin{cases} \rho e^{\rho(\varepsilon-\varepsilon_c)} & \varepsilon < \varepsilon_c \\ \\ 0 & \varepsilon > \varepsilon_c \end{cases} \tag{2.5}$$

where $\rho = T_c^{-1}$ is the inverse critical temperature of the REM and ε_c is a cutoff energy. This is shown explicitly in appendix A, but see also Ref. [7]. Hence the *low temperature phase* of the REM can be defined equivalently as

(i) a system of N energy levels ε_i

(ii) that are independent random variables

(iii) drawn from the exponential probability distribution (2.5).

The physical properties of the REM for temperatures $T < T_c$ take finite, cutoff independent values in the limit

(iv) $N \to \infty$, $\varepsilon_c \to \infty$, while $Ne^{-\rho\varepsilon_c} = v$ fixed (this is the limit in which the density of the levels ε_i at any given value of the energy becomes a constant).

For the dynamics of this model De Dominicis et al. [8] postulated a master equation. The master equation describes the time evolution of the probability $P_i(t)$ of finding the system in level i at time t and reads

$$\frac{dP_i(t)}{dt} = \sum_{j=1}^{N} W_{ij}P_j(t) - \sum_{j=1}^{N} W_{ji}P_i(t) \quad (i = 1, 2, \ldots, N). \tag{2.6}$$

23

The transition rates W_{ij} for going from level j to level i obey the detailed balancing condition

$$W_{ij}P_j^{eq} = W_{ji}P_i^{eq} \quad (i \neq j) \tag{2.7}$$

so that in the limit $t \to \infty$ the equilibrium distribution $P_i^{eq} \sim e^{-\beta \epsilon_i}$ will be attained. This requirement forces the transition rates to take the form

$$W_{ij} = e^{-\beta \epsilon_i} e^{-\beta \Delta_{ij}} \quad (i \neq j), \tag{2.8}$$

where $e^{-\beta \Delta_{ij}}$ is symmetric in i and j and can be considered as due to a barrier Δ_{ij} between the levels i and j. In this work we consider $e^{-\beta \Delta_{ij}}$ that factorize so that the transition rates assume the form

$$W_{ij} = e^{-\beta \epsilon_i} V_i V_j. \tag{2.9}$$

We will restrict ourselves to some particular choices of barriers, namely

(i) Homogeneous barriers, $V_i = V$, which is the special case studied by De Dominicis et al. [8].

(ii) Level dependent barriers, $V_i = v_0 e^{\beta q \epsilon_i}$ where $0 < q < 1$. The case $q = 0$ is identical to (i).

(iii) Independent random barriers.

The solution of the master equation can be expressed in terms of a Green function. Eq. (2.6) with transition rates (2.9) can be written as

$$\frac{dP_i(t)}{dt} = V_i B_i \sum_{j=1}^{N} V_j P_j(t) - V_i P_i(t) \zeta \tag{2.10}$$

where we define Boltzmann factors $B_i \equiv \exp(-\beta \epsilon_i)$ and where

$$\zeta \equiv \sum_{j=1}^{N} V_j B_j. \tag{2.11}$$

By demanding that

$$\frac{dP_i(t)}{dt} = -\lambda P_i(t) \tag{2.12}$$

one obtains the eigenvalue problem. Let ϕ_i^λ be the i-th component of the right-hand eigenfunction with eigenvalue $-\lambda$. Then one can write

$$\phi_i^\lambda = \frac{V_i B_i}{V_i \zeta - \lambda} \sum_{j=1}^{N} V_j \phi_j^\lambda. \tag{2.13}$$

Multiplying Eq. (2.13) by V_i and summing over all i yields the eigenvalue equation

24

$$\sum_{j=1}^{N} \frac{V_i^2 B_i}{V_i \zeta - \lambda} = 1 \quad \text{iff} \quad \sum_{j=1}^{N} V_j \phi_j^{\lambda} \neq 0. \tag{2.14}$$

The condition mentioned in Eq. (2.14) is fulfilled when $V_i \neq V_j$ for at least one pair $i \neq j$. When all V_i's are different so that they can be ordered, $0 < V_1 < V_2 \ldots < V_N$, then it is easy to see that the solutions of Eq. (2.14) satisfy

$$0 = \lambda_1 < V_1 \zeta < \lambda_2 < V_2 \zeta < \ldots < \lambda_N < V_N \zeta. \tag{2.15}$$

Besides the right-hand eigenfunctions ϕ_i^{λ} one can also compute the left-hand eigenfunctions ψ_i^{λ}; detailed balance (2.7) implies $\psi_i^{\lambda} = \phi_i^{\lambda}/B_i$ [10]. From the left and right-hand eigenfunctions the Green function can be calculated as

$$
\begin{aligned}
G_{ij}(t) &= \sum_{\ell=1}^{N} e^{-\lambda_\ell t} \frac{\phi_i^{\lambda_\ell} \psi_j^{\lambda_\ell}}{\sum_{k=1}^{N} \phi_k^{\lambda_\ell} \psi_k^{\lambda_\ell}} \\
&= \sum_{\ell=1}^{N} e^{-\lambda_\ell t} \frac{\dfrac{V_i B_i}{V_i \zeta - \lambda_\ell} \dfrac{V_j}{V_j \zeta - \lambda_\ell}}{\displaystyle\sum_{k=1}^{N} \frac{V_k^2 B_k}{(V_k \zeta - \lambda_\ell)^2}}.
\end{aligned}
\tag{2.16}
$$

3 The magnetization autocorrelation function

The kinetic REM can be extended to a magnetic model by associating with each level i an independent random magnetization μ_i drawn from a probability distribution with zero mean and variance N. The equilibrium autocorrelation function of the magnetization is then

$$C_\mu(t) = \frac{1}{N} \sum_{i,j=1}^{N} \mu_i \{G_{ij}(t) - G_{ij}(\infty)\} \mu_j P_j^{eq} \tag{3.1}$$

In zero magnetic field we can average Eq. (3.1) with regard to the distribution of the magnetizations, which yields

$$C(t) \equiv \sum_{j=1}^{N} \{G_{jj}(t) - G_{jj}(\infty)\} P_j^{eq}. \tag{3.2}$$

By the same argument it can be shown that $C(t)$ is proportional to the equilibrium autocorrelation function of any physical quantity that does not couple to the energies of the REM. It is the purpose of this work to calculate $C(t)$ using the kinetics defined in section 2.

Eq. (3.2) involves the Green function (2.16) and we cannot evaluate it without knowing the eigenvalues $-\lambda_\ell$. In order to cast Eq. (3.2) into a form that allows for an average over disorder we consider its Laplace transform,

25

$$\tilde{C}(s) \equiv \int\limits_0^\infty dt \, e^{-st} C(t) \quad (\mathrm{Re}\, s > 0), \tag{3.3}$$

which with (2.16) and (3.2) can be written as

$$\tilde{C}(s) = \frac{1}{Z} \sum_{\ell=2}^N \frac{H(\lambda_\ell)}{(s + \lambda_\ell) F'(\lambda_\ell)} \quad (\mathrm{Re}\, s > 0) \tag{3.4a}$$

where

$$H(z) \equiv \sum_{j=1}^N \frac{V_j^2 B_j^2}{(V_j \zeta - z)^2} \tag{3.4b}$$

and

$$F(z) \equiv \sum_{j=1}^N \frac{V_j^2 B_j}{V_j \zeta - z} - 1 \tag{3.4c}$$

and where $\lambda_1 = 0$ is the eigenvalue of the equilibrium probability distribution. Using Eq. (2.11) one can rewrite Eq. (3.4c) as

$$F(z) = \frac{z}{\zeta} \sum_{j=1}^N \frac{V_j B_j}{V_j \zeta - z}. \tag{3.5}$$

We now consider the function f defined by

$$f(z) \equiv \frac{H(z)}{(s+z) F(z)} \tag{3.6}$$

in the complex plane. This function has poles in $z = -s$, in $z = V_j \zeta$ and in the zeroes of $F(z)$. Since $F(z) = 0$ is equivalent to the eigenvalue equation (2.14) the residues of the poles in the zeroes of $F(z) = 0$, except for $z = 0$, exactly sum up to $\tilde{C}(s)$. The contour integral of $f(z)$ along a circle centered in the origin with radius R vanishes as $R \to \infty$ and hence we find from the total of the residues of $f(z)$ an expression that no longer contains the unknown eigenvalues but only the Boltzmann factors B_i and the random V_i:

$$\tilde{C}(s) = \frac{1}{s} \left\{ \frac{\zeta \displaystyle\sum_{j=1}^N \frac{V_j^2 B_j^2}{(V_j \zeta + s)^2}}{Z \displaystyle\sum_{j=1}^N \frac{V_j B_j}{V_j \zeta + s}} - \frac{1}{Z^2} \sum_{j=1}^N B_j^2 \right\} + \frac{1}{Z} \sum_{j=1}^N \frac{B_j}{V_j \zeta + s}. \tag{3.7}$$

It turns out to be more convenient to write Eq. (3.7) as

$$\tilde{C}(s) = -\frac{1}{s} \left\{ \frac{\zeta \sum_{j=1}^{N} \sum_{k \neq j}^{N} \dfrac{V_j B_j V_k B_k}{(V_j \zeta + s)(V_k \zeta + s)}}{Z \sum_{\ell=1}^{N} \dfrac{V_\ell B_\ell}{V_\ell \zeta + s}} + \frac{1}{Z^2} \sum_{j=1}^{N} B_j^2 - 1 \right\} \tag{3.8}$$

from which it is clear that $\tilde{C}(s)$ has poles only on the negative real axis: there is no pole for $s = 0$.

Applying an inverse Laplace transformation yields the equilibrium autocorrelation function

$$C(t) = -\oint_\Omega \frac{ds}{2\pi i} \left\{ \frac{e^{st} \zeta \sum_{j=1}^{N} \sum_{k \neq j}^{N} \dfrac{V_j B_j V_k B_k}{(V_j \zeta + s)(V_k \zeta + s)}}{sZ \sum_{\ell=1}^{N} \dfrac{V_\ell B_\ell}{V_\ell \zeta + s}} \right\} \tag{3.9}$$

where the contour Ω is oriented counterclockwise and encloses all the poles of $\tilde{C}(s)$ on the negative real axis. When we change from the integration variable s to $-s\zeta$ we obtain a form that is more convenient for further calculations

$$C(t) = \oint_\Omega \frac{ds}{2\pi i} \left\{ \frac{e^{-st\zeta} \sum_{j=1}^{N} \sum_{k \neq j}^{N} \dfrac{V_j B_j V_k B_k}{(s - V_j)(s - V_k)}}{sZ \sum_{\ell=1}^{N} \dfrac{V_\ell B_\ell}{s - V_\ell}} \right\} \tag{3.10}$$

where the contour Ω is still oriented counterclockwise and encloses all the poles of the integrand on the positive real axis. This expression will be used in the next sections where averages over various kinds of disorder are discussed.

4   Homogeneous barriers

The simplest situation arises when all barriers are of equal height. This is the case when all V_i are identical which corresponds to taking $e^{-\beta \Delta_{ij}} = w_0$ in Eq. (2.8). The transition rates then assume the simple form $W_{ij} = w_0 B_i$. When these rates are substituted in the master equation (2.6), the set of equations decouples and the solution can easily be found:

$$P_j(t) = P_j^{eq} + \left\{ P_j(0) - P_j^{eq} \right\} e^{-w_0 Z t}. \tag{4.1}$$

The Green function $G_{ij}(t)$ is the solution $P_i(t)$ of the master equation with initial condition $P_i(0) = \delta_{ij}$ and so the equilibrium autocorrelation function, Eq. (3.2), becomes

$$C(t) = \left(1 - \frac{1}{Z^2} \sum_{j=1}^{N} B_j^2 \right) e^{-w_0 Z t}. \tag{4.2}$$

27

Taking all V_i's in Eq. (3.10) equal to $\sqrt{w_0}$ yields exactly the same result.

The equation (4.2) is to be averaged over the disorder. It is expressed in terms of the random Boltzmann factors B_i rather than in the random energies ε_i. Their distribution function p_B is easily derived from Eq. (2.5):

$$p_B(B) = \begin{cases} \dfrac{v}{N}\dfrac{\rho}{\beta}B^{-1-\frac{\rho}{\beta}} & \text{for } \left(\dfrac{v}{N}\right)^{\frac{\beta}{\rho}} < B < \infty \\ \\ 0 & \text{otherwise.} \end{cases} \tag{4.3}$$

For the squared partition function in the denominator of (4.2) we shall use the integral representation

$$\alpha^{-n} = \frac{1}{\Gamma(n)} \int\limits_0^\infty d\lambda\, \lambda^{n-1} e^{-\alpha\lambda} \quad (\operatorname{Re}\alpha > 0, n > 0). \tag{4.4}$$

The average of the equilibrium autocorrelation function (4.2) can then be written as

$$\begin{aligned}
\overline{C(t)} &= \lim_{N\to\infty} \left\{ \frac{v}{N}\frac{\rho}{\beta} \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_i\, B_i^{-1-\frac{\rho}{\beta}} e^{-w_0 B_i t} \right\}^N \\
&\quad - \lim_{N\to\infty} \left[v\frac{\rho}{\beta} \int\limits_0^\infty d\lambda \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_i\, \lambda B_i^{1-\frac{\rho}{\beta}} e^{-\lambda B_i - w_0 B_i t} \right. \\
&\quad \left. \times \left\{ \frac{v}{N}\frac{\rho}{\beta} \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_j\, B_j^{-1-\frac{\rho}{\beta}} e^{-\lambda B_j - w_0 B_j t} \right\}^{N-1} \right].
\end{aligned} \tag{4.5}$$

The limit $N \to \infty$ cannot be performed immediately due to the divergences at the lower bounds of the integrals. Consider for instance the first term. After a partial integration an expansion can be made in negative powers of N:

$$\frac{v}{N}\frac{\rho}{\beta} \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_i\, B_i^{-1-\frac{\rho}{\beta}} e^{-w_0 t B_i} = \exp\left\{ -w_0 t\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}} \right\} - \frac{v}{N}w_0 t \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_i\, B_i^{-\frac{\rho}{\beta}} e^{-w_0 t B_i}$$

$$= 1 - \frac{v}{N}(w_0 t)^{\frac{\rho}{\beta}} \Gamma\left(1 - \frac{\rho}{\beta}\right) + \mathcal{O}\left(N^{-\frac{\beta}{\rho}}\right). \tag{4.6}$$

The N-th power of this expression yields, in the limit of large N, an exponential. The same procedure is to be applied to the B_j-integral of the second term. The

28

other two integrals pose no particular problem and one finds

$$\overline{C(t)} = \frac{\rho}{\beta} \exp\left\{-v\Gamma\left(1 - \frac{\rho}{\beta}\right)(w_o t)^{\frac{\rho}{\beta}}\right\}$$

$$+ \left(1 - \frac{\rho}{\beta}\right)w_0 t \left\{v\Gamma\left(1 - \frac{\rho}{\beta}\right)\right\}^{\frac{\beta}{\rho}} \Gamma\left(1 - \frac{\beta}{\rho}; v\Gamma\left(1 - \frac{\rho}{\beta}\right)(w_0 t)^{\frac{\rho}{\beta}}\right), \quad (4.7)$$

where the incomplete gamma function [11] is defined by

$$\Gamma(\alpha; x) = \int_x^\infty dt \; t^{\alpha-1} e^{-t} \tag{4.8}$$

which for large x behaves as

$$\Gamma(\alpha; x) \simeq x^{\alpha-1} e^{-x}. \tag{4.9}$$

For large times t one can write Eq. (4.7), by means of (4.9), as

$$\overline{C(t)} \simeq \exp\left\{-v\Gamma\left(1 - \frac{\rho}{\beta}\right)(w_0 t)^{\frac{\rho}{\beta}}\right\}. \tag{4.10}$$

This result has been obtained along the lines of the analysis of De Dominicis et al. [8]. They, however, calculated the time-dependence of the overlap $y(q_M)$ at the maximum value q_M of the Parisi [5,6] order parameter $q(x)$

$$y(q_M) = \sum_{i=1}^N \overline{P_i^2(t)} \tag{4.11}$$

whereas we calculate the equilibrium autocorrelation function (3.10).

5 Level dependent barriers

In this section we shall calculate $\overline{C(t)}$ for the case where the barrier factors V_i in the transition rates (2.9) depend on the energy levels according to

$$V_i = N^{-p} B_i^{-q} \tag{5.1}$$

where q is a parameter in the interval $[0, 1]$, and the factor N^{-p} anticipates upon a possible need for scaling the transition rates with N. We shall deal with situations where $\frac{\rho}{\beta} \in (0, 1)$, that is, temperatures below the freezing temperature. With this choice for V_i we can write our starting point, Eq. (3.10), as

$$C(t) = \oint_\Omega \frac{ds}{2\pi i} \frac{\displaystyle\sum_{j=1}^N \sum_{k\neq j}^N \frac{B_j B_k}{(sB_j^q - 1)(sB_k^q - 1)}}{sZ \displaystyle\sum_{\ell=1}^N \frac{B_\ell}{sB_\ell - 1}} \exp\left\{-st N^{-2p} \sum_{i=1}^N B_i^{1-q}\right\}. \tag{5.2}$$

29

After introducing integral representations (see Eq.(4.4)) for the partition function and for the sum in the denominator we can perform the average over all sets of random levels, using the probability distribution (4.3) for the Boltzmann factors:

$$
\overline{C(t)} = \lim_{N \to \infty} v^2 \left(\frac{\rho}{\beta}\right)^2 \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_i \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_j \int\limits_{0}^{\infty} d\lambda \int\limits_{0}^{\infty} d\mu \oint\limits_{\Omega} \frac{ds}{2\pi i}
$$

$$
\times \ \frac{B_i^{-\frac{\rho}{\beta}} B_j^{-\frac{\rho}{\beta}}}{s(sB_i^q - 1)(sB_j^q - 1)}
$$

$$
\times \ \exp\left\{ -\lambda(B_i + B_j) - \mu\left(\frac{B_i}{sB_i^q - 1} + \frac{B_j}{sB_j^q - 1}\right) - stN^{-2p}\left(B_i^{1-q} + B_j^{1-q}\right) \right\}
$$

$$
\times \ \left[\frac{v}{N}\frac{\rho}{\beta} \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_k B_k^{-1-\frac{\rho}{\beta}} \exp\left\{ -\lambda B_k - \frac{\mu B_k}{sB_k^q - 1} - stN^{-2p}B_k^{1-q} \right\} \right]^{N-2} \quad (5.3)
$$

As in the case of uniform barriers (section 4) the limit $N \to \infty$ cannot be taken immediately in Eq. (5.3), but first a partial integration over B_k is needed:

$$
I^{\frac{1}{N-2}} \equiv \frac{v}{N}\frac{\rho}{\beta} \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_k B_k^{-1-\frac{\rho}{\beta}} \exp\left\{ -\lambda B_k - \frac{\mu B_k}{sB_k^q - 1} - stN^{-2p}B_k^{1-q} \right\}
$$

$$
= \ \exp\left\{ -\lambda\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}} - \frac{\mu\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}{s\left(\frac{v}{N}\right)^{\frac{q\beta}{\rho}} - 1} - stN^{-2p}\left(\frac{v}{N}\right)^{\frac{(1-q)\beta}{\rho}} \right\}
$$

$$
- \ \frac{v}{N} \int\limits_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^{\infty} dB_k B_k^{-\frac{\rho}{\beta}} \left\{ \lambda - \mu\frac{1 - s(1-q)B_k^q}{(1 - sB_k^q)^2} + stN^{-2p}(1-q)B_k^{-q} \right\}
$$

$$
\times \ \exp\left\{ -\lambda B_k - \frac{\mu B_k}{sB_k^q - 1} - stN^{-2p}B^{1-q} \right\} \quad (5.4)
$$

For $\frac{\rho}{\beta} > 1-q$ we adjust p such that $-2p - (1-q)\frac{\beta}{\rho} = -1$, which gives $p = \frac{1}{2} - \frac{1}{2}(1-q)\frac{\beta}{\rho}$. (Other choices for p would lead to a trivial result for $\overline{C(t)}$: either I is infinite or it becomes independent of t). This case will be treated first in the next subsection. Then we shall treat the simpler case of $\frac{\rho}{\beta} < 1 - q$ where we can take $p = 0$ since the exponential expands as $1 + o(N^{-1})$.

30

5.1 The case $\frac{\rho}{\beta} > 1 - q$

For this case, where $p = \frac{1}{2} - \frac{1}{2}(1-q)\frac{\beta}{\rho}$, we find in the limit $N \to \infty$:

$$I \simeq \exp\left[-stv^{\frac{\beta}{\rho}(1-q)} - v\int_0^\infty dB_k B_k^{-\frac{\rho}{\beta}}\left(\lambda + \mu\frac{s(1-q)B_k^q - 1}{(sB_k^q - 1)^2}\right)\right.$$
$$\left. \times \exp\left(-\lambda B_k - \frac{\mu B_k}{sB_k^q - 1}\right)\right]. \tag{5.5}$$

This is substituted in Eq. (5.3). In order to evaluate $\overline{C(t)}$ for long times we set $B_i = xt^{\frac{1}{q}}$, $B_j = yt^{\frac{1}{q}}$, $B_k = zt^{\frac{1}{q}}$, $s = s't^{-1}$, $\lambda = \lambda' t^{-\frac{1}{q}}$ and $\mu = \mu' t^{-\frac{1}{q}}$. Then, after dropping primes again, we expand in powers of $t^{-\frac{\rho}{q\beta}}$. The leading term in this expansion becomes, after one performs the λ- and μ-integrations,

$$\overline{C(t)} = v^2\left(\frac{\rho}{\beta}\right)^2 t^{-\frac{2\rho}{q\beta}}\int_0^\infty dx \int_0^\infty dy \oint_\Omega \frac{ds}{2\pi i}$$
$$\times \frac{x^{-\frac{\rho}{\beta}}y^{-\frac{\rho}{\beta}}}{s(x+y)\{s(xy^q + x^q y) - x - y\}}\exp\left\{-sv^{\frac{\beta}{\rho}(1-q)}\right\}. \tag{5.6}$$

Inside the contour Ω there is one simple pole, viz. at $s = (x+y)/(xy^q + x^q y)$, so that we obtain power law relaxation for this regime

$$\overline{C(t)} \simeq A\left(\frac{\rho}{\beta}, q\right) t^{-\frac{2\rho}{q\beta}} \qquad\qquad (\frac{\rho}{\beta} > 1 - q) \tag{5.7a}$$

where

$$A\left(\frac{\rho}{\beta}, q\right) = \frac{\rho}{2\beta}v^{4-\frac{2}{q}}\Gamma\left(1 + \frac{2\rho}{q\beta}\right)\int_0^\infty du \frac{u^{\frac{\rho}{\beta}}(1 + u^{1-q})^{\frac{2\rho}{q\beta}}}{(1+u)^{2+\frac{2\rho}{q\beta}}}. \tag{5.7b}$$

The integral in the latter expression converges. For $q = 1$ this expression reduces to the result published earlier [9].

5.2 The case $\frac{\rho}{\beta} < 1 - q$

In the case $\frac{\rho}{\beta} < 1 - q$ there is no need for a scaling of the transition rates with N, so $p = 0$. The I as defined by Eq. (5.4) then becomes, in the limit of $N \to \infty$, equal to

$$I \simeq \exp\left[-v\int_0^\infty dB_k B_k^{-\frac{\rho}{\beta}}\left(\lambda + \mu\frac{s(1-q)B_k^q - 1}{(sB_k^q - 1)^2} + st(1-q)B_k^{-q}\right)\right.$$
$$\left. \times \exp\left\{-\lambda B_k - \mu\frac{B_k}{sB_k^q - 1} - stB_k^{1-q}\right\}\right] \tag{5.8}$$

This can be substituted in Eq. (5.3). We can evaluate $\overline{C(t)}$ for long times by changing the integration variables according to $B_i = xt^{\frac{1}{2q-1}}$, $B_j = yt^{\frac{1}{2q-1}}$, $B_k = zt^{\frac{1}{2q-1}}$, $\mu = \mu' t^{-\frac{1}{2q-1}}$, $\lambda = \lambda' t^{-\frac{1}{2q-1}}$ and $s = s' t^{-\frac{q}{2q-1}}$. Then, after dropping primes again, we expand for long times in powers of $t^{-\frac{\rho}{\beta(2q-1)}}$. *This means that the expansion is valid only for $q > \frac{1}{2}$.* For the leading term of this expansion one obtains, after performing the λ- and the μ-integrations,

$$
\overline{C(t)} \simeq v^2 \left(\frac{\rho}{\beta}\right)^2 t^{-\frac{2\rho}{\beta(2q-1)}} \int\limits_0^\infty dx \int\limits_0^\infty dy \oint\limits_\Omega \frac{ds}{2\pi i} \exp\left\{-s\left(x^{1-q} + y^{1-q}\right)\right\}
$$
$$
\times \quad \frac{x^{-\frac{\rho}{\beta}} y^{-\frac{\rho}{\beta}}}{s(x+y)\{s(xy^q + x^q y) - x - y\}} \tag{5.9}
$$

The contour Ω only encloses a simple pole at $s = (x+y)/(xy^q + x^q y)$, so that we also obtain power law relaxation for this regime:

$$
\overline{C(t)} \simeq B_+ \left(\frac{\rho}{\beta}, q\right) t^{-\frac{2\rho}{\beta(2q-1)}} \qquad \left(q > \frac{1}{2}, \frac{\rho}{\beta} < 1 - q\right) \tag{5.10a}
$$

where

$$
B_+ \left(\frac{\rho}{\beta}, q\right) = v^2 \frac{\rho}{2\beta} \frac{\Gamma^2\left(1 + \frac{\rho}{\beta(2q-1)}\right)}{1 + \frac{2\rho}{\beta(2q-1)}} \tag{5.10b}
$$

Here also the limit $q \to 1$ can be taken for comparison of the result with [9]. Note however that the condition $\frac{\rho}{\beta} < 1 - q$ only allows zero temperature, $\frac{\rho}{\beta} = 0$.

The result for $q < \frac{1}{2}$ can be obtained by the following change of integration variables: $B_i = xt^\alpha$, $B_j = yt^\alpha$, $\lambda = \lambda' t^{-\alpha}$, $\mu = \mu' t^{-\alpha}$, $s = s' t^{-q\alpha}$ and $B_k = z^{\frac{1}{1-q}} (s')^{-\frac{1}{1-q}} (t')^{-\frac{1}{1-q} + \alpha \frac{q}{1-q}}$ where

$$
\alpha = \frac{\frac{\rho}{\beta(1-q)} - 1}{1 - 2q + \frac{\rho q}{\beta(1-q)}}. \tag{5.11}
$$

$\overline{C(t)}$ can then be evaluated for long times. Expanding in powers of t we find, after dropping primes and evaluating the λ- and the μ-integrals, for the leading term

$$
\overline{C(t)} \simeq v^2 \left(\frac{\rho}{\beta}\right)^2 t^{-2\gamma\left(\frac{\rho}{\beta(1-q)} - 1\right)} \int\limits_0^\infty dx \int\limits_0^\infty dy \oint\limits_\Omega \frac{ds}{2\pi i}
$$
$$
\times \quad \frac{x^{-\frac{\rho}{\beta}} y^{-\frac{\rho}{\beta}}}{s(x+y)\{s(x^q y + x y^q) - x - y\}}
$$
$$
\times \quad \exp\left[-t^\gamma \left\{s(x^{1-q} + y^{1-q}) + v\Gamma\left(1 - \frac{\rho}{\beta(1-q)}\right) s^{\frac{\rho}{\beta(1-q)}}\right\}\right] \tag{5.12a}
$$

where

$$\gamma = \frac{\rho}{\beta \left(1 - 2q + \frac{\rho q}{\beta(1-q)}\right)}. \tag{5.12b}$$

Again we only have a simple pole inside the contour Ω, at $s = (x+y)/(xy^q + x^q y)$. After setting $y = \lambda x$ we have

$$\overline{C(t)} = v^2 \left(\frac{\rho}{\beta}\right)^2 t^{-2\left(\frac{\rho}{\beta(1-q)}-1\right)\gamma} \int\limits_0^\infty dx \int\limits_0^\infty d\lambda \frac{x^{-1-\frac{2\rho}{\beta}}\lambda^{-\frac{\rho}{\beta}}}{(1+\lambda)^2} \exp\{-f(x,\lambda)t^\gamma\} \tag{5.13a}$$

where

$$f(x,\lambda) = x^{1-2q}\frac{1+\lambda}{\lambda^q} + v\Gamma\left(1 - \frac{\rho}{\beta(1-q)}\right) x^{-\frac{\rho q}{\beta(1-q)}} \left(\frac{1+\lambda}{\lambda^q+\lambda}\right)^{\frac{\rho}{\beta(1-q)}}. \tag{5.13b}$$

We shall now apply the method of steepest descent around the minimum $f(x_0,\lambda_0)$ of f, which occurs at

$$x_0 = \left\{\frac{\rho q v}{\beta(1-q)(1-2q)}\Gamma\left(1 - \frac{\rho}{\beta(1-q)}\right)\right\}^{1/\left(1-2q+\frac{\rho q}{\beta(1-q)}\right)} \tag{5.14a}$$

and

$$\lambda_0 = 1. \tag{5.14b}$$

Both the trace and the determinant of the Hessian of $f(x,\lambda)$ are positive at (x_0,λ_0). Hence we find in the long time limit

$$\overline{C(t)} \simeq B_-\left(\frac{\rho}{\beta},q\right) t^{\gamma_t} e^{-D\left(\frac{\rho}{\beta},q\right)t^\gamma} \qquad (q < \tfrac{1}{2}, \tfrac{\rho}{\beta} < 1-q) \tag{5.15a}$$

where

$$\gamma_t = -\left(\frac{2\rho}{\beta(1-q)} - 1\right)\gamma \tag{5.15b}$$

and γ has been given by Eq. (5.12b). Furthermore $D(\frac{\rho}{\beta},q) = f(x_0,\lambda_0)$ and

$$B_-\left(\frac{\rho}{\beta},q\right) = \frac{\rho\pi v(1-q)}{\beta q \Gamma\left(1 - \frac{\rho}{\beta(1-q)}\right)} x_0^{\frac{\rho q}{\beta(1-q)}}$$

$$\times \left\{\left(1 + \frac{\rho q}{\beta(1-q)(1-2q)}\right)(2q-3)(q-1)\right\}^{-\frac{1}{2}}. \tag{5.16}$$

This analysis strictly holds for $\frac{1}{2} > q > 0$. For $q \to 0$ we have $\gamma \to \frac{\rho}{\beta}$ and $D(\frac{\rho}{\beta},q) \to v\Gamma(1-\frac{\rho}{\beta})$, corresponding to the case of homogeneous barriers (section 4), but $\gamma_t \to -\gamma(\frac{2\rho}{\beta}-1) \neq 0$ and $B_-(\frac{\rho}{\beta},q)$ diverges for $q \to 0$.
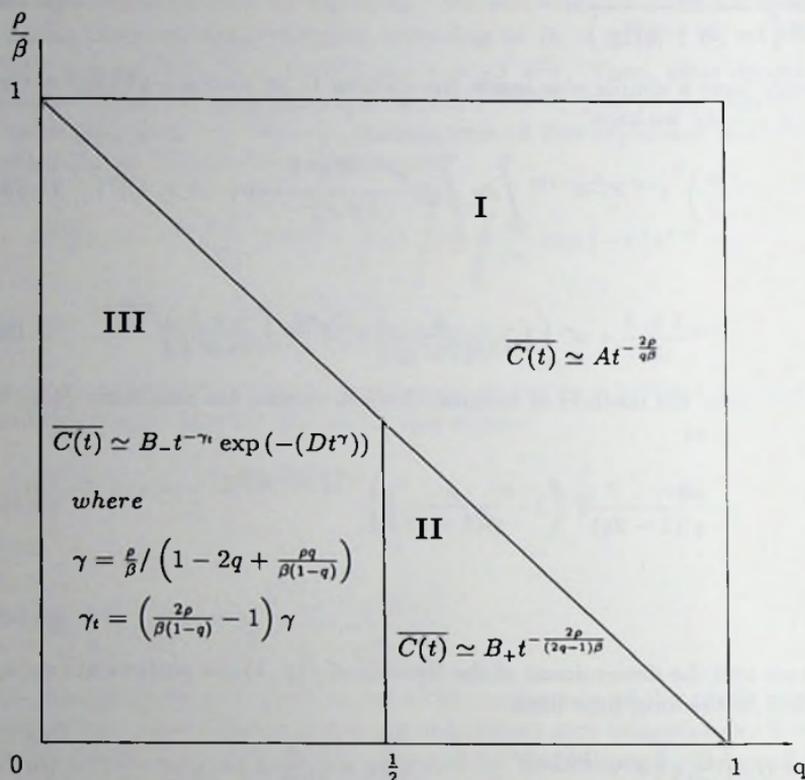
Fig. 1. Exact results for the asymptotic behaviour of the equilibrium auto-correlation function in the kinetic random energy model. The values of the constants A, $B_{\pm}$, and D are given in the text.

5.3 Discussion and qualitative understanding of the results

The results of this section are summarized in Fig. 1. In region **I**, where $\frac{\rho}{\beta} > 1 - q$, we find power law relaxation after scaling the transition rates with N. In region **II** we also find power law behaviour but with a different exponent. The limit $q \to 1$ for both regimes corresponds to the case considered elsewhere [9]. Finally, in region **III**, we find stretched exponential behaviour. The above results can be understood from the following qualitative argument. The Markov process described by the master equation (2.6) describes the motion of the system in the space of energy levels: the system arrives at some instant in level i and stays for some time at that level before jumping to the next level j, etc. The time that the system spends in a level, the *waiting time*, is distributed exponentially with a characteristic time

34

$$\tau_i^{-1} = \sum_{k \neq i} W_{ki}. \tag{5.17}$$

The probability to jump from level i to level j is given by the *transition probability*

$$p_{ji} = \frac{W_{ji}}{\sum\limits_{k \neq i} W_{ki}} \qquad (j \neq i). \tag{5.18}$$

We shall first analyze the waiting times. With (5.17) and (2.9) we have for $V_i = N^{-p} B_i^{-q}$:

$$\tau_i^{-1} = N^{-2p} \sum_{k \neq i} e^{-(1-q)\beta \epsilon_k + \beta q \epsilon_i}. \tag{5.19}$$

We can approximate the sum by its average (see (2.5))

$$\sum_{k \neq i} e^{-(1-q)\beta \epsilon_k} \sim \rho \int_{\epsilon_-}^{\epsilon_c} d\epsilon' e^{\{\rho-(1-q)\beta\}\epsilon'} = \rho \frac{e^{\{\rho-(1-q)\beta\}\epsilon_c} - e^{\{\rho-(1-q)\beta\}\epsilon_-}}{\rho - (1-q)\beta} \tag{5.20}$$

where ϵ_- denotes the lowest, negative, energy in the given realization of the model. The result depends on the sign of $\rho - (1-q)\beta$. For $\frac{\rho}{\beta} > 1 - q$ we find

$$\tau_i^{-1} \sim \rho \frac{N^{-2p+1-(1-q)\frac{\rho}{\beta}} e^{\beta q \epsilon_i}}{\rho - (1-q)\beta} \tag{5.21}$$

where we used $e^{-\rho \epsilon_c} = v/N$. So unless $p = \frac{1}{2} - \frac{1}{2}(1-q)\frac{\rho}{\beta}$ the relaxation time becomes zero or infinite in the limit of $N \to \infty$. For $\frac{\rho}{\beta} < 1 - q$ we have

$$\tau_i^{-1} \sim \rho \frac{N^{-2p} e^{\{\rho-(1-q)\beta\}\epsilon_- + \beta q \epsilon_i}}{|\rho - (1-q)\beta|} \tag{5.22}$$

which is finite in the limit $N \to \infty$ provided $p = 0$. In both cases the longest waiting times belong to the lowest energy levels.

Then we turn to the transition probabilities. With (5.18) and (2.9) one finds for $V_i = N^{-p} B_i^{-q}$:

$$p_{ji} = \frac{e^{-(1-q)\beta \epsilon_j}}{\sum\limits_{k \neq i} e^{-(1-q)\beta \epsilon_k}} \tag{5.23}$$

The scaling with N of the transition rates does not enter these probabilities. So the probability $p(\epsilon)d\epsilon$ to jump to a level with energy in the range $(\epsilon, \epsilon + d\epsilon)$ can be approximated by

$$p(\epsilon)d\epsilon \sim e^{\{\rho-(1-q)\beta\}\epsilon} d\epsilon \tag{5.24}$$

for $\epsilon > \epsilon_-$. Again we see that the result is determined by the sign of $\rho - (1-q)\beta$. For $\frac{\rho}{\beta} > 1 - q$ most jumps will be towards energy levels around ϵ_c and so transitions between low-lying energy levels, where the system spends most of its time, almost always occur via the highest energy levels. For $\frac{\rho}{\beta} < 1-q$ most jumps will be towards the lowest levels (near ϵ_-) and so jumps between low energy levels occur directly, not via the high energy levels.

6 Random barriers

Finally we shall treat the case where the V_i are independent random variables distributed according to some distribution $p_V(V_i)$. In particular they do not depend on the energies ε_i of the levels.

When we introduce integral representations (see Eq. (4.4)) for the partition function and for the sum in the denominator, Eq. (3.10), which is again our starting point, becomes

$$
\begin{aligned}
C(t) &= \oint_\Omega \frac{ds}{2\pi i s} \int_0^\infty d\lambda \int_0^\infty d\mu \sum_{i=1}^N \sum_{j\neq i}^N \frac{V_i B_i V_j B_j}{(s-V_i)(s-V_j)} \\
&\times \exp\left\{ -\sum_{k\neq i,j}^N B_k \left(stV_k + \lambda + \frac{\mu V_k}{s-V_k} \right) \right\}
\end{aligned}
\tag{6.1}
$$

We first calculate the average over all sets of random levels using the probability density function (4.3) for the Boltzmann factors:

$$
\begin{aligned}
\overline{C(t)}^B &= \lim_{N\to\infty} \oint_\Omega \frac{ds}{2\pi i s} \int_0^\infty d\lambda \int_0^\infty d\mu \\
&\times \frac{v}{N}\frac{\rho}{\beta} \sum_{i=1}^N \int_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^\infty dB_i B_i^{-\frac{\rho}{\beta}} \frac{V_i}{s-V_i} \exp\left\{ -B_i\left(stV_i+\lambda+\mu\frac{V_i}{s-V_i}\right) \right\} \\
&\times \frac{v}{N}\frac{\rho}{\beta} \sum_{j\neq i}^N \int_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^\infty dB_j B_j^{-\frac{\rho}{\beta}} \frac{V_j}{s-V_j} \exp\left\{ -B_j\left(stV_j+\lambda+\mu\frac{V_j}{s-V_j}\right) \right\} \\
&\times \prod_{k\neq i,j}^N \frac{v}{N}\frac{\rho}{\beta} \int_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^\infty dB_k B_k^{-1-\frac{\rho}{\beta}} \exp\left\{ -B_k\left(stV_k+\lambda+\mu\frac{V_k}{s-V_k}\right) \right\}
\end{aligned}
\tag{6.2}
$$

In order to evaluate this average we need two types of integrals,

$$
\frac{v}{N}\frac{\rho}{\beta} \int_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^\infty dB B^{-\frac{\rho}{\beta}} e^{-\alpha B} = \frac{v}{N}\frac{\rho}{\beta}\alpha^{\frac{\rho}{\beta}-1}\Gamma\left(1-\frac{\rho}{\beta}\right) + \mathcal{O}\left(N^{-\frac{\beta}{\rho}}\right)
\tag{6.3a}
$$

and

$$
\frac{v}{N}\frac{\rho}{\beta} \int_{\left(\frac{v}{N}\right)^{\frac{\beta}{\rho}}}^\infty dB B^{-1-\frac{\rho}{\beta}} e^{-\alpha B} = 1 - \frac{v}{N}\alpha^{\frac{\rho}{\beta}}\Gamma\left(1-\frac{\rho}{\beta}\right) + \mathcal{O}\left(N^{-\frac{\beta}{\rho}}\right).
\tag{6.3b}
$$

36

The latter integral is calculated as in Eq. (4.6). Substitution of (6.3a) and (6.3b) in Eq. (6.2) gives:

$$
\overline{C(t)}^B = \lim_{N \to \infty} \oint_\Omega \frac{ds}{2\pi i s} \int_0^\infty d\lambda \int_0^\infty d\mu
$$

$$
\times \frac{v}{N} \frac{\rho}{\beta} \Gamma\left(1 - \frac{\rho}{\beta}\right) \sum_{i=1}^N \frac{V_i}{s - V_i} \left(s t V_i + \lambda + \frac{\mu V_i}{s - V_i}\right)^{\frac{\rho}{\beta} - 1}
$$

$$
\times \frac{v}{N} \frac{\rho}{\beta} \Gamma\left(1 - \frac{\rho}{\beta}\right) \sum_{j \neq i}^N \frac{V_j}{s - V_j} \left(s t V_j + \lambda + \frac{\mu V_j}{s - V_j}\right)^{\frac{\rho}{\beta} - 1}
$$

$$
\times \exp\left\{-\frac{v}{N} \Gamma\left(1 - \frac{\rho}{\beta}\right) \sum_{k \neq i,j}^N \left(s t V_k + \lambda + \frac{\mu V_k}{s - V_k}\right)^{\frac{\rho}{\beta}}\right\} \tag{6.4}
$$

The next step is to analyze $\overline{C(t)}^B$ for long times. To this purpose we set $\lambda = \lambda' t$ and $\mu = \mu' t$ and drop the primes afterwards. The integrand then shows a boundary maximum for $\lambda = 0$ and $\mu = 0$. We apply the method of steepest descent and find

$$
\overline{C(t)}^B \simeq \lim_{N \to \infty} \oint_\Omega \frac{ds}{2\pi i s} \sum_{i=1}^N \frac{V_i^{\frac{\rho}{\beta}}}{s - V_i} \sum_{j \neq i}^N \frac{V_j^{\frac{\rho}{\beta}}}{s - V_j} \left(\sum_{k \neq i,j}^N V_k^{\frac{\rho}{\beta} - 1} \sum_{k \neq i,j}^N \frac{V_k^{\frac{\rho}{\beta}}}{s - V_k}\right)^{-1}
$$

$$
\times \exp\left\{-\frac{v}{N} \Gamma\left(1 - \frac{\rho}{\beta}\right) (st)^{\frac{\rho}{\beta}} \sum_{k \neq i,j}^N V_k^{\frac{\rho}{\beta}}\right\} \tag{6.5}
$$

The final step involves the average over all possible sets of barriers. In order to perform this average we again introduce integral representations (see Eq. (4.4)) for the sums in the denominator:

$$
\overline{C(t)}^B \simeq \lim_{N \to \infty} \oint_\Omega \frac{ds}{2\pi i s} \int_0^\infty d\lambda \int_0^\infty d\mu \frac{1}{N} \sum_{i=1}^N \frac{V_i^{\frac{\rho}{\beta}}}{s - V_i} \frac{1}{N} \sum_{j \neq i}^N \frac{V_j^{\frac{\rho}{\beta}}}{s - V_j}
$$

$$
\times \exp\left[-\frac{1}{N} \sum_{k \neq i,j}^N \left\{v\Gamma\left(1 - \frac{\rho}{\beta}\right) (st)^{\frac{\rho}{\beta}} V_k^{\frac{\rho}{\beta}} + \lambda V_k^{\frac{\rho}{\beta} - 1} + \frac{\mu V_k^{\frac{\rho}{\beta}}}{s - V_k}\right\}\right] \tag{6.6}
$$

The sums of functions of the random barriers V_i are self-averaging:

$$
\overline{C(t)}^{B,V} \simeq \oint_\Omega \frac{ds}{2\pi i s} \int_0^\infty d\lambda \int_0^\infty d\mu \left(\overline{\frac{V^{\frac{\rho}{\beta}}}{s - V}}^V\right)^2
$$

$$
\times \exp\left\{-v\Gamma\left(1 - \frac{\rho}{\beta}\right) (st)^{\frac{\rho}{\beta}} \overline{V^{\frac{\rho}{\beta}}}^V - \lambda \overline{V^{\frac{\rho}{\beta} - 1}}^V - \mu \overline{\frac{V^{\frac{\rho}{\beta}}}{s - V}}^V\right\} \tag{6.7}
$$

The λ- and μ-integrations can be performed and one finds

37

$$\overline{C(t)}^{B,V} \simeq \oint_{\Omega} \frac{ds}{2\pi i s} \frac{\overline{\frac{V^{\frac{\rho}{\beta}}}{s-V}}^{V}}{s-V} \left(\overline{V^{\frac{\rho}{\beta}-1}}^{V} \right)^{-1} \exp\left\{ -v\Gamma\left(1-\frac{\rho}{\beta}\right)(st)^{\frac{\rho}{\beta}} \overline{V^{\frac{\rho}{\beta}}}^{V} \right\}. \quad (6.8)$$

We shall now discuss some barrier distributions. A special distribution is

$$p_V(V) = \delta(V - \sqrt{w_0}). \quad (6.9)$$

Using this distribution in (6.8) one recovers Eq. (4.10) of section 4. More generally we expect all distributions that are centered around some mean value to yield stretched exponential relaxation as is the case in Eq. (4.10). More interesting results might be expected from a barrier distribution with a large spread. For ease of calculations we choose

$$p_V(V) = \begin{cases} C_{ab} V^{-\frac{\rho}{\beta}} & \text{for} \quad 0 < a < V < b < \infty \\ 0 & \text{otherwise} \end{cases} \quad (6.10a)$$

where the normalization constant is given by

$$C_{ab}^{-1} = \frac{b^{1-\frac{\rho}{\beta}} - a^{1-\frac{\rho}{\beta}}}{1 - \frac{\rho}{\beta}}. \quad (6.10b)$$

This distribution leads to the following averages,

$$\overline{V^{\frac{\rho}{\beta}}}^{V} = C_{ab}(b-a), \quad (6.11a)$$

$$\overline{V^{\frac{\rho}{\beta}-1}}^{V} = C_{ab}\log\left(\frac{b}{a}\right), \quad (6.11b)$$

$$\overline{\frac{V^{\frac{\rho}{\beta}}}{s-V}}^{V} = C_{ab}\log\left(\frac{s-a}{s-b}\right). \quad (6.11c)$$

Substituting these in Eq. (6.8) gives

$$\overline{C(t)}^{B,V} \simeq \log^{-1}\left(\frac{b}{a}\right) \oint_{\Omega} \frac{ds}{2\pi i s} \log\left(\frac{s-a}{s-b}\right) \exp\left\{ -w(st)^{\frac{\rho}{\beta}} \right\} \quad (6.12a)$$

where Ω encloses all the poles of the integrand on the positive real axis and where we set

$$w = v\Gamma\left(1-\frac{\rho}{\beta}\right) C_{ab}(b-a). \quad (6.12b)$$

Due to the logarithm in the integrand there is a branch cut inside the contour Ω on the positive real axis between a and b. Integrating around this cut yields

$$\overline{C(t)}^{B,V} \simeq \log^{-1}\left(\frac{b}{a}\right) \int_a^b \frac{dx}{x} \exp\left\{ -w(xt)^{\frac{\rho}{\beta}} \right\}. \quad (6.13)$$

38

Finally we apply the method of steepest descent, the integrand being maximal for $x = a$. Hence we again find, for large times, stretched exponential relaxation:

$$\overline{C(t)}^{B,V} \simeq w^{-1} \log^{-1}\left(\frac{b}{a}\right) (at)^{-\frac{\rho}{\beta}} \exp\left\{-w(at)^{\frac{\rho}{\beta}}\right\} \tag{6.14}$$

In the limit $a \to 0$ the above expression diverges. This corresponds to the situation where the barriers may be infinitely high and this of course frustrates the relaxation.

Performing the calculations for the milder distribution $p(V) \sim V^{n-\frac{\rho}{\rho}}$ $(n > 0)$ one also obtains stretched exponential relaxation. Hence we conclude that there is stretched exponential behaviour of the equilibrium autocorrelation function whenever there are no infinite barriers.

Appendix A: Derivation of an exponential from a Gaussian distribution

In this appendix we shall derive the exponential probability distribution $P_N(E)$ for the lowest N energy levels of a 2^N level random energy model with a Gaussian level distribution $P(E)$ given by Eq. (2.1). Let $\mathcal{E}_N$ be an energy such that the expected number of energy levels below $\mathcal{E}_N$ is equal to N, that is

$$2^N \int_{-\infty}^{\mathcal{E}_N} dE\, P(E) = N. \tag{A.1}$$

Using the asymptotic expansion for the complementary error function [11] we find for large negative $\mathcal{E}_N$:

$$N = 2^N \frac{\sqrt{NJ^2}e^{-\mathcal{E}_N^2/NJ^2}}{2\sqrt{\pi}(-\mathcal{E}_N)}\left\{1 + \mathcal{O}\left(\frac{NJ^2}{\mathcal{E}_N^2}\right)\right\}. \tag{A.2}$$

Solve this expression for $\mathcal{E}_N$:

$$\mathcal{E}_N = E_0(N) + \frac{J\log\left\{2N\sqrt{\pi\log 2}\right\}}{2\sqrt{\log 2}} + o(1) \qquad \text{as } N \to \infty \tag{A.3}$$

where

$$E_0(N) = -NJ\sqrt{\log 2} + \frac{J\log N}{4\sqrt{\log 2}}. \tag{A.4}$$

The first term of the right hand side of Eq. (A.4) is the extensive part of the REM free energy in the frozen phase. In the limit of large N we may consider the N lowest energy levels $E_1, E_2, \ldots, E_N$ as independent random variables drawn from the probability distribution

$$P_N(E) = \begin{cases} \dfrac{P(E)}{\int_{-\infty}^{\mathcal{E}_N} dE'\, P(E')} & \text{for } E \le \mathcal{E}_N \\[3mm] 0 & \text{for } E > \mathcal{E}_N \end{cases} \tag{A.5}$$

Since these levels differ from $E_0(\mathcal{N})$ by at most an amount of order 1 we can put

$$E_i = E_0(\mathcal{N}) + \varepsilon_i, \qquad\qquad (i = 1, 2, \ldots, N) \qquad\qquad \text{(A.6a)}$$

$$\mathcal{E}_N = E_0(\mathcal{N}) + \varepsilon_c \qquad\qquad\qquad \text{(A.6b)}$$

and expand

$$\frac{E_i^2}{\mathcal{N}J^2} = \frac{(E_0(\mathcal{N}) + \varepsilon_i)^2}{\mathcal{N}J^2}$$

$$= \mathcal{N}\log 2 + \tfrac{1}{2}\log\mathcal{N} - \frac{2\sqrt{\log 2}}{J}\varepsilon_i + o(1) \qquad \text{as } \mathcal{N} \to \infty \qquad \text{(A.7)}$$

Upon using this expansion for $P(E)$ (see Eq. (2.1)) in (A.5) and with (A.1) we find

$$p(\varepsilon) \equiv P_N(E_0 + \varepsilon) = \begin{cases} \rho e^{\rho(\varepsilon - \varepsilon_c)} & \varepsilon \le \varepsilon_c \\[2mm] 0 & \varepsilon > \varepsilon_c \end{cases} \qquad\qquad \text{(A.8)}$$

where

$$\rho^{-1} \equiv T_c = \frac{J}{2\sqrt{\log 2}}, \qquad\qquad\qquad \text{(A.9)}$$

is the critical temperature of the REM. From (A.1) and (A.6b) it also follows that

$$v \equiv N e^{-\rho\varepsilon_c} = \frac{1}{2\sqrt{\pi\log 2}}. \qquad\qquad\qquad \text{(A.10)}$$

Equations (A.9) and (A.10) relate the parameters ρ and ε_c of the exponential distribution to the parameter J of the original Gaussian distribution. The thermodynamic limit now corresponds to taking $N \to \infty$ and $\varepsilon_c \to \infty$ while keeping $N e^{-\rho\varepsilon_c} = v$ constant.

References

[1] Derrida, B., Phys. Rev. Lett. **45** (1980) 79.

[2] Derrida, B., Phys. Rev. **B 24** (1981) 2613.

[3] Sherrington, D. and Kirkpatrick, S., Phys. Rev. Lett. **35** (1975) 1792.

[4] Kirkpatrick, S. and Sherrington, D., Phys. Rev. **B 17** (1978) 4384.

[5] Parisi, G., J. Phys. **A 13** (1980) 1101.

[6] Parisi, G., Phys. Rev. Lett. **50** (1983) 1946.

[7] Mézard, M., Parisi, G., and Virasoro, M. A., J. Phys. (Paris) Lett. **46** (1985) L217.

[8] De Dominicis, C., Orland, H., and Lainée, F., J. Phys. (Paris) Lett. **46** (1985) L463.

[9] Koper, G. J. M. and Hilhorst, H. J., Europhys. Lett. **3** (1987) 1213.

[10] Van Kampen, N. G., *Stochastic processes in physics and chemistry* (North-Holland, Amsterdam, 1981).

[11] Abramowitz, M. and Stegun, I. A., *Handbook of mathematical functions* (National Bureau of Standards, Washington DC, 1964).

Chapter III

A Domain Theory for
Aging Effects in Spin Glasses *

In this chapter a theory of aging in spin glasses, subjected to a time-dependent temperature and magnetic field, is presented. It is shown how the theory explains a variety of aging effects as have been observed experimentally.

1 Introduction

The phenomenon of aging in spin glasses was discovered in 1983 by Lundgren et al. [1]. It has since then been confirmed and investigated by many workers on a variety of spin glasses. Here a theory of aging in spin glasses is presented. In subsection 1.1 the context within which aging is observed is briefly discussed and in subsection 1.2 a summary of this chapter is given.

1.1 Aging in spin glasses

In a typical spin glass relaxation experiment ("field jump" experiment) one rapidly cools a sample in a magnetic field H to a temperature T below the freezing temperature T_f. One then cuts the field and observes the relaxation of the magnetization $M(t)$, which in that case is called the *thermoremanent* magnetization: $M(t) = M_{TRM}(t)$. Alternatively one may cool the sample in zero field to the temperature T, then switch a field on and watch the increase of $M(t)$, then called the *zero field cooled* magnetization: $M(t) = M_{ZFC}(t)$. Obviously

$$\lim_{t \to \infty} M_{TRM}(t) = 0, \qquad \lim_{t \to \infty} M_{ZFC}(t) = M_{eq}(H), \qquad (1.1)$$

where $M_{eq}(H)$ denotes the equilibrium magnetization in a field H. Experimentally one finds that $M_{TRM}(t)$ and $M_{ZFC}(t)$ relax extremely slowly (the relaxation is spread over a microscopic, a macroscopic and an astronomical time scale!) and nonexponentially. Except for temperatures very close to T_f, the system is still far from equilibrium even after the longest possible observation time, which is of the order of days.

Several theories have attempted to find the functional dependence of $M(t)$ on time (see e.g. the review article [2]). In particular, logarithmic, power law and stretched exponential (or Kohlrausch) relaxation have been proposed and used to fit experimental data. It gradually appeared, however, and was first pointed out by Lundgren et al. [1], that in very small magnetic fields these relaxation curves are not uniquely determined: they turn out to depend on the *waiting time* t_w that has elapsed between the moment $t = 0$ at which the temperature T_f was reached, and the moment $t = t_w$ at which the field was cut (in a TRM experiment) or switched on (in a ZFC experiment).

During the waiting time one has not detected experimentally any changes in the system; in particular its magnetization remains constant (except for a slight ($\sim 1\%$) decrease after field cooling). Nevertheless, the *response* of the system to a jump in the magnetic field at $t = t_w$ becomes *slower* as t_w gets larger [1]. This indicates that during the waiting time the system is not in equilibrium but only slowly evolves towards it: it *ages*. Aging effects also occur when not the field but the temperature is varied (see Nordblad et al. [3] and Refregier et al. [4]), and can be detected by measurements not only of the magnetization but also of the ac susceptibility (see Lundgren et al. [5,6], Svedlindh et al. [7], and Refregier et al. [4]) and of magnetic noise (see Ocio et al. [8]).

1.2 Summary

In section 2 we succinctly review the phenomenological linear response description of aging and some of its implications. We argue that there are experimentally observed aging phenomena for which this description fails. In the remainder of this work we then develop a theory which is also capable of describing a variety of experiments in which nonlinearities dominate.

The theory describes the magnetic response of the system in the presence of an arbitrary (but small) time-dependent magnetic field $H(t)$ and temperature $T(t)$. Its general framework is given in section 3. The theory postulates a mesoscopic picture of time-dependent domains of correlated spins and assumes that the magnetic response behavior of a spin depends on the size of the domain in which it is located. A fundamental feature of the theory is the interplay between two lengths. The first one is the time-dependent linear dimension of a domain. The second one is an overlap length $\ell(\Delta T, \Delta H)$ which indicates up until which length scale we cannot distinguish two thermodynamic equilibrium states whose temperature and magnetic field differ by amounts ΔT and ΔH, respectively; this length generalizes a length scale introduced by Bray and Moore [9] to nonzero field.

In section 4 we present the simplest application of the theory, viz. to the field jump experiment at a constant temperature. In the limit of small magnetic field jumps one recovers the linear response phenomenology. We then turn towards nonlinear phenomena. These are due to restrictions on the domain growth imposed by the overlap length. Our results for both the linear and the nonlinear effects are found to agree qualitatively with the experimental data [10]. In section 5 we apply the theory to more complicated recent relaxation experiments [3,4] in which one carries out both magnetic field and temperature jumps. We review the experimental data and show how the theory accounts for the observed phenomena. In section 6 we consider experiments [4,6] in which one observes the time dependence of the ac magnetic susceptibility following temperature jumps. We obtain again theoretical curves in good agreement with the experiments.

To our knowledge, the only earlier theoretical work which specifically focusses on aging in spin glasses is a mean field approach by Ginzburg [11]. In section 7 we comment on the relation of our work to Ginzburg's as well as to some more general work on spin glass dynamics, and we make a few concluding remarks.

2 Linear and nonlinear response

In subsection 2.1 we briefly expose the phenomenological linear response description of a spin glass. We emphasize that all relevant quantities can be expressed in terms of a single response function $R(t, t')$. This is done in particular for the time-dependent ac magnetic susceptibility in subsection 2.2. Although most of the ideas of these two subsections occur and have been used in the literature, a coherent presentation has, to our knowledge, not been given. In subsection 2.3 we indicate how experimentally the limits to the validity of the linear response description

manifest themselves.

2.1 Linear response

The waiting time effect mentioned in section 1.1 has been the subject of detailed studies. In field jump experiments one has varied the waiting time t_w from a few seconds to a day and over. One has also studied the response $M(t)$ to time-dependent fields $H(t)$ more general [12,13] than the step functions of the TRM and ZFC experiments.

It has been firmly established by several groups and in a variety of experiments that for reasonably small field values (say $\lesssim 10$ Gauss) the response $M(t)$, $t > 0$, is *linear* in the applied field $H(t')$, $t' > 0$. (We recall that $t = 0$ is the instant of time at which the sample was cooled to the temperature T below T_f.) The most general relationship which expresses this fact is

$$M(t) = M(0) + N\chi_{eq} \int_0^t dt' G(t,t')\dot{H}(t') \tag{2.1a}$$

$$= N\chi_{eq} \left(H(t) - \int_0^t dt' R(t,t')\dot{H}(t') \right). \tag{2.1b}$$

Here χ_{eq} is the equilibrium dc susceptibility in zero field and N the number of spins in the sample. The response function $G(t,t')$ measures the response at time t to a unit increase in magnetic field at time t'. The two expressions (2.1a) and (2.1b) are equivalent, the relaxation function $R(t,t')$ being defined as

$$R(t,t') \equiv 1 - G(t,t'). \tag{2.2}$$

We have the properties

$$R(t,t) = 1, \quad \lim_{t-t' \to \infty} R(t,t') = 0. \tag{2.3}$$

Furthermore, when the time following the quench tends to infinity, R tends to the equilibrium relaxation function:

$$\lim_{t \to \infty} R(t, t-\tau) \equiv R_{eq}(\tau) \equiv 1 - G_{eq}(\tau) \tag{2.4}$$

The relations (2.1a) and (2.1b) essentially already occur in Ref. [12], but their significance has been insufficiently stressed: in the linear regime the function $G(t,t')$ gives a complete description of the spin glass response to small magnetic field changes, including the aging behavior.

In what follows we shall frequently consider the field jump experiment described by

$$H(t) = \begin{cases} H_0 & (t \le t_w) \\ H_1 = H_0 - \Delta H & (t > t_w). \end{cases} \tag{2.5}$$

44

According to the linear response relationship (2.1b) the excess magnetization

$$\Delta M(t) \equiv M(t) - N\chi_{eq}H_1 \tag{2.6}$$

decays as

$$\Delta M(t) = N\chi_{eq}\Delta H R(t, t_w) \quad (t > t_w). \tag{2.7}$$

The thermoremanent magnetization $M_{TRM}(t)$ and the zero field cooled magnetization $M_{ZFC}(t)$ discussed in section 1.1 are special cases of equations (2.5) and (2.7) obtained by setting $H_1 = 0$, $H_0 = H$ and $H_1 = H$, $H_0 = 0$, respectively. As a consequence

$$M_{TRM}(t) + M_{ZFC}(t) = N\chi_{eq}H, \tag{2.8}$$

a relation satisfied to a high degree of accuracy in experimental results [13,14].

2.2 Time-dependent ac susceptibility

Within the above formalism one can discuss the time-dependent ac susceptibility. We suppose that a sample is cooled in zero field at time $t = 0$ and that after a waiting time t_w an ac field with frequency ω is applied:

$$H(t) = \begin{cases} 0 & (t \leq t_w) \\ H_0 e^{-i\omega t} & (t > t_w). \end{cases} \tag{2.9}$$

Upon applying a partial integration to (2.1b) and using (2.9) we find

$$
\begin{aligned}
M(t) &= N\chi_{eq}\left(R(t,0)H(0) + \int_0^t dt' \frac{dR(t,t')}{dt'}H(t')\right) \\
&= -N\chi_{eq}H(t)\int_0^{t-t_w} d\tau \frac{dR(t,t-\tau)}{d\tau}e^{i\omega\tau} \quad (t > t_w).
\end{aligned} \tag{2.10}
$$

If the coefficient of $H(t)$ in (2.10) varies only little over a period $2\pi/\omega$, then it can be identified with the *time-dependent ac susceptibility*,

$$\chi(\omega; t) = -\chi_{eq}\int_0^{t-t_w} d\tau \frac{dR(t,t-\tau)}{d\tau}e^{i\omega\tau}. \tag{2.11}$$

Lundgren et al. [6] were the first to report on such time-dependence of the ac-susceptibility. They found power law decay towards the equilibrium value with an exponent ≈ 0.5 in their experiments on Cu(4at%Mn). (This power is identical to the one occurring in R if R is assumed to decay as a stretched exponential; see Ocio et al. [8].)

In the limit $t \to \infty$ the expression (2.11) reduces to the ac susceptibility of the system in equilibrium,

$$
\begin{aligned}
\chi(\omega) &\equiv \lim_{t \to \infty} \chi(\omega; t) \\
&= -\chi_{eq} \int_0^\infty d\tau \frac{dR_{eq}(\tau)}{d\tau} e^{i\omega\tau},
\end{aligned}
\tag{2.12}
$$

where (2.4) has been used; with the aid of (2.3) one sees that $\chi(0) = \chi_{eq}$.

In order to proceed we now exploit the fact that the experimental data (e.g. [10,15]) are well described by a relaxation function $R(t, t')$ of the form

$$
R(t, t') = \left(1 + \frac{t - t'}{t_0}\right)^{-\alpha} F(t, t'),
\tag{2.13}
$$

in which α is a temperature dependent exponent in the range from 0.05 to 0.2 and F is a function for which $F(t, t) = 1$ and

$$
\lim_{t \to \infty} F(t, t - \tau) = 1 \qquad (\tau \text{ fixed}).
\tag{2.14}
$$

The nontrivial implication is that from (2.4), (2.13) and (2.14) one has

$$
R_{eq}(\tau) = \left(1 + \frac{\tau}{t_0}\right)^{-\alpha}.
\tag{2.15}
$$

Such power law relaxation in a system in equilibrium has also been found, e.g., in Monte Carlo simulations [16]. The expression (2.15) satisfies $R_{eq}(0) = 1$; however, the time constant t_0 is so small that for all times τ for which data are available (i.e. $\tau \gtrsim 10^{-6}$ s [12]) one has $\tau/t_0 \gg 1$, and hence $R_{eq}(\tau) \simeq (\tau/t_0)^{-\alpha}$.

In the remainder of this work the form (2.15) of the equilibrium relaxation function will be taken for granted, and we shall be concerned with the mechanisms by which deviations from it are generated when the system is out of equilibrium. These deviations are represented by the function F; in the following sections explicit expressions for F will be derived on the basis of an underlying physical picture.

Upon using that $|\omega| t_0 \ll 1$ for all frequencies of interest we find from Eq. (2.12) and Eq. (2.15)

$$
\chi(\omega) \simeq \chi_{eq} \left\{ 1 - (|\omega| t_0)^\alpha \Gamma(1 - \alpha) \left(\cos \frac{\alpha\pi}{2} - i \operatorname{sgn} \omega \sin \frac{\alpha\pi}{2} \right) \right\}.
\tag{2.16}
$$

It was first noticed by Lundgren et al. [5] that the real part χ' and the imaginary part χ'' of the ac susceptibility approximately satisfy a "$\frac{\pi}{2}$ law":

$$
\frac{\pi}{2} \frac{\partial \chi'(\omega)}{\partial \log \omega} \simeq -\chi''(\omega) \quad (\omega > 0).
\tag{2.17}
$$

In Ref. [5] the relation (2.17) was proved to hold approximately for a spin system having a broad spectrum of relaxation times. It was subsequently derived and discussed by Pytte and Imry [17] in the wider context of scaling in random systems governed by thermally activated processes. In the present case we may verify explicitly that (2.17) is also valid for the equilibrium susceptibility (2.16), provided α is not too large with respect to unity.

46

2.3 Nonlinear response

We shall especially be interested in explaining what happens beyond the limits of the linear response regime. Experimentally these limits appear, for example, in the field jump experiment described by Eq. (2.5) as we shall briefly discuss now. Eq. (2.7) shows that in the linear response regime $\Delta M(t)/\Delta H$ is independent of both H_0 and ΔH. When ΔH grows one expects, of course, nonlinearities associated with those of the static susceptibility, i.e. with the higher-order terms in the expansion

$$\frac{1}{N}M_{eq}(H) = \chi_{eq}H + \chi_{3,eq}H^3 + \ldots \tag{2.18}$$

These nonlinear terms can no longer be neglected when ΔH becomes comparable to $\sqrt{|\chi_{eq}/\chi_{3,eq}|}$. Recent experimental evidence [12,13,14] indicates, however, that when ΔH increases a different and more important nonlinearity manifests itself in the dynamics well before ΔH becomes comparable to $\sqrt{|\chi_{eq}/\chi_{3,eq}|}$. Its effect is to accelerate the relaxation in a way which *does* depend on ΔH.

In the following sections we shall construct a theory capable of explaining both the linear and the nonlinear observed behavior. We shall do so on the basis of a few physical hypotheses, in particular the one of time-dependent domains. Our hypotheses will lead us to a theory for the spin glass response which is in form identical to the linear response relation (2.1b). There is again a "relaxation function" $R(t,t')$ which contains all basic information. The theory is, however, linear only in appearance, since $R(t,t')$ will in general depend on the size of the magnetic field jumps.

In section 3 we present our general formalism. In section 4 we apply it to experiments with field jumps at a constant temperature, and in section 5 to experiments with both field and temperature jumps. In each case we describe the relevant experiments and show how these can be explained by the present theory.

3 A mesoscopic theory of spin glass response

In this section we present the theoretical framework capable of explaining, at least qualitatively, all the observed phenomena. As we shall see when discussing the experimental results (in particular in sections 5 and 6), past values of temperature and magnetic field leave traces in the structure of the spin glass which at a later time may manifest themselves in the measurements. The main difficulty to be resolved is how to characterize this structure at the mesoscopic level. Our theory will be constructed in agreement with a few postulated principles that we shall list first, in section 3.1. In sections 3.2 to 3.4 these principles will then be converted into an operational tool.

3.1 Principles

(i) A thermodynamic state of a spin glass at a temperature T_1 and in a magnetic field H_1 is characterized by a specific set of spin correlations. We shall,

for short, refer to these as the (T_1, H_1)-correlations. In a system in equilibrium at (T_1, H_1) the range of the (T_1, H_1)-correlations is infinite.

(ii) The correlations of two different thermodynamic equilibrium states, one at (T_1, H_1) and one at (T_2, H_2), are nearly identical up to their "overlap" length $\ell(T_1 - T_2, H_1 - H_2)$. This length scale diverges as $(T_2, H_2) \rightarrow (T_1, H_1)$. In this sense, in a system (whether or not in equilibrium) which possesses (T_1, H_1)-correlations there also exist (T_2, H_2)-correlations, and vice versa.

(iii) At any time, a nonequilibrium spin glass may be analyzed with respect to its (T_1, H_1)-correlations for an arbitrary choice of (T_1, H_1). One then finds, in general, that the system is composed of domains within which there exist (T_1, H_1)-correlations, but beyond which the (T_1, H_1)-correlations are destroyed. We shall, for short, refer to these as the (T_1, H_1)-domains. For a system in equilibrium at (T, H) one finds a single infinite (T, H)-domain. However in the same system one finds a distribution of (T_1, H_1)-domains centered around $\ell(T_1 - T, H_1 - H)$. In a general nonequilibrium system there will be, at a time t, a *distribution* $p(s, t, T_1, H_1)$ of the sizes s of the (T_1, H_1)-domains. (This distribution does not itself play a predominant role in what follows, but we will comment on it in the appendix.)

(iv) At the time $t = 0$, i.e. just after the quench to a temperature $T < T_f$, the system is totally disordered and all domain sizes are zero (i.e. of the order of the typical spin spacing).

(v) In a nonequilibrium system placed at a certain time t in a heat bath of constant temperature T and in a constant magnetic field H, the (T, H)-domains will start growing without limit. Furthermore, for any $(T_1, H_1) \neq (T, H)$, the (T_1, H_1)-domains will grow until they reach the overlap length $\ell(T_1 - T, H_1 - H)$. If at time t any (T_1, H_1)-domains larger than this size are present, these will subsequently be broken up into domains of this maximum size. The laws governing the growth and breaking-up have to be specified.

(vi) We describe magnetic relaxation of the spins by a linear response relation. At time t the maximum relaxation time for a spin is determined by the size of the $(T(t), H(t))$-domain to which the spin – at that time – belongs. The response function of a spin in a domain with a time-varying size $s(t)$ is a complicated functional of $s(t)$, to be specified below.

The necessity to consider, *simultaneously and within one and the same system*, (T, H)-domains for more than one value of (T, H) (or even for a continuum of such values) appears when there are time-dependent fields or temperatures. Such a description is essential for a correct qualitative understanding of e.g. the modified field jump experiment of section 5 and the susceptibility experiments of section 6. But it is also logically unavoidable: if e.g. the field $H(t)$ keeps alternating between two values H_1 and H_2, then neither the (T, H_1)-domain structure nor the (T, H_2)-domain structure should be privileged.

We have tacitly assumed, for simplicity, that only a single equilibrium state exists for each pair (T, H). If more states exist, labeled by an index α, then we have to denote the equilibrium by a triplet of parameters (T, H, α), and the present theory has to be generalized accordingly. It would remain true, however, that for

48

$t \to \infty$ at constant T and H the system will tend to form a single infinite domain with correlations which are characteristic of a triplet (T, H, α) for some α.

3.2 The overlap length $\ell(\Delta T, \Delta H)$

We first discuss a length which was introduced by Bray and Moore [9,18] (but see also Ref. [19]) within the context of the Fisher and Huse [19] droplet model for the spin glass phase. In this droplet model one supposes that spin glass behaviour can be understood in terms of reversals ("excitations") of spin clusters ("droplets"). For large ℓ, the free energy F_ℓ associated with a typical low-lying excitation of a droplet of linear size ℓ is assumed to scale as $F_\ell \sim J(\ell/a)^y$, where J is the typical coupling strength between neighbouring spins and a the typical spin spacing.

Bray and Moore, on the basis of this droplet model, associate with any temperature change ΔT of the system in the spin glass phase a characteristic "overlap" length $\ell_{\Delta T}$. For small ΔT this length behaves as

$$\ell_{\Delta T} \sim a \left| \frac{J}{\Delta T} \right|^{1/\zeta} \tag{3.1}$$

where

$$\zeta = \frac{d_s}{2} - y, \tag{3.2}$$

with y a constant and d_s the fractal dimension of the droplets. The meaning of $\ell_{\Delta T}$ is that two identical systems, one in equilibrium at a temperature T and one at a temperature $T + \Delta T$, have nearly identical spin-spin correlations over distances less than $\ell_{\Delta T}$, but completely different correlations beyond $\ell_{\Delta T}$.

Next we consider the more general case where there is both a change of temperature ΔT and a change of magnetic field ΔH. Our argument generalizes the one given by Bray and Moore [9]. We focus on a particular low-lying droplet excitation of excitation energy F_ℓ. Due to the change ΔT the free energy of this same excitation will change by an amount $\Delta F_\ell^{(T)}$ which is proportional to $\Delta T(\ell/a)^{d_s/2}$, being the sum of $\sim (\ell/a)^{d_s}$ contributions with uncorrelated signs. Similarly, the change ΔH in the magnetic field will bring about a change $\Delta F_\ell^{(H)}$ which is proportional to $\mu \Delta H(\ell/a)^{d/2}$, being the sum of the Zeeman energies of $\sim (\ell/a)^d$ randomly oriented spins. The droplet excitation in question loses its property of being a low-lying excitation if ΔT or ΔH becomes so large that

$$F_\ell \sim \left| \Delta F_\ell^{(T)} \right| + \left| \Delta F_\ell^{(H)} \right|, \tag{3.3}$$

that is, if

$$c_1 \frac{|\Delta T|}{J} \left(\frac{\ell}{a} \right)^{\frac{d_s}{2} - y} + c_2 \frac{\mu |\Delta H|}{J} \left(\frac{\ell}{a} \right)^{\frac{d}{2} - y} = 1, \tag{3.4}$$

where c_1 and c_2 are numerical constants. Our length $\ell(\Delta T, \Delta H)$ is defined as the solution of Eq. (3.4). For $\Delta H = 0$ it reduces to the Bray and Moore length (3.1).

3.3 Magnetic relaxation in time-dependent domains

We start from the idea that a spin in a hypothetical *finite* domain in *equilibrium* responds linearly to magnetic field variations, the response being described by Eq. (2.1b) with a relaxation function $R(t, t')$ which depends on the domain size s and can be expressed as

$$R(t, t') = R_{eq}(t - t')F(t, t'). \tag{3.5}$$

Here R_{eq} is the relaxation function for an infinite domain in equilibrium, and $F(t, t')$ is a cutoff reflecting the system size. For an equilibrium domain of size s a plausible choice is

$$F(t, t') = e^{-(t-t')/\tau_{max}(s)} \tag{3.6}$$

where $\tau_{max}(s)$ is the maximum relaxation time in the relaxation spectrum of a size s domain. The equations (3.5) and (3.6) are of the same form as (2.13) and (2.15), except that now the function F has been given a physical interpretation. Due to the spin coherence within a domain, $\tau_{max}(s)$ will increase with s. In this work we shall use the form

$$\tau_{max}(s) \simeq t_1 \left(\frac{s}{a}\right)^z, \tag{3.7}$$

where t_1 is a microscopic time and z a constant. A dependence of this kind is used in various other discussions of spin glass dynamics (see e.g. [20,21,22]).

When the domain size varies with time it is natural to put, in analogy to (3.5),

$$R(t, t'; [s(t'')]) = R_{eq}(t - t')F(t, t'; [s(t'')]) \quad (t' < t'' < t) \tag{3.8}$$

while a plausible generalization of (3.6) is

$$F(t, t'; [s(t'')]) = \exp\left[-\int_{t'}^{t} dt'' \tau_{max}^{-1}(s(t''))\right]. \tag{3.9}$$

In a nonequilibrium spin glass the relaxation time of a spin at time t'' is determined by the size of the $(T(t''), H(t''))$-domain to which the spin – at that time – belongs (see (*vi*) of section 3.1). Consequently the relaxation function $R(t, t')$ of the whole system is a weighted sum on the $(T(t''), H(t''))$-domain sizes for times $t' \leq t'' \leq t$:

$$R(t, t') = \int \mathcal{D}[s_{T(t''),H(t'')}(t'')] f(t, t'; [s_{T(t''),H(t'')}(t'')]) R(t, t'; [s_{T(t''),H(t'')}(t'')]). \tag{3.10}$$

Here $s_{T,H}(t)$ denotes the size of the (T, H)-domain to which a spin belongs at time t. Eq. (2.1b) together with Eq. (3.10) yields a relation between magnetization and magnetic field which is linear only in appearance. Nonlinearities come in implicitly via the mechanism which governs the evolution of the domain sizes.

3.4 Domain dynamics

The weight function $f(t, t'; [s(t'')])$ introduced in (3.10) follows from the domain dynamics. We shall develop this dynamics for some practical situations where the weight function $f(t, t'; [s(t'')])$ is centered around one or two characteristic domain sizes.

The simplest situation arises immediately after a quench to a temperature $T < T_f$ at $t = 0$ in a magnetic field H. The (T, H)-domains will then start to grow and we shall postulate that the characteristic size $\bar{s}_{T,H}(t)$ of these domains grows as a power law of time

$$\bar{s}_{T,H}(t) \simeq a \left(\frac{t}{t_2} \right)^p, \tag{3.11}$$

where t_2 is a microscopic time and p a constant. (Growth laws of domains in spatially uniform as well as disordered systems have been reviewed by Binder et al. [23].) The (T', H')-domains, for arbitrary T' and H', will grow as well but up to the overlap length $\ell(T - T', H - H')$. As long as they grow their characteristic size increases according to

$$\bar{s}_{T',H'}(t) \simeq a \left(\frac{t}{t_2(T, T')} \right)^p, \tag{3.12}$$

This is just Eq. (3.11), but with t_2 temperature dependent. The first temperature argument, T, is the actual temperature and the second one, T', indicates that we are considering (T', H')-domains ($t_2(T, T)$ is equal to the t_2 in Eq. (3.11)). For what follows it is necessary to remark that (3.12) is a solution of the autonomous differential equation

$$\frac{d\bar{s}_{T',H'}(t)}{dt} = p t_2^{-1}(T, T') a^{\frac{1}{p}} \bar{s}_{T',H'}^{\frac{p-1}{p}}(t). \tag{3.13}$$

When the environmental temperature T is an arbitrary time-dependent function $T(t)$, the solution of Eq. (3.13), with initial condition $\bar{s}_{T',H'}(t_i) = s_i$, is given by

$$\bar{s}_{T',H'}(t | t_i, s_i) = \left[s_i^{\frac{1}{p}} + a^{\frac{1}{p}} \int_{t_i}^{t} dt'' \, t_2^{-1}(T(t''), T') \right]^p. \tag{3.14}$$

Now consider the (T', H')-domains in the same system when at time $\hat{t}_1$ the heat bath temperature is changed from T to T_1 and the field is switched from H to H_1. In this situation the rule (v) of section 3.1 applies:
(i) Whenever $\bar{s}_{T',H'}(\hat{t}_1) < \ell(T_1 - T', H_1 - H')$, the characteristic domain size increases according to (3.13).
(ii) When $\bar{s}_{T',H'}(\hat{t}_1) > \ell(T_1 - T', H_1 - H')$, the domain size distribution will start to break up into two populations, one with the former characteristic size $\bar{s}_{T',H'}(\hat{t}_1)$ and one with a new characteristic size $\ell(T_1 - T', H_1 - H')$. In each time interval Δt the population of the larger domain size decreases by an amount $\tau_b^{-1} \Delta t$ while

51

the population of the smaller domain size increases by the same amount. For τ_b we take the maximum relaxation time of a domain of size $\ell(T_1 - T', H_1 - H')$,

$$\tau_b \equiv \tau_{max}(\ell(T_1 - T', H_1 - H'); T_1, T')$$
$$\equiv t_1(T_1, T') \left(\frac{\ell(T_1 - T', H_1 - H')}{a} \right)^z. \tag{3.15}$$

Here the relaxation time $\tau_{max}(s; T_1, T')$ is a generalization of (3.7) where the temperature dependence of the time constant t_1 is accounted for ($t_1(T, T)$ is equal to the t_1 of Eq. (3.7)).

With the above rules one can, in principle, describe the evolution of (T', H')-domains, after temperature and magnetic field jumps at time instants $\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_n$. The most general case, with arbitrary time-dependent temperature and magnetic field, is described in the appendix. In the examples that we shall treat in the next sections it suffices to consider one or two characteristic domain sizes. This simplifies the evaluation of (3.10) significantly.

4 Field jump experiments at constant temperature

We shall apply the theory of section 3 to field jump experiments performed at a constant temperature $T < T_f$. We consider the experiment described by Eq. (2.5), where after a waiting time t_w the magnetic field is changed from H_0 to $H_1 \equiv H_0 - \Delta H$. Of interest is the decay of the excess magnetization $\Delta M(t)$ (see Eq. (2.6)). In a constant temperature experiment the general theory simplifies considerably. In particular, the time constants $t_1(T'', T)$ and $t_2(T'', T)$, defined by Eqs.(3.15) and (3.12) respectively, remain unaltered during the experiment and hence can simply be denoted by t_1 and t_2. The overlap length discussed in section 3.2 will be denoted by $\ell_{\Delta H}$ as it only depends on ΔH:

$$\ell_{\Delta H} \sim |\Delta H|^{-2/(d-2y)}. \tag{4.1}$$

The calculation of the response $M(t)$ after the field jump requires the knowledge of the relaxation function $R(t, t_w)$ for $t > t_w$ (Eq. 3.10) which, in turn, only requires the knowledge of the (T, H_1)-domains for times after the field jump. In order to proceed we shall assume that the weight function $f(t, t'; [s_{T(t''), H(t'')}(t'')])$ is centered around an average size $\bar{s}_{T, H_1}(t)$, and that therefore the functional integral in Eq. (3.10) is well approximated by the integrand evaluated in $\bar{s}_{T, H_1}(t)$. Together with Eq. (3.8) this leads to

$$R(t, t_w) = R_{eq}(t - t_w) F(t, t_w; [\bar{s}_{T, H_1}(t'')]). \tag{4.2}$$

According to the domain dynamics postulated in section 3.4, in the time interval $(0, t_w)$ the domain size $\bar{s}_{T, H_1}(t)$ cannot grow larger than the overlap length $\ell_{\Delta H}$. The interplay between these two lengths leads to the distinction of two cases that will turn out to correspond to linear and to nonlinear behavior, respectively: small field jumps (i.e. ΔH small enough so that $\bar{s}_{T, H_1}(t_w) < \ell_{\Delta H}$), discussed in subsection 4.1, and larger field jumps, discussed in subsection 4.2.

4.1 Small field jumps, $\bar{s}_{T,H_1}(t_w) < \ell_{\Delta H}$

After the quench to $T < T_f$ in a field H_0 the characteristic dimension $\bar{s}_{T,H_1}(t)$ will begin to increase according to Eq. (3.11). We consider here field jumps which by hypothesis are small enough so that at $t = t_w$ the characteristic dimension $\bar{s}_{T,H_1}(t_w)$ is still less than $\ell_{\Delta H}$. For $t > t_w$ the growth law (3.11) will continue to hold, and from Eqs. (2.7), (4.2) and (3.9) we have that the excess magnetization $\Delta M(t)$ will decay as

$$\Delta M(t_w + \tau) = N\chi_{eq}\Delta H R_{eq}(\tau) \exp\left[-\int\limits_{t_w}^{t_w+\tau} dt'' \tau_{max}^{-1}(\bar{s}_{T,H_1}(t''))\right]. \qquad (4.3)$$

Using the experimentally obtained equilibrium relaxation function (2.15) and the power law dependencies (3.7) and (3.11) we find from (4.3) that

$$\Delta M(t_w + \tau) = N\chi_{eq}\Delta H \left(1 + \frac{\tau}{t_0}\right)^{-\alpha} \qquad (4.4)$$

$$\times \exp\left[-\frac{t_2^{pz}}{t_1(1 - pz)}\left\{(t_w + \tau)^{1-pz} - t_w^{1-pz}\right\}\right].$$

Eq. (4.4) is the result of our theory. It predicts a definite waiting time effect on the magnetic relaxation. The corresponding relaxation curves have been plotted in Fig. 1a for a number of values of the waiting time t_w. (From Alba et al. [10] we estimate $\alpha = 0.05$, $t_0 = 1.5 \times 10^{-17}$ min., $pz = 0.9$ and $t_1 t_2^{-pz} = 1$ min^{1-pz}.)

In the analysis of experimental data on single jump experiments it has become customary to plot not only $M(t)$, but also the "logarithmic" decay rate $S(\tau)$ here defined by

$$S(\tau) = \left|\frac{1}{N\chi_{eq}\Delta H}\frac{dM(t_w + \tau)}{d\log\tau}\right|. \qquad (4.5)$$

In Fig. 1b we show the decay rates corresponding to the curves of Fig. 1a. It appears that the function $S(\tau)$ exhibits a clear maximum for a value of τ to be denoted as τ_m. Hence τ_m can be considered as the crossover point for $\Delta M(t_w + \tau)$ between the slow power law decay ("equilibrium relaxation") due to the function R_{eq}, and the more rapid decay imposed by F (see Eq. (4.2)). The experimental $S(\tau)$ curves indeed exhibit a similar maximum [1,15,24]. Moreover, for several different types of spin glasses, such as Cu(5at%Mn) [14,15], Ag(2.6at%Mn) [25], and CdCr$_{1.7}$In$_{0.3}$S$_4$ [4], experiments with waiting times t_w ranging up to 10^4 s show that this crossover takes place when the time τ_m is roughly equal to the waiting time t_w, i.e.,

$$\tau_m = C(t_w)t_w, \qquad (4.6)$$

with C a constant of order unity or at most a weakly varying function of t_w.

From Eq. (4.4) it follows that for the theoretical curves of Fig. 1b the proportionality (4.6) holds with C given by
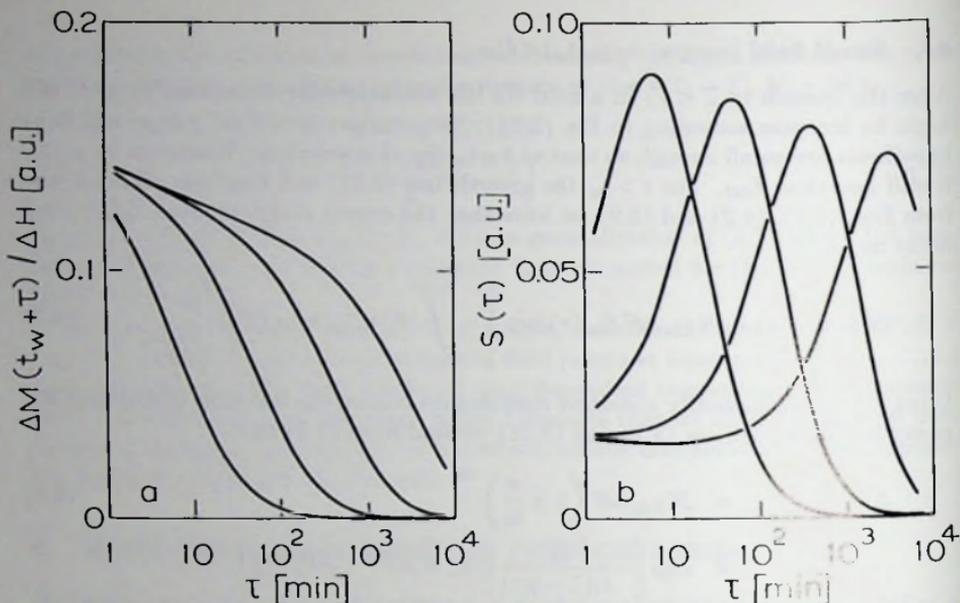
Fig. 1. Theoretical curves for (a) the relaxation of the excess magnetization $\Delta M(t_w + \tau)/\Delta H$, Eq. (4.4), and (b) the logarithmic relaxation rate $S(\tau)$, Eq. (4.5), for small magnetic field jumps ΔH and waiting times $t_w = 10, 100,$ 1000, and 10,000 min (from left to right). See the text for the parameter values.

$$C(t_w) \simeq (1 - 2\alpha)/(pz + t_w^{1-pz} t_2^{pz} t_1^{-1}) \tag{4.7}$$

for small α, $pz \approx 1$ and not too small t_w ($t_w \gtrsim 1$). For the parameter values given above and t_w in the experimental range we have that $C(t_w)$ is of order unity and weakly varying with t_w. Hence we conclude that this kind of field jump experiments are well described by our theory.

4.2 Larger field jumps

The second case to be considered is when the linear size of the (T, H_1)-domains reaches its upper limit $\ell_{\Delta H}$ in the time interval $(0, t_w)$, so that $\bar{s}_{T,H_1}(t_w) = \ell_{\Delta H}$. After the field jump this limit is removed and the domain growth will proceed. With initial condition $\bar{s}_{T,H_1}(t_w) = \ell_{\Delta H}$ the solution of Eq. (3.13) reads

$$\bar{s}_{T,H_1}(t) = a \left[\left(\frac{\ell_{\Delta H}}{a} \right)^{\frac{1}{p}} + \frac{t - t_w}{t_2} \right]^p \quad (t > t_w). \tag{4.8}$$

The theoretical decay of the excess magnetization is again given by Eq. (4.3), but now with the expression (4.8) for $\bar{s}_{T,H_1}(t)$. Hence the effect of the larger field jumps is reflected in the dependence of the characteristic domain size on ΔH and this is the way in which the relaxation is *nonlinear* in the field jump ΔH.

54

For this case one can also calculate the maximum in the logarithmic decay rate $S(\tau)$ (Eq. (4.5)) and finds that now it is dependent on the size of the field jump ΔH through

$$\tau_m = C(\tilde{t}_{\Delta H})\tilde{t}_{\Delta H}. \tag{4.9}$$

where the function C is as in Eq. (4.7) and

$$\tilde{t}_{\Delta H} \equiv t_2 \left(\frac{\ell_{\Delta H}}{a} \right)^{\frac{1}{p}}. \tag{4.10}$$

The equations (4.6), (4.9) and (4.10) can be merged into a single expression for the time τ_m at which the logarithmic decay rate reaches its maximum, viz.

$$\tau_m = C(t_{\Delta H})t_{\Delta H}, \tag{4.11}$$

where

$$t_{\Delta H} \equiv \min \left(t_w, t_2 \left(\frac{\ell_{\Delta H}}{a} \right)^{\frac{1}{p}} \right). \tag{4.12}$$

Hence, this picture of growing domains together with a domain size dependent relaxation process leads to
(i) a *small ΔH regime*, in which the relaxation is linear: curves for $\Delta M(t)/\Delta H$ obtained at a fixed t_w, but at different values of ΔH superimpose. In this regime the relaxation rate maximum occurs at a time $\tau_m = C(t_w)t_w$ independent of ΔH.
(ii) a *regime of larger ΔH*, in which the relaxation is nonlinear: curves obtained at different ΔH no longer superimpose. In particular, when ΔH becomes larger the location τ_m of the relaxation rate maximum begins to depend on ΔH and shifts to smaller values according to

$$\tau_m \approx t_{\Delta H} \sim |\Delta H|^{-2/p(d-2y)}. \tag{4.13}$$

The crossover between the two regimes occurs at a value of ΔH which satisfies $\bar{s}(t_w) = \ell_{\Delta H}$. This equation therefore divides the $t_w - \Delta H$ plane into a region of linear and a region of nonlinear relaxation. In particular it predicts that the *longer* is the waiting time, the *smaller* is the field jump needed to provoke nonlinear relaxation [10,13,26,27]. It is furthermore apparent from the above discussion that, if one accepts the power law dependencies (3.7) and (3.11) with exponents z and p, respectively, the *linear* relaxation depends only on the product pz, which should take a value close to unity. The *nonlinear* relaxation, however, allows for a determination of p and z individually via Eq. (4.13). We have attempted an estimate based on the work of Alba et al. [10] on the insulating spin glass compound $CdCr_{1.7}In_{0.3}S_4$. Alba et al. report on field jump (TRM) experiments performed at a fixed temperature of $0.72T_f$ (12 K) where the field jump ΔH is varied from effectively zero to 50 Gauss for a range of waiting times t_w between 10 and 900 minutes and where the relaxation curves of the magnetization for times $t > t_w$ are recorded. Subsequently their results are fitted to a master curve of which the parameter values for several t_w and ΔH are
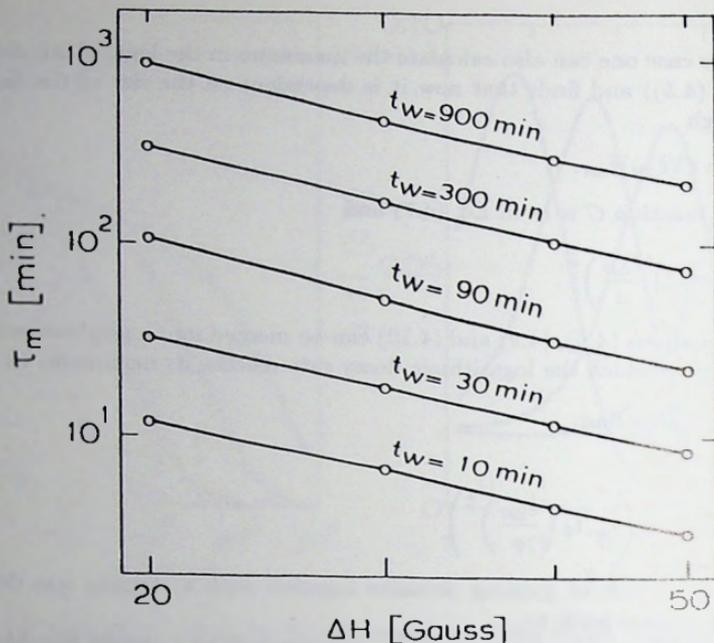
Fig. 2. Location of the maxima τ_m in the relaxation rate versus field jump size ΔH for data taken from Alba et al. [10].

given [10]. We have pursued this analysis in the following way. From the expression for the master curve we went back and calculated $S(\tau)$. We plotted $S(\tau)$ and located the maximum τ_m for the values of ΔH and t_w using the parameters from [10]. Fig. 2 shows a log-log plot of the τ_m versus ΔH for the given set of waiting times. We do not show τ_m for $\Delta H = 0$ and 10 Gauss; in this field regime τ_m levels off to a value independent of ΔH. For each waiting time, the four values of τ_m belonging to 20, 30, 40, and 50 Gauss lie on a straight line whose slope is approximately -1.55. With this information we find the approximate value $p \approx 0.5$ if we take $y \approx 0.2$ for $d = 3$ from Ref. [9]. This value of p for a real spin glass is, somewhat surprisingly, closer to the value $\frac{1}{2}$ applicable to domain growth in spatially uniform Ising ferromagnets (see Binder et al. [23]) than to the slow logarithmic growth law often found for random systems by theory [28] and by Monte Carlo simulation [29]. A possible explanation is hindered by the fact that, whereas ferromagnetic domains are easily observed experimentally, there is no direct way to see spin glass domains in the laboratory.

5 Relaxation experiments with field and temperature jumps

In this section we shall consider a modification of the field jump experiment. The time dependence of the magnetic field is still given by Eq. (2.5), but in a time interval $(\hat{t}_1, \hat{t}_2)$ during the waiting time period the temperature is kept at $T + \Delta T$ instead of T:

$$T(t) = \begin{cases} T & (0 < t < \hat{t}_1) \\ T + \Delta T & (\hat{t}_1 < t < \hat{t}_2; \quad \hat{t}_2 < t_w) \\ T & (\hat{t}_2 < t) \end{cases} \quad (5.1)$$

Both positive and negative ΔT will be of interest. In section 5.1 we shall first discuss the experimental data. Then we shall show, in section 5.2, how the theory developed above can account qualitatively for the observed phenomena. The formalism is *exactly the same* as was applied to the constant temperature experiments, except that we now have to allow for a temperature dependence of the times t_1 and t_2. In section 5.3 we shall show how the extra parameters in the case of nonconstant temperature can be set to fit the experimental curves.

5.1 Experimental results

The experiment described above has been performed by Nordblad et al. [3] and by Refregier et al. [4]. In the limit of sufficiently small ΔT (for the examples of $CdCr_{1.7}In_{0.3}S_4$ [4] and Cu(10at%Mn) [3] this means $|\Delta T| \lesssim 0.1 K$) the magnetic relaxation curves coincide with the ones obtained at constant temperature T for a waiting time t_w. For sufficiently large *positive* ΔT (which means in practice $\Delta T \gtrsim 2K$ for $CdCr_{1.7}In_{0.3}S_4$ when $\hat{t}_2 - \hat{t}_1 = 300$ s [4]) the relaxation curve appears to coincide with the one at constant temperature T for a waiting time $t_w - \hat{t}_2$: at the time $\hat{t}_2$ the system is "reborn" in the terminology of Ref. [3]. However, for intermediate values of ΔT the situation is less simple: the relaxation curves obtained there do not coincide with the curves at constant T for any t_w, as demonstrated both by Nordblad et al. [3] and, in a particularly clear way, by Refregier et al. [4]. The former authors speak of an "incompletely reborn system" and of the "coexistence of two separate and distinct aging states" in the same system. Our conclusion is that in this intermediate regime the system cannot be described by a single characteristic domain size, but that the distribution of domain sizes has more structure. This is what motivated the development of the full theory of section 3.

The modified field jump experiment with *negative* values of ΔT [3,4] shows a different picture. The situation is not symmetric with respect to $\Delta T = 0$. When ΔT is sufficiently large negative, the relaxation curves for $t > t_w$ coincide with those obtained at constant temperature T for a waiting time $t_w - (\hat{t}_2 - \hat{t}_1)$, i.e. relaxation takes place as though the interval $(\hat{t}_1, \hat{t}_2)$ of a decreased temperature had been completely nonexistent [4]. The correlations characteristic of the temperature T that were built up during the interval $(0, \hat{t}_1)$ are not lost during the interval $(\hat{t}_1, \hat{t}_2)$ but seem to remain frozen; then, once the temperature T is reestablished, their buildup continues. This shows that although a temperature jump ΔT (or, by the

same token, a field jump ΔH [27]) may be large enough to require the breakup of existing correlations, this *breakup is not instantaneous* but should be described by a relaxation time.

5.2 Application of the theory

We shall analyze the modified field jump experiments for values of ΔH so small with respect to ΔT that for all practical purposes we have

$$\ell(\Delta T, \Delta H) \approx \ell(\Delta T, 0) \equiv \ell_{\Delta T} \tag{5.2}$$

We are again interested in the decay of the excess magnetization $\Delta M(t)$ for $t > t_w$ and hence only the (T, H_1)-domain structure is needed to calculate the weight function $f(t, t'; [s_{T(t'')}, H(t'')(t'')])$ (see (3.10) and (2.7)).

We shall again assume that in the first time interval, $0 < t < \hat{t}_1$, the distribution of the sizes of the (T, H_1)-domains is well represented by a single characteristic length $\bar{s}(t) \equiv \bar{s}_{T,H_1}(t)$. This length then grows according to the solution (3.14) of Eq. (3.13) and is given by

$$\bar{s}(t) = a \left(\frac{t}{t_2(T, T)} \right)^p \qquad 0 < t < \hat{t}_1. \tag{5.3}$$

In the second time interval $\hat{t}_1 < t < \hat{t}_2$, there are two possibilities. The first one is that $|\Delta T|$ is not too large so that $\bar{s}(\hat{t}_1)$, as given by Eq. (5.3), is less than $\ell_{\Delta T}$. In that case, $\bar{s}(t)$ will continue to grow according to Eq. (5.3), albeit at a different rate due to the temperature dependence of t_2. Moreover, if $\bar{s}(t)$ reaches the upper limit $\ell_{\Delta T}$ before the time $\hat{t}_2$, then it will stick to that value. From $\hat{t}_2$ on the domain size evolves again without limit, and hence we have for $t > \hat{t}_2$

$$\bar{s}(t) = a \left[\min \left\{ \left(\frac{\ell_{\Delta T}}{a} \right)^{\frac{1}{p}}, \frac{\hat{t}_1}{t_2(T, T)} + \frac{\hat{t}_2 - \hat{t}_1}{t_2(T + \Delta T, T)} \right\} + \frac{t - \hat{t}_2}{t_2(T, T)} \right]^p. \tag{5.4}$$

The second possibility is that $|\Delta T|$ is so large that $\bar{s}(\hat{t}_1) > \ell_{\Delta T}$. In that case, according to the theory of section 3, there appear two different domain sizes: the "old" one $\bar{s}_{old}(t') = \bar{s}(\hat{t}_1)$ and a "young" one $\bar{s}_{young}(t') = \ell_{\Delta T}$ for $t' > \hat{t}_1$. The fraction $f_{young}(t'; \hat{t}_1)$ of spins in domains of size $\ell_{\Delta T}$ increases with time whereas the fraction $f_{old}(t'; \hat{t}_1)$ of spins in domains of size $\bar{s}(\hat{t}_1)$ decreases. At time $t = \hat{t}_2$ we have

$$f_{young}(\hat{t}_2; \hat{t}_1) = 1 - \exp \left\{ -\frac{\hat{t}_2 - \hat{t}_1}{\tau_b} \right\}, \tag{5.5a}$$

and

$$f_{old}(\hat{t}_2; \hat{t}_1) = \exp \left\{ -\frac{\hat{t}_2 - \hat{t}_1}{\tau_b} \right\}. \tag{5.5b}$$

Here τ_b is given by Eq. (3.15) as the maximum relaxation time of a domain of size $\ell_{\Delta T}$

$$\tau_b = \tau_{max}(\ell_{\Delta T}; T + \Delta T, T) = t_1(T + \Delta T, T) \left(\frac{\ell_{\Delta T}}{a}\right)^z. \tag{5.6}$$

From $\hat{t}_2$ on both characteristic domain sizes evolve again without upper limit according to the rate equation (3.13). This yields

$$\bar{s}_{young}(t) = a \left(\left(\frac{\ell_{\Delta T}}{a}\right)^{\frac{1}{p}} + \frac{t - \hat{t}_2}{t_2(T, T)} \right)^p \quad (t > \hat{t}_2) \tag{5.7a}$$

and

$$\bar{s}_{old}(t) = a \left(\frac{\hat{t}_1 + t - \hat{t}_2}{t_2(T, T)} \right)^p \quad (t > \hat{t}_2). \tag{5.7b}$$

Since the breaking-up process stops at $t = \hat{t}_2$, the fractions f_{young} and f_{old} remain constant for $t > \hat{t}_2$.

In the present case with *two* characteristic domain sizes the basic equations (3.10) and (3.8) for the relaxation function R reduce to

$$R(t, t_w) = R_{eq}(t - t_w) \sum_{i=young,old} f_i(\hat{t}_2; \hat{t}_1) F(t, t_w; [\bar{s}_i(t'')]) \quad (t > t_w) \tag{5.8}$$

This expression also holds for the case $\bar{s}(\hat{t}_1) < \ell_{\Delta T}$ provided that one take for $\bar{s}_{old}(t)$ the expression (5.3), and put $f_{old} \equiv 1$ and $f_{young} \equiv 0$.

The decay of the excess magnetization can be obtained by substituting Eq. (5.8) in Eq. (2.7) and using Eq. (3.9). In the next subsection we shall show how this leads to the theoretical curves for $\Delta M(t)$ corresponding to the modified field jump experiment, Eq. (5.1).

5.3 Fit to experiment

In order to obtain explicit theoretical curves it is necessary to specify the temperature dependence of t_1 and t_2 (Eqs. (3.15) and (3.13)), and of the constants in Eq. (3.4). We shall use the data from experiments carried out by Refregier et al. [4] on the insulating spin glass compound $CdCr_{1.7}In_{0.3}S_4$ to obtain values for the parameters that govern these temperature dependencies.

As discussed in section 3 the time constants $t_i(T; T_1)$, $i = 1, 2$, refer to the dynamics of (T_1, H_1)-domains in a system at temperature T. For $T = T_1$ we shall use the abbreviation $t_{i0}(T_1) \equiv t_i(T_1; T_1)$, $i = 1, 2$. If domain growth is a thermally activated process, one would expect the T-dependence of the dynamics to manifest itself as an Arrhenius factor in the t_i. The experiments [4] show, however, a rather stronger asymmetry between positive and negative ΔT. We therefore put, for general T and T_1

$$t_i(T; T_1) = t_{i0}(T_1) A(T - T_1) \tag{5.9}$$

in which $A(0) = 1$ and A increases steeply when $T-T_1$ becomes negative. All strongly varying functions $A(\Delta T)$ give qualitatively similar results, and we have obtained a good fit with the three parameter function
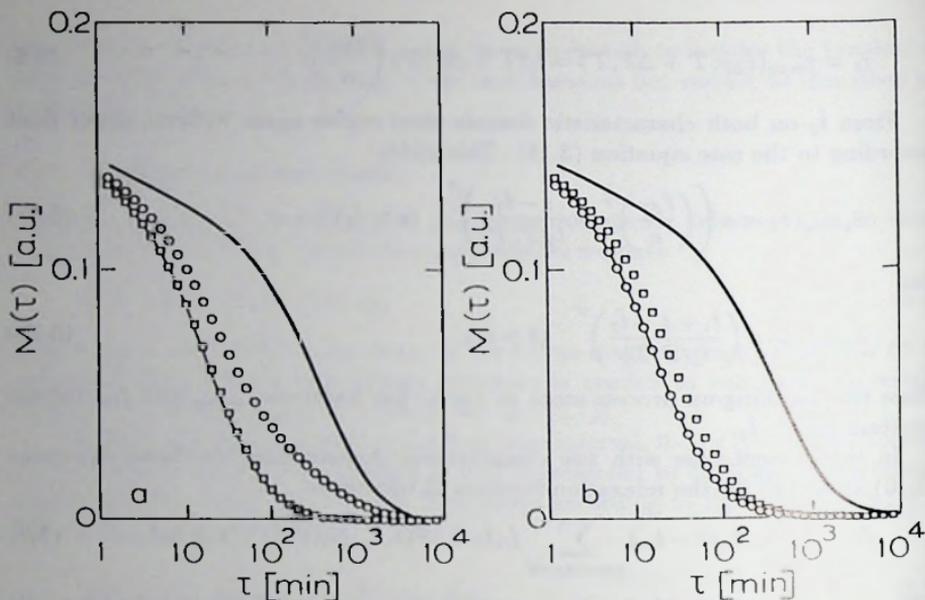
Fig. 3. Theoretical curves for the relaxation of the excess magnetization where in a time interval $(\hat{t}_1, \hat{t}_2)$ during the waiting time period $(0, t_w)$ the temperature is kept at $T + \Delta T$. (a) shows the curves for positive ΔT where $\hat{t}_1 = 895$ min, $\hat{t}_2 = 900$ min and $t_w = 930$ min. The solid lines correspond to TRM curves obtained for waiting times of 30 min and 930 min without temperature jumps. The squares ($\square$) refer to the curve for $\Delta T = 2$K and the circles ($\circ$) to the curve for $\Delta T = 1$K. (b) shows the curves for negative ΔT where $\hat{t}_1 = 15$ min, $\hat{t}_2 = 915$ min and $t_w = 930$ min. Here the circles ($\circ$) refer to $\Delta T = -1$K and the squares ($\square$) to $\Delta T = -0.3$K. See the text for the parameter values.

$$A(\Delta T) = x e^{-\gamma_- \Delta T} + (1 - x) e^{-\gamma_+ \Delta T}, \qquad (5.10)$$

where $\gamma_- = 18$ K^{-1} is large compared to $\gamma_+ = 2.4$ K^{-1} and where $x = 0.1$. We set $t_{10}(T) t_{20}^{-pz}(T) = 1$ min^{1-pz}, $\alpha = 0.05$ and $t_0 = 1.5 \times 10^{-17}$ min as in section 4.

Only two combinations of parameters from Eq. (3.4) play a role, viz.

$$k \equiv t_{20}(T) \left(\frac{J}{c_1} \right)^q \qquad (5.11)$$

and

$$q^{-1} \equiv \left(\frac{d_s}{2} - y \right) p. \qquad (5.12)$$

A fit to an analysis of the experiment by Refregier et al. [4] yields the values $q = 1.66$ and $k = 4.77$ K^{q}min.

With the parameters listed above and the choice for $A(\Delta T)$ given by Eq. (5.10) the theoretical curves of Fig. 3 are obtained. Fig. 3a shows the curves for positive

60

ΔT. For sufficiently large ΔT the fraction of spins with the "young" characteristic domain size is dominant wheras for not too large positive ΔT domains of the "young" and the "old" size coexist. This explains why the theoretical curve for $\Delta T = 1$K does not look like a "normal" TRM curve. A decomposition into two "distinct aging states" as performed by Nordblad et al. [3] clearly is possible here. Fig. 3b shows the curves for negative ΔT, where there is no breaking-up but where the domain growth simply freezes for sufficiently large ΔT. For not too large negative ΔT there is no freezing but domain growth at a lower rate. Hence the resulting decay curve is still similar to the "normal" TRM curves. All theoretical curves of Fig. 3 are qualitatively the same as the experimental curves obtained by Refregier et al. [4] as well as those of Nordblad et al. [3]. We conclude that the theory correctly accounts for this class of experimental data.

6 The ac susceptibility after temperature jumps

A third type of experiment that has been performed by Lundgren et al. [6] and in greater detail by Refregier et al. [4] consists of observing the behavior of the zero field ac susceptibility $\chi(\omega; t)$ after temperature jumps. As discussed in section 2.2, this quantity relaxes towards an equilibrium value. In this section we shall consider the behavior of the time-dependent ac susceptibility for three cases: (i) immediately after the quench to a temperature $T < T_f$ at $t = 0$, (ii) after a second temperature quench to $T_1 = T - \Delta T$ (ΔT positive) at some later time $\hat{t}_1$, and (iii) after reheating the system to T at $\hat{t}_2 > \hat{t}_1$. This sequence of temperature jumps corresponds to an experiment performed by Refregier et al. on $CdCr_{1.7}In_{0.3}S_4$ [4]. Experimentally one observes a relaxation of the susceptibility after the first two temperature changes, but *not* after the third one. We shall show how our theory predicts a similar time dependence of $\chi(\omega; t)$. The amplitude of the ac magnetic field will be taken infinitesimally small.

The analysis of case (i), the behavior of the ac susceptibility immediately after the quench to the temperature T, makes use of results from subsection 4.1. We describe the system again by a single characteristic domain size $\bar{s}_{T,0}(t)$. There is no upper limit on the domain growth so from Eq. (4.4) we have the response function

$$R(t, t - \tau) = \left(1 + \frac{\tau}{t_0}\right)^{-\alpha} \exp\left\{-\frac{t_2^{pz}}{t_1(1 - pz)}\left(t^{1-pz} - (t - \tau)^{1-pz}\right)\right\}. \qquad (6.1)$$

Substitution of (6.1) into (2.11) and evaluation of the integral for $t > 0$ results in the first curve shown in Fig. 4. The values for α, t_0, pz and $t_1 t_2^{-pz}$ are the same as in the sections 4 and 5.

Just like in section 2.2 we can calculate the asymptotic behavior of this curve for long times. To this purpose we compute the derivative of Eq. (6.1) and expand for large t and τ fixed and find

$$\frac{dR(t, t - \tau)}{d\tau} \simeq -\frac{\alpha}{t_0}\left(1 + \frac{\tau}{t_0}\right)^{-\alpha-1}$$

61
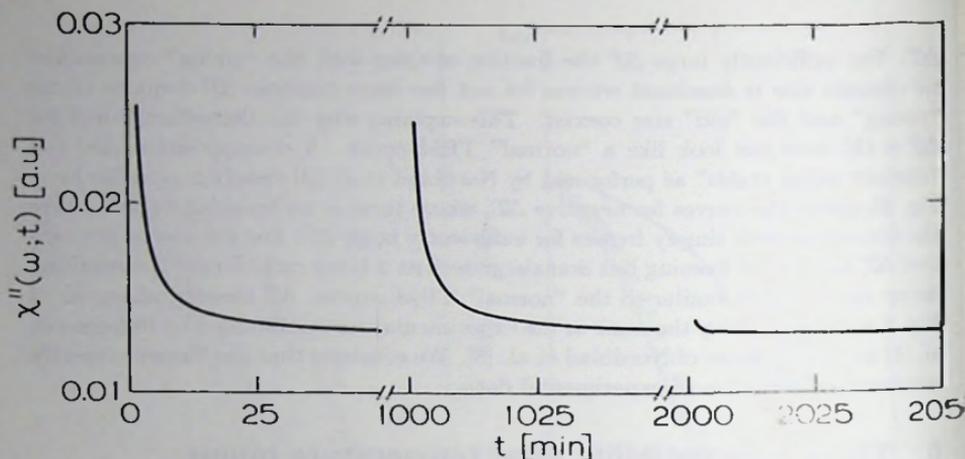
Fig. 4. Theoretical curve for the imaginary part of the time-dependent ac susceptibility $\chi''(\omega; t)$. Shown is the relaxation to equilibrium immediately after the quench to $T < T_f$ at $t = 0$, after a second quench to $T - \Delta T$ at $t = 1000$ min and after reheating to T at $t = 2000$ min. The equilibrium values of χ'' at T and $T - \Delta T$ have been assumed equal. See the text for the parameter values.

$$-\frac{1}{ct^{pz}}\left\{\left(1 + \frac{\tau}{t_0}\right)^{-\alpha} - \frac{\alpha\tau}{t_0}\left(1 + \frac{\tau}{t_0}\right)^{-\alpha-1}\right\}, \tag{6.2}$$

where $c = t_1 t_2^{-pz}$. When (6.2) is substituted in (2.11) the first term gives rise to the equilibrium susceptibility $\chi(\omega)$ that we calculated in Eq. (2.16) but due to the finite upper limit in the integral there will be an additional, rapidly decaying, oscillating part. The second term gives rise to a similar integral, so

$$\begin{aligned}
\frac{\chi(\omega; t)}{\chi_{eq}} &\simeq 1 - (|\omega| t_0)^\alpha \Gamma(1-\alpha)\left\{\cos\frac{\alpha\pi}{2} - i\,\text{sgn}\,\omega\sin\frac{\alpha\pi}{2}\right\} \\
&+ c^{-1}t^{-pz}|\omega|^{-\alpha-1}t_0^\alpha\Gamma(2-\alpha)\left\{\sin\frac{\alpha\pi}{2} + i\,\text{sgn}\,\omega\cos\frac{\alpha\pi}{2}\right\} \\
&- i\alpha\omega^{-1}t_0^{-1}\left(1 + \frac{t}{t_0}\right)^{-\alpha-1}e^{i\omega t}.
\end{aligned} \tag{6.3}$$

In practice the oscillating part is suppressed by averaging over one or more periods $2\pi/\omega$ and subsequent low-pass filtering. (For the theoretical curves of Fig. 4 this suppression has been achieved by sampling $\chi''(\omega, t)$ at times $2\pi/\omega$ apart.) Hence we find power law decay towards equilibrium with a power pz.

In case (ii), after the second temperature quench at time $\hat{t}_1$ to a temperature $T_1 = T - \Delta T$ (ΔT positive), only the $(T_1, 0)$-domains are to be considered. We can describe the system by means of a characteristic domain size $\bar{s}_{T_1,0}(t)$. During the interval $(0, \hat{t}_1)$ the growth of $\bar{s}_{T_1,0}(t)$ is restricted to the overlap length. We will

62

assume that we deal with the case, where the domain growth is actually limited by the overlap length $\bar{s}_{T_1,0}(\hat{t}_1) = \ell_{\Delta T}$, where we used again the abbreviation $\ell_{\Delta T} = \ell(\Delta T, 0)$. This is the case for sufficiently large ΔT. For times $t > \hat{t}_1$ the domain growth is unrestricted and we read from Eq. (3.14):

$$\bar{s}_{T_1,0}(t) = a \left[\left(\frac{\ell_{\Delta T}}{a} \right)^{\frac{1}{p}} + \frac{t - \hat{t}_1}{t_2} \right]^{p} \qquad (t > \hat{t}_2) \qquad (6.4)$$

The response function $R(t, t')$ can be calculated by substituting Eq. (6.4) into Eq. (4.2). For sufficiently large ΔT the overlap length $\ell_{\Delta T}$ is so small that $\bar{s}_{T_1,0}(t)$ is only influenced at very short times $t - \hat{t}_1$: the domain growth for the $(T_1, 0)$-domains will effectively set in at $t = \hat{t}_1$. Therefore the second theoretical curve in Fig. 4 is very similar to the first curve for the quench at $t = 0$. (The parameters used are the same as in section 5). For simplicity we have assumed χ_{eq} not to depend on temperature.

Finally we will discuss the case (iii) where the system is reheated to T at time $\hat{t}_2$. As was the case in section 5, breaking-up of the domains will occur in the interval $(\hat{t}_1, \hat{t}_2)$ for sufficiently large ΔT, and two fractions of spins will emerge after $\hat{t}_2$: a "young" fraction and an "old" fraction. As in section 5, the "young" fraction will be very small for ΔT large enough and hence we can very well describe the system for $t > \hat{t}_2$ with one characteristic domain size, which equals $\bar{s}_{T,0}(\hat{t}_1)$ at $t = \hat{t}_2$. From Eq. (3.14) we then have

$$\bar{s}_{T,0}(t) = a \left[\frac{\hat{t}_1}{t_2} + \frac{t - \hat{t}_2}{t_2} \right]^{p} \qquad (t > \hat{t}_2). \qquad (6.5)$$

The third theoretical curve in Fig. 4 clearly shows the absence of the relaxation behaviour of the previous curves. Similar curves have been obtained by Refregier et al. [4] for their experiments on $CdCr_{1.7}In_{0.3}S_4$. Our results for $\chi''(\omega; t)$ are furthermore fully consistent with the experimental results of Lundgren et al. [6] on $Cu(4at\%Mn)$ obtained during a sequence of temperature increases. A small discrepancy remains, nevertheless, when the relaxation behaviors of χ'' and χ' are compared: Lundgren et al. [6] observe that in the nonstationary regime the $\frac{\pi}{2}$ law (2.17) remains valid, whereas our asymptotic behavior (6.3) does not satisfy this law.

We conclude this section by pointing out an interesting although only partial analogy. In an Ising ferromagnet in zero field, after a quench to the low-temperature phase, ferromagnetic domains appear whose average size increases with time [23]. Under these circumstances one would expect to find for the frequency dependent *staggered* susceptibility a relaxation behavior analogous to the first curve in Fig. 4 for $\chi''(\omega; t)$.

7 Final remarks

We wish to discuss the relation of the present study to existing work on spin glass dynamics. Virtually no theoretical work focusses specifically on the aging phenomena in spin glasses. An exception is the paper by Ginzburg [11], who, within a mean

field approach, does address aging, starting from the dynamical theory of the spin glass by Sompolinsky and Zippelius [30]. Our work differs from Ginzburg's in that it does not start from a microscopic spin model, but rests on a set of assumptions at the mesoscopic level; it attempts to describe finite-dimensional systems possessing one or more characteristic lengths; it presents a quantitative comparison with recent experimental findings, in particular by Refregier et al. [4] and by Nordblad et al. [3]; and it goes beyond the linear response regime.

Recently Sibani and Hoffmann [31] have shown that linear aging effects can be reproduced by a model based on the picture of motion in phase space as thermally activated hopping in an ultrametric space [32]. It is, however, not clear how this theory can be extended to account for nonlinear aging phenomena as well.

A great deal of attention has been given to aging in polymers (see, in particular ref. [33]). Alba et al. [25] and Ocio et al. [34], in order to fit their magnetic relaxation data in spin glasses, succesfully use the same parametrization as employed for polymers. A physical picture underlying this procedure is, however, still lacking.

A theory which, like ours, addresses the spin glass dynamics at the mesoscopic level, is the "fractal cluster model". This model was introduced and studied recently by Malozemoff and coworkers [20,35] as well as by Lundgren et al. [21]. The fractal cluster model has been used to derive static and dynamic scaling laws in the vicinity of T_f, but has not been applied to aging. It postulates the existence of "clusters" inside of which the spins are randomly oriented but rigidly coupled together. In response to magnetic field changes such a cluster may be visualized as rotating in a frozen matrix [36]. With each cluster a relaxation time is associated via an equation identical to our Eq. (3.7). In this model the distribution law for the cluster sizes is assumed to be the equilibrium distribution, characterized by a temperature-dependent length. For $T < T_f$ there is also an infinite cluster [20].

In this work we speak of *domains* of correlated spins rather than of "clusters". The two concepts cannot be identified. Firstly, each pair (T, H) has a different domain structure associated with it. Secondly, a domain represents a less rigid and permanent aspect of the spin glass structure than does a cluster: it responds to a change of magnetic field or temperature by starting to grow or to break up. Hence the distribution of the domain sizes is *not* in equilibrium but evolves with time (it can tend towards an equilibrium distribution only when $t \to \infty$ at fixed T and H). It is precisely this feature which enabled us to explain the great variety of experimental aging effects.

A description of the nonequilibrium spin glass state as consisting of equilibrium domains separated by domain walls was given also by Kinzel [37]. In this work the domain size increases with time and Kinzel argues that there is a power law relation between the excess energy and magnetization during the equilibration process.

A cluster of spins that can flip between only two configurations is a two-level system. Models of spin glass dynamics involving a distribution of two-level systems and an associated distribution of relaxation times were considered by many authors, e.g. McMillan [22], Préjean and Souletie [38] and Hüser et al. [39]. In all these cases the distribution of relaxation times is fixed once for all. In contrast, the present

64

theory associates with each of the time-dependent distributions $p(s, t; T_1, H_1)$ (see section 3 and the appendix) a *time-dependent* distribution of relaxation times via the relation (3.7) between s and $\tau_{max}(s)$.

The notion of a time-dependent spectrum to describe spin glass relaxation was used earlier by Lundgren et al. [1,21,24]. Our work bases this notion on a picture of domains that grow and break up, and makes the evolution equation for the spectrum explicit. Along the way we have needed several assumptions, which however we feel are all secondary. The agreement found with experiment does not prove that all these hypotheses are right; however it means that a simple picture like the one proposed here is capable of explaining the large collection of experimental data.

Appendix

In section 3.1 we have introduced the distribution $p(s, t; T_1, H_1)$ of the (T_1, H_1)-domain sizes at time t. Although this distribution has not played an explicit role in the subsequent calculations, it is nevertheless interesting to remark that is satisfies the master equation

$$\frac{\partial}{\partial t} p(s, t; T_1, H_1) = -\frac{\partial}{\partial s} \left[\lambda(s; T(t), T_1) p(s, t; T_1, H_1) \right]$$
$$- \tau_b^{-1} \theta \left(s - \ell(T(t) - T_1, H(t) - H_1) \right) p(s, t; T_1, H_1)$$
$$+ \tau_b^{-1} \delta \left(s - \ell(T(t) - T_1, H(t) - H_1) \right) \int_{\ell(T(t) - T_1, H(t) - H_1)}^{\infty} ds' p(s', t; T_1, H_1), \qquad (A.1)$$

in which λ is the growth rate of a domain of size s at temperature T, which, according to Eq. (3.13), equals

$$\lambda(s; T, T_1) = \frac{p}{t_2(T, T_1)} a^{\frac{1}{p}} s^{\frac{p-1}{p}}. \qquad (A.2)$$

The time constant τ_b is defined by Eq. (3.15). One easily checks that Eq. (A.1) is compatible with the normalization

$$\int_0^{\infty} ds \, p(s, t; T, H) = 1 \qquad (A.3)$$

According to (iv) in section 3.1, if at time $t = 0$ the system is quenched to below T_f, we have that the initial condition is a delta peak at $s = 0$, regardless of (T, H):

$$p(s, 0; T, H) = \delta(s) \qquad (A.4)$$

It is clear that in experiments in which after the quench only a finite number (say n) of temperature or magnetic field jumps are applied, the function $p(s, t; T, H)$ can be the sum of at most n delta peaks.

References

[1] Lundgren, L., Svedlindh, P., Nordblad, P., and Beckman, O., Phys. Rev. Lett. **51** (1983) 911.

[2] Binder, K. and Young, A. P., Rev. Mod. Phys. **58** (1986) 801.

[3] Nordblad, P., Svedlindh, P., Sandlund, L., and Lundgren, L., Phys. Lett. A **120** (1987) 475.

[4] Refregier, P., Vincent, E., Hammann, J., and Ocio, M., J. Physique **48** (1987) 1533.

[5] Lundgren, L., Svedlindh, P., and Beckman, O., J. Magn. Magn. Mater. **25** (1981) 33.

[6] Lundgren, L., Svedlindh, P., and Beckman, O., J. Magn. Magn. Mater. **31-34** (1983) 1349.

[7] Svedlindh, P., Granberg, P., Nordblad, P., Lundgren, L., and Chen, H. S., Phys. Rev. **B 35** (1987) 268.

[8] Ocio, M., Bouchiat, H., and Monod, P., J. Phys. (Paris) Lett. 46 (1985) L647.

[9] Bray, A. J. and Moore, M. A., Phys. Rev. Lett. **58** (1987) 57.

[10] Alba, M., Bouchiat, H., Hammann, J., Ocio, M., and Refregier, P., J. Appl. Phys. **61** (1987) 61.

[11] Ginzburg, S. L., Sov. Phys. JETP **63** (1986) 439, [Zh. Eksp. Teor. Fiz. **63** (1986) 754].

[12] Lundgren, L., Nordblad, P., and Sandlund, L., Europhys. Lett. 1 (1986) 529.

[13] Refregier, P., Vincent, E., Hammann, J., and Ocio, M., private communication.

[14] Nordblad, P., Lundgren, L., and Sandlund, L., J. Magn. Magn. Mater. **54-57** (1986) 185.

[15] Nordblad, P., Svedlindh, P., Lundgren, L., and Sandlund, L., Phys. Rev. B **33** (1986) 645.

[16] Ogielski, A. T., Phys. Rev. **B 32** (1985) 7384.

[17] Pytte, E. and Imry, Y., Phys. Rev. **B 35** (1987) 1465.

[18] Bray, A. J. and Moore, M. A., in: *Heidelberg Colloquium on Glassy Dynamics*, eds. J. L. van Hemmen and I. Morgenstern (Springer, Heidelberg, 1987).

[19] Fisher, D. S. and Huse, D. A., Phys. Rev. Lett. **56** (1986) 1601.

[20] Continentino, M. A. and Malozemoff, A. P., Phys. Rev. **B 33** (1986) 3591.

[21] Lundgren, L., Nordblad, P., and Svedlindh, P., Phys. Rev. **B 34** (1986) 8164.

[22] McMillan, W. L., J. Phys. C **17** (1984) 3179.

[23] Binder, K., Heermann, D. W., Michev, A., and Sadiq, A., in: *Heidelberg Colloquium on Glassy Dynamics*, eds. J. L. van Hemmen and I. Morgenstern (Springer, Heidelberg, 1987).

[24] Lundgren, L., Nordblad, P., Svedlindh, P., and Beckman, O., J. Appl. Phys. **57** (1985) 3371.

[25] Alba, M., Ocio, M., and Hammann, J., Europhys. Lett. **2** (1986) 45.

[26] Nordblad, P., Lundgren, L., and Sandlund, L., Europhys. Lett. **3** (1987) 235.

[27] Nordblad, P., Svedlindh, P., Granberg, P., and Lundgren, L., Phys. Rev. **B 35** (1987) 7150.

[28] Villain, J., Phys. Rev. Lett. **52** (1984) 1543.

[29] Grest, G. and Srolovitz, D. J., Phys. Rev. **B 32** (1985) 3014.

[30] Sompolinski, H. and Zippelius, A., Phys. Rev. **B 25** (1982) 6860.

[31] Sibani, P. and Hoffmann, K. H., preprint (1989).

[32] Sibani, P., Phys. Rev. **B 35** (1987) 8572.

[33] Struik, L. C. E., *Physical aging in amorphous polymers and other materials* (Elsevier Scient. Publ., Amsterdam, 1978).

[34] Ocio, M., Alba, M., and Hammann, J., J. Phys. (Paris) Lett. **46** (1985) L1101.

[35] Malozemoff, A. P. and Barabara, B., J. Appl. Phys. **57** (1985) 3410.

[36] Malozemoff, A. P., Barnes, S. E., and Barbara, B., Phys. Rev. Lett. **51** (1983) 1704.

[37] Kinzel, W., J. Phys. C **21** (1988) L381.

[38] Préjean, J. J. and Souletie, J., J. Physique **41** (1980) 1335.

[39] Hüser, D., Van Duijneveldt, A. J., Nieuwenhuys, G. J., and Mydosh, J. A., J. Phys. C **19** (1986) 3697.

Chapter IV

Nonequilibrium Dynamics and Aging in a One-Dimensional Ising Spin Glass *

In this chapter it is shown for the example of a one-dimensional model system how the domain theory of Chapter III can be given a microscopic basis.

1 Introduction

In Chapter III a theory is presented that explains the observed aging phenomena on the basis of a domain picture of the nonequilibrium spin glass state. A somewhat different theory that embodies the same ideas is given by Fisher and Huse [1]. The aim of this chapter is to show how in a specific case, viz. for a modified Edwards-Anderson model [2] endowed with Glauber dynamics [3], such a domain theory can be developed from a microscopic level. The model is one-dimensional, but despite its simplicity it contains the essential features of a spin glass. It is expected – and indeed found – that on length scales shorter than the correlation length this model shows behaviour characteristic of a true spin glass phase.

In subsections 1.1 and 1.2 two preliminary concepts, viz. the notion of (time-dependent) domains, and the measurement of the dynamic susceptibility and fluctuations in arbitrary time-dependent systems, are considered. In section 2 the model is defined, and in the remaining sections a theoretical description of the "temperature jump experiment", in which at an initial time the heat bath temperature is suddenly changed from T_i to T, is given. In sections 3, 4 and 5 the time-dependent overlap with the final equilibrium state at T, the time-dependent magnetic response, and the time-dependent magnetic fluctuations of the system, are considered.

In particular the question of the validity of the Fluctuation-Dissipation Theorem (FDT) in nonequilibrium is addressed because up till now there is no agreement between theory and experiment. Theoretical investigations of a mean field "soft spin" model endowed with Langevin dynamics [4] yield a violation of the FDT in the nonequilibrium regime [5]. On the experimental side one [6] has only very recently succeeded in measuring the magnetic fluctuations in and out of equilibrium. Many groups [6,7,8,9,10,11] report the validity of the FDT in equilibrium and Refregier [12] reports also its validity in nonequilibrium.

1.1 Overlap between two states and domains

The standard spin glass model is the Edwards-Anderson model [2] which has the Hamiltonian

$$\mathcal{H}(\{s_j\}) = -\sum_{(j,k)} J_{jk} s_j s_k, \tag{1.1}$$

where the Ising spins, $s_j = \pm 1$, are located at sites j of a regular d-dimensional lattice and where the sum runs over all pairs of neighbouring sites. Disorder is introduced by a suitable probability distribution $\mathcal{P}(J_{jk})$ for the bonds J_{jk}. The Edwards-Anderson correlation function for this model is given by (see e.g. [13]):

$$C^{eq}_{|j-k|}(T) \equiv \overline{\left(\langle s_j s_k \rangle^{eq}_T \right)^2}, \tag{1.2}$$

where $|j - k|$ is the distance between sites j and k. Throughout this paper $\langle \ldots \rangle^{eq}_T$ denotes a thermal average at a temperature T and the overbar $\overline{\ldots}$ denotes an average over the disorder. In the paramagnetic phase, for temperatures $T > T_f$, this

correlation function decays exponentially and one may define a spin glass correlation length $\xi_{eq}(T)$ by

$$C_r^{eq}(T) \simeq \exp\{-2r/\xi_{eq}(T)\} \quad \text{for} \quad r \to \infty. \tag{1.3}$$

The correlation length $\xi_{eq}(T)$ diverges at the freezing temperature T_f and is infinite for temperatures $T < T_f$.

A recent observation by Bray and Moore [14,15,16] has been that the correlations in the spin glass phase are very sensitive to small changes in temperature. The similarity between the correlations of two thermodynamic states of the same system at different temperatures T and T' is expressed by means of the *overlap function* that essentially is the *projection* of one of the states onto the other:

$$C_{|j-k|}^{eq}(T, T') \equiv \overline{\langle s_j s_k \rangle_T^{eq} \langle s_j s_k \rangle_{T'}^{eq}}. \tag{1.4}$$

For the spin glass phase, where $T, T' < T_f$ and hence both $\xi_{eq}(T)$ and $\xi_{eq}(T')$ are infinite, the analysis of Bray and Moore demonstrates the existence of an *overlap length* $\ell_{eq}(T, T')$ such that

$$C_r^{eq}(T, T') \simeq \exp\{-2r/\ell_{eq}(T, T')\} \quad \text{for} \quad r \to \infty. \tag{1.5}$$

They used a description of the spin glass phase in terms of a $T = 0$ scaling theory [16] to investigate the scaling behaviour of $\ell_{eq}(T, T')$ and found

$$\ell_{eq}(T, T') \sim \left| \frac{T - T'}{\mathcal{J}} \right|^{-\frac{1}{\zeta}} \quad \text{for} \quad |T - T'| \to 0 \tag{1.6}$$

with positive exponent ζ and $\mathcal{J}$ the typical coupling strength. So in the limit of vanishing temperature difference, $|T - T'| \to 0$, when the two thermodynamic states become identical, the overlap length in the spin glass phase diverges as it should. For finite temperature differences the correlations of the two thermodynamic states are nearly identical on length scales less than $\ell_{eq}(T, T')$ and different on larger length scales. Eq. (1.4) is also of interest when the correlation length ξ_{eq} for one or both of the states is finite. Let for instance $T > T_f$ so that the correlation length $\xi_{eq}(T)$ in the thermodynamic state at temperature T is no longer infinite. As a consequence the correlations of the two states cannot be equal over distances longer than $\xi_{eq}(T)$. The more general expression for the overlap length (1.6) is therefore

$$\ell_{eq}(T, T') \simeq \left\{ c \left| \frac{T - T'}{\mathcal{J}} \right|^{\frac{1}{\zeta}} + \frac{1}{2\xi_{eq}(T)} + \frac{1}{2\xi_{eq}(T')} \right\}^{-1} \quad \text{for} \quad |T - T'| \to 0 \tag{1.7}$$

with c a constant. For $\xi_{eq}(T)$ and $\xi_{eq}(T')$ both infinite Eq. (1.7) leads to Eq. (1.6). If $\xi_{eq}(T)$ is large but finite and $\xi_{eq}(T')$ is infinite, the scaling behaviour (1.6) for larger temperature differences $|T - T'|$ levels off to a constant value $\ell_{eq}(T, T') = \xi_{eq}(T)$ as $T' \to T$. Eq. (1.7) is exact for the model that we shall treat in the next sections and we expect it to hold for other spin glass models as well.

Let T' be an arbitrary reference temperature. Then, within this framework, we can give a definition of what is meant by T'-*domains* in a thermodynamic state

71

at temperature T: these are regions where the correlations are identical to the correlations of the same system in equilibrium at the reference temperature T'. According to the above theory these domains have a characteristic size $\ell_{eq}(T, T')$. We emphasize that for a particular thermodynamic state at temperature T there is an infinite set of domain structures each of which belongs to a distinct reference temperature T'.

In a nonequilibrium state one can again distinguish T'-domains, whose linear size then depends on time. The overlap function (1.4) can be generalized to include the nonequilibrium case as follows:

$$C_{|j-k|}(t; T') \equiv \overline{\langle s_j s_k \rangle_t \langle s_j s_k \rangle_{T'}^{eq}}, \tag{1.8}$$

where $\langle \ldots \rangle_t$ denotes an average over a time-dependent ensemble. The time-dependent characteristic length of the overlap function (1.8) is referred to as the time-dependent characteristic linear size $\ell(t; T')$ of the T'-domains. It describes up until which length scale correlations in the time-dependent state are identical to correlations of the same system in equilibrium at temperature T'. A nonequilibrium system evolving towards equilibrium in a heat bath of temperature T may at any time be analyzed in terms of T'-domains for an arbitrary reference temperature T'. One then finds that the characteristic domain size $\ell(t; T')$ changes with time t until if finally reaches the overlap length $\ell_{eq}(T, T')$. For the special choice $T' = T$ this upper limit is the correlation length $\xi_{eq}(T)$ and so in a system that for temperature T exhibits long range order the characteristic T-domain size grows without limit.

1.2 Fluctuations and dissipation: theory and experiment

In this work we shall address the question of the validity of the FDT in nonequilibrium systems within the context of the model of section 2. Here we present some preliminary considerations.

1.2.1 General

The presence of a (time-dependent) magnetic field $H(t)$ gives rise to a Zeeman energy

$$\mathcal{H}_1 = -H(t) \sum_j s_j. \tag{1.9}$$

For sufficiently small magnetic field values this additional energy can be treated as a perturbation to the interaction energy (1.1) and the magnetization per spin $m(t) \equiv \overline{\langle s_j \rangle_t}$ responds linearly to the applied magnetic field:

$$m(t) = \int\limits_0^t dt' \chi(t, t - t') H(t - t') \quad (t > 0), \tag{1.10}$$

where we took the initial magnetization to vanish, $m(0) = 0$. The *magnetic response function* $\chi(t, t - t')$ gives the response at time t to a unit magnetic field impulse at time $t - t'$. Causality implies that $\chi(t, t - t')$ vanishes for $t' < 0$. In thermal equilibrium the response function depends only on the time difference t' and we write

$$\chi(t, t - t') \overset{eq}{\to} \chi^{eq}(t'). \tag{1.11}$$

Furthermore the response to a unit magnetic field step at time $t = t_w$ tends to the equilibrium susceptibility at temperature T,

$$\chi_{eq}(T) = \lim_{t \to \infty} \int_{t_w}^{t} dt' \chi(t, t - t'). \tag{1.12}$$

The *equilibrium dynamic susceptibility* $\tilde{\chi}^{eq}(\omega)$ is defined as the Fourier transformed response function (1.11)

$$\tilde{\chi}^{eq}(\omega) \equiv \int_{0}^{\infty} dt' \chi^{eq}(t') e^{i\omega t'}. \tag{1.13}$$

Experimentally the dynamic susceptibility of an arbitrary nonequilibrium system is measured by applying to it an oscillating magnetic field

$$H(t) = \begin{cases} H_0 e^{-i\omega t} & \text{for } t > 0 \\ \\ 0 & \text{otherwise} \end{cases} \tag{1.14}$$

which when substituted in (1.10), yields the response

$$m(t) = H(t) \int_{0}^{t} dt' \chi(t, t - t') e^{i\omega t'}. \tag{1.15}$$

Then one filters out the Fourier component of this signal with frequency ω by multiplying it by $e^{i\omega t}$ and by subsequent averaging during a time window τ_o that is centered around t and that comprises one or more full periods of the frequency ω. The amplitude of this component divided by H_0 is the experimentally measured *time-dependent dynamic susceptibility* $\tilde{\chi}_{\tau_o}(\omega; t)$, given by

$$\tilde{\chi}_{\tau_o}(\omega; t) = \frac{1}{\tau_o} \int_{t - \frac{1}{2}\tau_o}^{t + \frac{1}{2}\tau_{o'}} dt' \int_{0}^{t'} dt'' \chi(t', t' - t'') e^{i\omega t''}. \tag{1.16}$$

Of course this time-dependent dynamic susceptibility is a useful concept only when the observation time τ_o is sufficiently small compared to the time scale on which the nonequilibrium behaviour of the system manifests itself. For stationary systems $\tilde{\chi}_{\tau_o}(\omega; t)$ is independent of the observation time τ_o and for $t \to \infty$ it tends to $\tilde{\chi}^{eq}(\omega)$.

The autocorrelation function of the magnetization,

$$C_M(t_1, t_2) \equiv \lim_{L \to \infty} \frac{1}{L} \sum_{j,k=1}^{L} \overline{\langle s_j(t_1) s_k(t_2) \rangle} \tag{1.17}$$

can in thermal equilibrium situations be written as

$$C_M(t, t - t') \xrightarrow{eq} C_M^{eq}(t'). \tag{1.18}$$

Whereas $C_M^{eq}(t')$ is an even function of the time difference t' we have to expect that $C_M(t, t - t')$ in general is not. In equilibrium systems the autocorrelation function (1.18) is related to the fluctuation spectrum $S^{eq}(\omega)$ by the Wiener-Khinchin theorem

$$S^{eq}(\omega) = \frac{2}{\pi} \int_0^\infty dt' \, C_M^{eq}(t') \cos \omega t' \tag{1.19}$$

and the fluctuation spectrum is in turn related to the dissipative part of the dynamic susceptibility by the FDT

$$S^{eq}(\omega) = \frac{2}{\pi \beta \omega} \operatorname{Im} \tilde{\chi}^{eq}(\omega). \tag{1.20}$$

Experimentally one determines the fluctuation spectrum of the magnetization of an arbitrary nonequilibrium system from an observation during a finite time window τ_o, centered around t. The result is

$$S_{\tau_o}(\omega; t) \equiv \frac{1}{\pi \tau_o} \int_{t - \frac{1}{2}\tau_o}^{t + \frac{1}{2}\tau_o} dt_1 \int_{t - \frac{1}{2}\tau_o}^{t + \frac{1}{2}\tau_o} dt_2 \, C_M(t_1, t_2) \cos \omega(t_1 - t_2). \tag{1.21}$$

In equilibrium the expression (1.21) is independent of t; moreover for sufficiently large observation times τ_o it should also become independent of τ_o. The question now is whether the experimentally obtained nonequilibrium quantities (1.16) and (1.21) also satisfy the FDT (1.20). In section 5 we shall come back to this question and answer it within the context of the model to be defined in section 2.

1.2.2 Example

It is easy to illustrate that the FDT (1.20) is not automatically satisfied even in an equilibrium system when the time window τ_o is only finite. We consider a simple stationary system with a single relaxation time α^{-1}, so that

$$C_M(t, t - t') = \frac{\chi_{eq}(T)}{\beta} e^{-\alpha |t'|} \tag{1.22}$$

and

$$\chi(t, t - t') = \chi_{eq}(T) \alpha e^{-\alpha t'} \quad \text{for } t' > 0, \text{ otherwise } 0. \tag{1.23}$$

Let τ_o comprise n_o periods of length $2\pi \omega^{-1}$. From the expressions (1.16, 1.21) one obtains the equilibrium dynamic susceptibility

74

$$\bar{\chi}_{\tau_o}(\omega; t) = \chi_{eq}(T)\frac{\alpha}{\alpha - i\omega}\left\{1 - \frac{e^{-(\alpha - i\omega)t}}{\alpha - i\omega}(-1)^{n_o}\frac{\sinh(\alpha\tau_o/2)}{\tau_o/2}\right\} \tag{1.24}$$

and the fluctuation spectrum

$$S_{\tau_o}(\omega; t) = \frac{\chi_{eq}(T)}{\pi\beta}\frac{2\alpha}{\alpha^2 + \omega^2}\left\{1 + \frac{\omega^2 - \alpha^2}{\omega^2 + \alpha^2}\frac{1 - e^{-\alpha\tau_o}}{\alpha\tau_o}\right\}. \tag{1.25}$$

In the expression for the dynamic susceptibility (1.24), the second term in the curly brackets is an exponentially decaying transient due to the fact that the ac magnetic field was switched on at $t = 0$; it is negligible if $t \gg \alpha^{-1}$. The expression for the fluctuation spectrum (1.25) is independent of time. However, it assumes its equilibrium value only in the limit of an infinitely long observation time $\tau_o \gg \alpha^{-1}$. Hence the FDT holds only for $t > \tau_o \gg \alpha^{-1}$.

If the relaxation time of the system, α^{-1}, is very long, then it is experimentally relevant to consider in (1.24) and (1.25) the limit $\alpha^{-1} \gg \tau_o$ (observation time much shorter than the decay time of the fluctuations). In that limit the measured fluctuation spectrum (for frequencies $\omega = 2\pi n_o/\tau_o$) appears to acquire a factor two compared to the equilibrium expression. Hence the experimental data will satisfy a proportionality as in the FDT but with a proportionality factor almost twice as large.

2 A renormalized Edwards-Anderson model

2.1 Derivation of the model

We shall imagine that originally we are interested in a d-dimensional lattice of $(bL) \times b^{d-1}$ sites, i.e. remaining finite in all but one directions as $L \to \infty$. The system is supposed to be described by the Edwards-Anderson Hamiltonian (1.1) with short-range random couplings of typical strength $\mathcal{J}$. The system is at a temperature T_{bare}. We assume that we can apply to this system a renormalization procedure in which L block spins s_k, with $k = 1, 2, \ldots, L$, are introduced, that represent blocks of b^d original spins. The system of spins s_k is at a (renormalized) temperature T which is a smooth function of T_{bare},

$$T = f(T_{bare}), \tag{2.1}$$

and we suppose that it can be described by the Hamiltonian (1.1) for dimension $d = 1$,

$$\mathcal{H}(\{s_k\}; \{J_k(T)\}) = -\sum_{k=1}^{L} J_k(T)s_k s_{k+1} \quad (s_{L+k} \equiv s_k) \tag{2.2}$$

in which the renormalized random couplings J_k are again of typical strength $\mathcal{J}$. Due to the effects of frustration in the original system, however, the precise value of the individual couplings J_k will depend very sensitively on the original temperature T_{bare}, or, equivalently, on T. This has been indicated explicitly in the notation.

An exact calculation of the $J_k(T)$ is not feasible. We shall therefore replace such a calculation with a set of assumptions concerning these couplings which brings out their sensitivity to temperature changes.

We stipulate that the bonds $J_j(T)$ and $J_k(T)$ are independent for $j \neq k$ and that furthermore $J_k(T)$ is a Markov process of the following type (this process is called the "random telegraph process"):

(i) For $T = 0$ the bond $J_k(0)$ is either $+\mathcal{J}$ or $-\mathcal{J}$ with equal probability.

(ii) For a temperature increment ΔT the bond $J_k(T)$ changes sign with probability $\Gamma \Delta T / 2$, where Γ is a constant that arises in the renormalization process.

Consequently, for all temperatures T, the bond distribution is of the "$\pm \mathcal{J}$-type" [13]:

$$\mathcal{P}(J_k, T) = \tfrac{1}{2} \left\{ \delta(J_k + \mathcal{J}) + \delta(J_k - \mathcal{J}) \right\}. \tag{2.3}$$

The bond correlation is given by

$$\overline{J_j(T)J_k(T')} = \mathcal{J}^2 \delta_{j,k} e^{-\Gamma|T-T'|}. \tag{2.4}$$

The system dimension d and the finite size b must obviously enter into the renormalization relations linking T and Γ to T_{bare}. It is not here our aim, however, to study this dependence. In particular, for our purpose of demonstrating how aging phenomena can arise, it suffices to consider Γ as a temperature independent constant.

2.2 Statics

The starting point for the remainder of this paper is the Hamiltonian (2.2) together with the assumed properties of the couplings $J_k(T)$. The static properties of this system are easily obtained.

For a given realization $\{J_k(T)\}$ of the bonds the spatial correlation function is, in the thermodynamic limit of chain length $L \to \infty$, equal to

$$\langle s_j s_{j+n} \rangle_T^{eq} = \prod_{l=0}^{n-1} \tanh \beta J_{j+l}(T) \quad (n > 0), \tag{2.5}$$

where β is the inverse temperature (Boltzmann's constant is taken to be unity). The correlation length (1.3) is

$$\xi_{eq}^{-1}(T) = -\tfrac{1}{2} \log \overline{\tanh^2 \beta J_k(T)} = -\log \tanh \beta \mathcal{J}. \tag{2.6}$$

One can also calculate the equilibrium overlap function (1.4) and finds for the overlap length

$$\ell_{eq}^{-1}(T, T') = -\tfrac{1}{2} \log \overline{\tanh \beta J_k(T) \tanh \beta' J_k(T')}. \tag{2.7}$$

The disorder average in Eq. (2.7) can be performed with the aid of expression (2.4) for the bond correlation. The expression for the equilibrium overlap length is then indeed of the form (1.7):

$$\ell_{eq}(T,T') = 2\left\{\Gamma|T-T'| - \log\tanh\left(\frac{\mathcal{J}}{T}\right) - \log\tanh\left(\frac{\mathcal{J}}{T'}\right)\right\}^{-1} \tag{2.8}$$

Eq. (2.8) differs from the simple scaling form (1.6) found by Bray and Moore [14,15] in that for no finite temperature does the overlap length diverge when $|T-T'| \to 0$, it only increases up to the spin glass correlation length $\xi_{eq}(T)$. This is due to the absence of long range order in this model. We shall however take $\xi_{eq}(T)$ very large and study in the subsequent sections time-dependent phenomena that involve spatial scales much shorter than $\xi_{eq}(T)$. Such phenomena should be completely analogous to phenomena in the spin glass phase of systems with a freezing transition.

In this model all frustration effects have been absorbed in the temperature dependence of the couplings $J_k(T)$. Therefore, *at a fixed temperature T*, this model is just a Mattis spin glass [17] as can be seen after one transforms the spin variables according to

$$s_k = \tilde{s}_k(T) \prod_{l=1}^{k-1} \text{sgn}\, J_l(T) \tag{2.9}$$

in the Hamiltonian (2.2) which yields a Hamiltonian for a ferromagnetic chain of Ising spin variables $\bar{s}_k(T)$. The same transformation converts the Zeeman term (1.9) into a randomly site-dependent magnetic field, so that, again *at a fixed temperature*, the present model is also a random field Ising model.

2.3 Dynamics

We finally introduce the dynamics for this model. We follow Glauber [3] and postulate a master equation for the probability $P(\{s_k\}, t)$ of finding a particular realization of the chain with spin configuration $\{s_k\}$ at time t,

$$\frac{\partial P(\{s_1,\ldots,s_L\},t)}{\partial t} = \sum_{k=1}^{L} W(s_k|-s_k;T(t))P(\{s_1,\ldots,-s_k,\ldots,s_L\},t)$$

$$-\sum_{k=1}^{L} W(-s_k|s_k;T(t))P(\{s_1,\ldots,s_k,\ldots,s_L\},t). \tag{2.10}$$

Here $T(t)$ is the temperature at time t of the heat bath in which the system is placed. The transition rate $W(-s_k|s_k;T)$ for a spin s_k to flip to $-s_k$ depends on the values of the neighbouring spins s_{k-1} and s_{k+1} according to

$$\begin{aligned} W(-s_k|s_k;T) &= \tfrac{1}{2}\left\{1 - \tfrac{1}{2}s_k(s_{k-1}+s_{k+1})\tanh\beta(J_{k-1}(T)+J_k(T))\right. \\ &\left. -\tfrac{1}{2}s_k(s_{k-1}-s_{k+1})\tanh\beta(J_{k-1}(T)-J_k(T))\right\}. \end{aligned} \tag{2.11}$$

The transition rates are such that detailed balance is satisfied. Note that by this definition time is measured in units of the elementary spin flip time. One easily

verifies that the Mattis transformation (2.9) converts the master equation (2.10) for the disordered chain into a master equation for a ferromagnetic chain of spin variables $\tilde{s}_k(T)$ and that the transition rate for a spin $\tilde{s}_k(T)$ to flip to $-\tilde{s}_k(T)$ is given by

$$\tilde{W}(-\tilde{s}_k|\tilde{s}_k;T) = \tfrac{1}{2}\left\{1 - \tfrac{1}{2}\gamma\tilde{s}_k(\tilde{s}_{k-1} + \tilde{s}_{k+1})\right\}, \tag{2.12}$$

where $\gamma \equiv \tanh 2\beta\mathcal{J}$. These rates are identical those of Ref. [3] for a ferromagnetic Ising chain.

From the master equation we shall derive evolution equations for the spin correlation functions. The time-dependent correlation functions will be used in the following sections to analyze the nonequilibrium behaviour of the model. Because of the correspondence with the ferromagnetic chain some of the results from Ref. [3] can be used immediately.

3 The time-dependent overlap function

In this and the following sections we shall study the equilibration of the chain in a typical experimental setup. We shall assume that for $t < 0$ the chain is in equilibrium at an initial temperature $T_i = T + \Delta T$ and that at time $t = 0$ it is subjected to a temperature jump ΔT so that for times $t > 0$ it will tend to equilibrium at temperature T.

The aim of this section is to analyze the evolution of the overlap function $C_n(t;T)$ of Eq. (1.8) at temperature T. This quantity is not itself experimentally measurable, but its decay length $\ell(t;T)$ plays a fundamental role in the theory of aging proposed earlier [18]. The final result of this section will therefore be an expression for this length.

The time-dependent overlap function $C_n(t;T)$ of Eq. (1.8) contains an equilibrium correlation function $\langle s_j s_{j+n}\rangle_T^{eq}$ for which Eq. (2.5) is substituted and a time-dependent correlation function $\langle s_j s_{j+n}\rangle_t$ where $\langle\ldots\rangle_t$ denotes an ensemble average with Glauber dynamics at temperature T. After performing a Mattis transformation (2.9) and after a trivial disorder average that involves the bond correlation (2.4) we obtain

$$
\begin{aligned}
C_n(t;T) &= \overline{\langle s_j s_{j+n}\rangle_t \langle s_j s_{j+n}\rangle_T^{eq}} \\
&= \prod_{l=1}^{n-1}\overline{\operatorname{sgn}^2 J_{j+l}(T)}\langle \tilde{s}_j(T)\tilde{s}_{j+n}(T)\rangle_t \tanh^n \beta\mathcal{J} \\
&= \langle \tilde{s}_j(T)\tilde{s}_{j+n}(T)\rangle_t z^n \\
&= z^n c_n(t) \quad (n > 0),
\end{aligned} \tag{3.1}
$$

where we introduced the shorthand $z \equiv \tanh \beta\mathcal{J}$ and $c_n(t) \equiv \langle \tilde{s}_j(T)\tilde{s}_{j+n}(T)\rangle_t$. The experimental conditions given above imply that the initial value of the overlap function is equal to the projection of the equilibrium state at temperature T_i onto

the reference state and by means of Eqs. (2.5, 2.4) it can be written as

$$
\begin{aligned}
C_n(0;T) &= \overline{\langle s_j s_{j+n}\rangle_T^{eq}} \, \overline{\langle s_j s_{j+n}\rangle_T^{eq}} \\
&= \prod_{l=1}^{n-1} \overline{\operatorname{sgn} J_{j+l}(T_i) \operatorname{sgn} J_{j+l}(T)} \tanh^n \beta_i \mathcal{J} \tanh^n \beta \mathcal{J} \\
&= \left(z_i z e^{-\Gamma|\Delta T|}\right)^n \quad (n>0),
\end{aligned}
\tag{3.2}
$$

where $z_i = \tanh \beta_i \mathcal{J}$ and $\beta_i = T_i^{-1}$. The characteristic length of Eq. (3.2) is the initial value of the overlap length

$$
\ell(0;T) = \frac{2}{\Gamma|\Delta T| - \log z_i - \log z}.
\tag{3.3}
$$

The final, asymptotic value of the overlap function equals the projection of the equilibrium state at temperature T onto itself and so it is the equilibrium correlation function at temperature T:

$$
\begin{aligned}
\lim_{t\to\infty} C_n(t;T) &= C_n^{eq}(T) \\
&= \prod_{l=1}^{n-1} \overline{\operatorname{sgn}^2 J_{j+l}(T)} \tanh^{2n} \beta \mathcal{J} \\
&= z^{2n} \quad (n>0).
\end{aligned}
\tag{3.4}
$$

The final overlap length is therefore equal to the correlation length at temperature T,

$$
\lim_{t\to\infty} \ell(t;T) = \xi_{eq}(T) = |\log z|^{-1},
\tag{3.5}
$$

which for any finite ΔT is larger than the initial overlap length (3.3).

We shall first derive the time-evolution of the correlation function $c_n(t)$. From Eqs. (3.1) and (3.2) we obtain the initial condition

$$
c_n(0) = \left(z_i e^{-\Gamma|\Delta T|}\right)^n \equiv z_e^n \quad (n>0),
\tag{3.6}
$$

where we introduced an effective temperature T_e and put $z_e = \tanh \beta_e \mathcal{J}$. It can be shown that $z_e \leq z$, and that the equality holds only for $\Delta T = 0$: The change of the bonds induced by the temperature jump ΔT breaks up correlated regions and so the initial chain appears as a chain that was in equilibrium at a higher temperature T_e, regardless of whether the initial temperature T_i was smaller or larger than T.

From Eqs. (3.1) and (3.4) we can also obtain the asymptotic value

$$
\lim_{t\to\infty} c_n(t) = z^n \quad (n>0).
\tag{3.7}
$$

One can, by means of the master equation (2.10), derive an evolution equation (see Ref.[3]):

$$
\dot{c}_n(t) = \begin{cases} -2c_n(t) + \gamma c_{n+1}(t) + \gamma c_{n-1}(t) & (n \neq 0) \\ 0 & (n=0) \end{cases}
\tag{3.8}
$$

and it is easy to verify that the solution to (3.8) and (3.6) reads

$$c_n(t) = z^n - f_n(t, z; z) + f_n(t, z_e; z) \quad (n > 0). \tag{3.9}$$

The function $f_n(t, p; z)$ is defined as

$$f_n(t, p; z) \equiv e^{-2t} \sum_{l=1}^{\infty} p^l \left\{ I_{n-l}(2\gamma t) - I_{n+l}(2\gamma t) \right\}, \tag{3.10}$$

where I_n is the modified Bessel function of the first kind and of order n (see e.g. [19]), and where the z-dependence comes in via γ.

With the integral representation

$$I_n(x) = \frac{1}{\pi} \int_0^\pi d\theta \cos(n\theta) e^{x \cos \theta} \tag{3.11}$$

the sum in Eq. (3.10) can be performed by elementary methods and one finds

$$f_n(t, p; z) = \frac{2}{\pi} \int_0^\pi dk \, \frac{p \sin k \sin nk}{1 - 2p \cos k + p^2} e^{-2(1-\gamma \cos k)t} \quad (n > 0). \tag{3.12}$$

Now $\theta = k/\sqrt{t}$ is substituted and an expansion is made in powers of $1/\sqrt{t}$ and one finds for the leading term

$$f_n(t, p; z) \simeq \frac{2n e^{-2(1-\gamma)t}}{\pi \sqrt{t}} \int_0^\infty dk \, \frac{k^2}{t \frac{(1-p)^2}{p} + k^2} e^{-\gamma k^2} \quad (n > 0, t \to \infty) \tag{3.13}$$

with the first order corrections of relative order n^2/t.

We shall be interested in the low temperature regime where $\xi_{eq}(T)$ is large so that

$$z = e^{-1/\xi_{eq}(T)} \approx 1 - \xi_{eq}^{-1}(T) \tag{3.14}$$

and

$$\gamma = \frac{2z}{1 + z^2} \approx 1 - \tfrac{1}{2} \xi_{eq}^{-2}(T). \tag{3.15}$$

We shall also take the temperature jump sufficiently large, $\Gamma |\Delta T| \approx 1$, so that initially the chain appears as a chain with a very small correlation length $\xi_{eq}(T_e) \approx 1$. Experiments with smaller temperature jumps have also been carried out and one can readily extend our analysis to such situations. We can than write Eq. (3.13) as

$$f_n(t, p; z) \simeq \frac{n e^{-t/\xi_{eq}^2(T)}}{\sqrt{\pi t}} F \left(\frac{t(1-p)^2}{p} \right). \tag{3.16}$$

where

$$F(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty dk \, \frac{k^2}{x+k^2} e^{-k^2}. \tag{3.17}$$

One finds $F(0) = 1$ and $F(x) \simeq 1/2x$ for $x \to \infty$. For $p = z$ the argument of F in Eq.(3.16) equals $t/\xi_{eq}^2(T)$ and for $p = z_e$ it equals $(1 - z_e)^2 t/z_e$, which we shall write as $at/\xi_{eq}^2(T_e)$ with a nonsingular constant a. Substitution of (3.14) and (3.16) in Eq. (3.9) yields

$$c_n(t) \simeq 1 - \frac{n}{\xi_{eq}(T)} - \frac{ne^{-t/\xi_{eq}^2(T)}}{\sqrt{\pi t}} \left\{ F\left(\frac{t}{\xi_{eq}^2(T)}\right) - F\left(\frac{at}{\xi_{eq}^2(T_e)}\right) \right\}. \tag{3.18}$$

For times t with $1 \ll t \ll \xi_{eq}^2(T)$ Eq. (3.18) reduces to

$$c_n(t) \simeq 1 - \frac{n}{\sqrt{\pi t}} \qquad (1 \ll t \ll \xi_{eq}^2(T), n^2/t \ll 1). \tag{3.19}$$

Together with Eq. (3.1) this determines the behaviour of the overlap function $C_n(t;T)$. Comparison to the definition of the time-dependent characteristic length $\ell(t;T)$ which follows Eq.(1.8) shows that

$$\ell(t;T) \simeq 2\sqrt{\pi t}, \qquad (1 \ll t \ll \xi_{eq}^2(T)). \tag{3.20}$$

This is of the general power law form $\ell \sim t^p$ *postulated* in the phenomenological theory of Chapter III (see also Ref. [18]). Here such a law (with $p = \frac{1}{2}$) has been derived from a microscopic basis.

The above analysis can be extended to include the crossover to times where the final correlations become of importance: both $t \gg 1$ and $\xi_{eq}^2(T) \gg 1$ but with arbitrary ratio $t/\xi_{eq}^2(T)$. We first introduce a time-dependent correlation length $\xi(t;T)$ which is the characteristic length of $c_n(t)$. From Eq. (3.18) we read

$$\xi(t;T) = \xi_{eq}(T) \left\{ 1 + \frac{e^{-t/\xi_{eq}^2(T)}}{\sqrt{\pi t/\xi_{eq}^2(T)}} F\left(\frac{t}{\xi_{eq}^2(T)}\right) \right\}^{-1} \tag{3.21}$$

and one has

$$\xi(t;T) \simeq \begin{cases} \sqrt{\pi t} & \text{for} \quad 1 \ll t \ll \xi_{eq}^2(T) \\ \xi_{eq}(T) & \text{for} \quad t \gg \xi_{eq}^2(T). \end{cases} \tag{3.22}$$

Then, using Eq. (3.1), we find

$$\ell(t;T) = 2 \left\{ \xi_{eq}^{-1}(T) + \xi^{-1}(t;T) \right\}^{-1}. \tag{3.23}$$

The above derived behaviour of the overlap length can be understood from a simple physical picture. When at time $t = 0$ the temperature jumps to T a fraction of the bonds changes sign. The result is that at $t = 0$ the correlation length of the chain is reduced to $\xi_{eq}(T_e) \approx 1$, where the effective temperature T_e is defined by Eq. (3.6). From this initial condition the chain has to evolve towards a situation

where the correlated regions, these are the sequences of spins joined by satisfied bonds, are very large, $\xi_{eq}(T) \gg 1$. The temperature T is very low so that it is very unlikely that a spin inside a correlated region of size $\xi(t;T) \ll \xi_{eq}(T)$ flips. The boundaries of the correlated regions, these are the bonds that are not satisfied by their neighbouring spins, can move freely in both directions: they perform a *random walk*.

The number of random walkers decreases with time because when two of them meet they *annihilate* each other. The average distance between the random walkers increases with $\sqrt{t}$ and this is exactly the growth law for $\xi(t;T)$. It holds as long as $\xi(t;T) \ll \xi_{eq}(T)$, but when $\xi(t;T)$ gets closer to $\xi_{eq}(T)$ the creation of random walkers inside the correlated regions, caused by spontaneous spin flips, gets more probable and this limits the growth of $\xi(t;T)$ to $\xi_{eq}(T)$. The latter behaviour is found for times t larger than the maximum relaxation time $\frac{1}{2}(1 - \gamma)^{-1} \approx \frac{1}{2}\xi_{eq}^2(T)$ (see Eq. (3.12)).

4   The time-dependent response function $\chi(t, t - t')$

In this section we continue to consider the temperature jump experiment described in the first paragraph of section 3. There we considered the time-dependent internal domain structure of the spin glass chain. We now turn towards the experimentally observable magnetic response of the chain (cf. Eq.(1.10)) during equilibration after the temperature jump at time $t = 0$. We begin by calculating the time-dependent magnetization $m(t)$ for the case of an infinitesimally small magnetic field $H(t)$. We shall then see how the time-evolution of $m(t)$ is coupled to that of the internal domain structure.

As a preliminary the transition rates in a magnetic field have to be defined. In a constant field H there is a Zeeman energy term (1.9) to be added to the Hamiltonian (2.2). In order to satisfy detailed balance in the presence of the magnetic field the transition rates have to depend on H. The following choice is made:

$$W(-s_k|s_k; T, H) = W(-s_k|s_k; T, 0)(1 - s_k \tanh \beta H), \qquad (4.1)$$

where the transition rates in the absence of a magnetic field are given by Eq.(2.11). In the case of a time-dependent magnetic field $H(t)$ one can replace H in Eq.(4.1) by $H(t)$.

From the master equation (2.10) an evolution equation for the average of a spin can be derived (see Ref.[3]):

$$\frac{d\langle s_k \rangle_t}{dt} = -\langle s_k \rangle_t + \frac{\gamma}{2} \operatorname{sgn} J_{k-1}(T)\langle s_{k-1} \rangle_t + \frac{\gamma}{2} \operatorname{sgn} J_k(T)\langle s_{k+1} \rangle_t$$
$$+ \tanh(\beta H(t)) \left\{ 1 - \frac{\gamma}{2} \operatorname{sgn} J_{k-1}(T)\langle s_{k-1}s_k \rangle_t - \frac{\gamma}{2} \operatorname{sgn} J_k(T)\langle s_k s_{k+1} \rangle_t \right\} \quad (4.2)$$

which contains, besides the expected inhomogeneous term $\tanh \beta H(t)$, the spin correlations $\langle s_{k-1}s_k \rangle_t$ and $\langle s_k s_{k+1} \rangle_t$. In terms of the Mattis transformed spin variables

$\bar{s}_k(T)$ (see Eq.(2.9)) the evolution equation (4.2) reads

$$\frac{d\langle \bar{s}_k \rangle_t}{dt} = -\langle \bar{s}_k \rangle_t + \frac{\gamma}{2}\langle \bar{s}_{k-1} \rangle_t + \frac{\gamma}{2}\langle \bar{s}_{k+1} \rangle_t$$

$$+ \tanh(\beta H(t)) \left(\prod_{l=1}^{k-1} \operatorname{sgn} J_l(T) \right) \left\{ 1 - \frac{\gamma}{2}\langle \bar{s}_{k-1}\bar{s}_k \rangle_t - \frac{\gamma}{2}\langle \bar{s}_k \bar{s}_{k+1} \rangle_t \right\}. \qquad (4.3)$$

This equation describes the evolution of the average of a spin $\bar{s}_k$ in a ferromagnetic random field Ising model. For translationally invariant initial conditions it can be solved to linear order in the magnetic field. The spin correlations $\langle \bar{s}_{k-1}\bar{s}_k \rangle_t$ and $\langle \bar{s}_k \bar{s}_{k+1} \rangle_t$ are, to zeroeth order in $H(t)$, approximated by Eq.(3.9) and hence

$$\frac{d\langle \bar{s}_k \rangle_t}{dt} = -\langle \bar{s}_k \rangle_t + \frac{\gamma}{2}\langle \bar{s}_{k-1} \rangle_t + \frac{\gamma}{2}\langle \bar{s}_{k+1} \rangle_t + \beta h_k(t) \qquad (4.4)$$

where the random fields $h_k(t)$ are given by

$$h_k(t) \equiv H(t) \left(\prod_{l=1}^{k-1} \operatorname{sgn} J_l(T) \right) \left\{ 1 - \gamma c_1(t) \right\} \quad (k \geq 1). \qquad (4.5)$$

It can easily be verified that the solution of Eq.(4.4) is

$$\langle \bar{s}_k \rangle_t = e^{-t} \sum_{l=-\infty}^{\infty} \langle \bar{s}_l \rangle_{t=0} I_{k-l}(\gamma t)$$

$$+ \beta \int_0^t dt'\, e^{-t'} \sum_{l=-\infty}^{\infty} I_{k-l}(\gamma t') h_l(t - t'). \qquad (4.6)$$

It is interesting to remark that if one uses (4.6), (4.5), and (3.7), and the identities

$$\int_0^\infty dt\, e^{-t} I_n(\gamma t) = \frac{z^{|n|}}{\sqrt{1 - \gamma^2}} \qquad (4.7)$$

and $(1 - \gamma z)/\sqrt{1 - \gamma^2} = 1$, one finds that in a constant field $H(t) = H$ the spin averages $\langle \bar{s}_k \rangle_t$ approach for $t \to \infty$ the equilibrium values

$$\langle \bar{s}_k \rangle_\infty = \beta H \sum_{l=-\infty}^{\infty} \left(\prod_{j=1}^{l-1} \operatorname{sgn} J_j(T) \right) \tanh^{|k-l|} \beta \mathcal{J} \qquad (4.8)$$

(for l negative the product over j runs from l to 0). This is also the result given by a transfer matrix calculation in equilibrium if one formally expands to lowest order in βH. We expect it to be correct only if the distribution of $\langle \bar{s}_k \rangle_\infty$ has a width small compared to unity, i.e. if

$$\overline{\langle \bar{s}_k \rangle_\infty^2} \ll 1. \qquad (4.9)$$

By substituting (4.8) in (4.9) and averaging over the $J_j(T)$ one finds that this leads to the condition (for $H \ll 1$ and $\xi_{eq}(T) \gg 1$)

$$H\sqrt{\xi_{eq}(T)} \ll T \tag{4.10}$$

The left hand side of this inequality represents the Zeeman energy of a correlated region, and the right hand side its free energy with respect to the ground state. For the infinitesimal magnetic fields which we consider here, (4.10) represents no serious restriction, provided we do not also let T become equal to zero.

From the solution (4.6) the average magnetization (1.10) can be calculated in terms of the spin variables s_k. After the disorder average one obtains, taking $\langle \bar{s}_k \rangle_0 = 0$ for all spins

$$m(t) = \beta \int_0^t dt' e^{-t'} I_0(\gamma t') H(t - t')\{1 - \gamma c_1(t - t')\} \quad (t > 0). \tag{4.11}$$

For long times, $t \gg 1$, and low temperatures, $\xi_{eq}^2(T) \gg 1$, we may replace the correlation function $c_1(t)$ in Eq. (4.11) by Eq.(3.18) and with (3.21) we find

$$m(t) \simeq \beta \int_0^t dt' e^{-t'} I_0(\gamma t') \xi^{-1}(t - t'; T) H(t - t'). \tag{4.12}$$

The response function is read off directly from Eqs. (1.10) and (4.11):

$$\chi(t, t - t') = \beta e^{-t'} I_0(\gamma t')\{1 - \gamma c_1(t - t')\}, \tag{4.13}$$

and under the above assumptions

$$\chi(t, t - t') \simeq \beta e^{-t'} I_0(\gamma t') \xi^{-1}(t - t'; T). \tag{4.14}$$

The Eqs. (4.12) and (4.14) show how the magnetic response of the system is coupled to the time-dependent size $\xi(t; T)$.

The result derived above can readily be understood since only the boundary spins of the correlated regions of the chain are susceptible to the infinitesimally small field. This is because their local field is zero as compared to the local field of $2\mathcal{J}$ of the spins inside the correlated regions. The factor $\xi^{-1}(t; T)$ in Eq. (4.14) represents the fraction of susceptible spins at time t. Such predominance of domain boundary contributions to the magnetization has been observed experimentally not directly in spin glasses but in random field systems [20,21].

Suppose that following the temperature jump at $t = 0$ we keep the magnetic field zero until $t = t_w$, and at that time switch it on to a thereafter constant value H:

$$H(t) = H\theta(t - t_w). \tag{4.15}$$

Substitution of (4.15) in (4.12) shows that on the time interval $[t_w, \infty)$ the magnetization increases from $m(t_w) = 0$ to $m(\infty) = \beta H$ according to

84

$$m(t) \simeq \beta H \int_0^{t-t_w} dt' e^{-t'} I_0(\gamma t') \xi^{-1}(t-t';T).$$ (4.16)

A measure of the magnetic relaxation time $\tau(t_w)$ associated with the magnetic field step at time t_w is given by the initial slope of $m(t)$, normalized by the step size,

$$\tau^{-1}(t_w) = \frac{1}{m(\infty)} \left. \frac{dm(t)}{dt} \right|_{t=t_w}$$ (4.17)

with $m(t)$ given by (4.16). Working this out yields

$$\tau(t) = \xi(t;T)$$ (4.18)

In the time regime $1 \ll t \ll \xi_{eq}^2(T)$ one finds from (3.23) and (4.18)

$$\tau(t) \simeq 2\ell(t;T) \quad (1 \ll t \ll \xi_{eq}^2(T)).$$ (4.19)

This relation couples the instantaneous magnetic relaxation time to the domain size. It is precisely of the power law form $\tau \sim \ell^z$ *hypothesized* in Chapter III (see also Ref. [18]). However, here it has been derived (with $z = 1$) from a microscopic starting point.

5 The Fluctuation-Dissipation Theorem out of equilibrium

In this section we calculate the magnetic fluctuations of the chain as it equilibrates (in zero magnetic field). Then we shall calculate the fluctuation spectrum and the dynamic susceptibility and discuss the validity of the FDT. The experimental conditions are as in section 3, where at time $t = 0$ the temperature jumps from its initial value T_i to the value $T = T_i - \Delta T$. The magnetic fluctuations are derived from the time-delayed correlation function (cf. Eq.(1.17)) which for $t_2 \geq t_1$ is defined by

$$\langle s_j(t_2) s_k(t_1) \rangle \equiv \sum_{\{s\},\{s'\}} s_j G(s_1, \ldots, s_L; t_2 | s'_1, \ldots, s'_L; t_1) s'_k P(s'_1, \ldots, s'_L; t_1)$$ (5.1)

where $G(s_1, \ldots, s_L; t_2 | s'_1, \ldots, s'_L; t_1)$ is the conditional probability of finding the chain in the spin configuration $\{s\}$ at time t_2 given the spin configuration $\{s'\}$ at time t_1.

When one expresses the magnetic fluctuations (1.17) in terms of the Mattis transformed spin variables $\bar{s}_k(T)$ (see Eq.(2.9)) and subsequently averages over disorder using (2.4) one obtains

$$\begin{aligned} C_M(t_2, t_1) &= \lim_{L \to \infty} \frac{1}{L} \sum_{j,n=1}^{L} \overline{\left(\prod_{l=0}^{n-1} J_{j+l} \right)} \langle \bar{s}_j(t_2;T) \bar{s}_{j+n}(t_1;T) \rangle \\ &= \langle \bar{s}_j(t_2;T) \bar{s}_j(t_1;T) \rangle \end{aligned}$$ (5.2)

and one notices that only the spin autocorrelations contribute to the magnetic fluctuations since

$$\langle \tilde{s}_j(t_2; T) \tilde{s}_j(t_1; T) \rangle = \overline{\langle s_j(t_2) s_j(t_1) \rangle}. \tag{5.3}$$

This is not a peculiarity of this model but rather a common rule among spin glass models; see for instance the review by Fischer [22].

From the master equation (2.10) an evolution equation for the time-delayed spin correlation function is derived and from its solution one obtains (see Ref. [3])

$$\mathcal{C}_M(t_1, t_2) = e^{-(t_1 - t_2)} \sum_{n=-\infty}^{\infty} c_n(t_2) I_n(\gamma(t_1 - t_2)) \quad (t_1 \geq t_2) \tag{5.4}$$

where the spin correlations $c_n(t)$ are defined by Eq.(3.1) and given by Eq. (3.9).

The fluctuation spectrum can now be calculated by substituting the magnetic fluctuations (5.4) in Eq. (1.21). As in the previous sections we concentrate on the low temperature regime, $\xi_{eq}(T) \gg 1$, and on the behaviour for longer times, $t \gg 1$. The observation time τ_o comprises a number n_o of periods of the frequency ω, $\omega \tau_o = 2n_o\pi$, and we shall take this time window much larger than the elementary spin flip time and much smaller than the time lapse since the temperature jump:

$$1 \ll \omega^{-1} < \tau_o \ll t. \tag{5.5}$$

Under the above conditions we find (see appendix A)

$$S_{\tau_o}(\omega; t) \simeq \frac{\xi_{eq}(T)}{\xi(t; T)} \left\{ 1 + \frac{2}{\pi \sqrt{n_o}} \right\} S^{eq}(\omega) \tag{5.6}$$

and

$$S^{eq}(\omega) \simeq \frac{1}{\pi \xi_{eq}(T) \omega^{3/2}}. \tag{5.7}$$

First we note that the measurement result (5.6) is observation time dependent as was to be expected from the example in section 1.2.2. Secondly there is a time dependence via $\xi(t; T)$, the fraction of susceptible spins. For fixed n_o and in the regime $1 \ll t \ll \xi_{eq}^2(T)$ (see Eq.(3.22)) the fluctuation spectrum (5.6) decays towards its asymptotic value as a power low, $\sim t^{-1/2}$. Precisely such decay of the fluctuation spectrum has been reported by Refregier [12].

In order to test the validity of the FDT we calculate the dynamic susceptibility (1.16) with the response function (4.14). With the same assumptions (5.5) on the time scales as above, we find (see appendix B)

$$\tilde{\chi}_{\tau_o}(\omega; t) \simeq \frac{\xi_{eq}(T)}{\xi(t; T)} \tilde{\chi}^{eq}(\omega) \tag{5.8}$$

with the equilibrium value

$$\tilde{\chi}^{eq}(\omega) \simeq \frac{\beta(1 + i)}{2 \xi_{eq}(T) \sqrt{\omega}}. \tag{5.9}$$

We first note that the measured dynamic susceptibility (5.8) is not observation time dependent. This is in qualitative agreement with the example of simple exponential relaxation of section 1.2.2. The time-dependence of the dynamic susceptibility is governed by $\xi(t;T)$ as was the case with the fluctuation spectrum (5.6). As a consequence, the ratio

$$\frac{\beta \omega S_{\tau_o}(\omega;t)}{\mathrm{Im}\, \chi_{\tau_o}(\omega;t)} \simeq \frac{2}{\pi} \left\{ 1 + \frac{2}{\pi \sqrt{n_o}} \right\} \tag{5.10}$$

is a constant independent of t and ω. According to the FDT this constant should be $2/\pi$. Here we find the exact validity of the FDT only in the limit of large n_o: for long observation times.

Many groups [6,7,8,9,10,11] report a proportionality of the kind (5.10) for the "equilibrium" values of the fluctuation spectrum and the dynamic susceptibility. Refregier [12] reports a constant ratio (5.10) also for the time-dependent values. To date the experimental value of the constant, which is reported to be independent of frequency and temperature, has, to our knowledge, not appeared in the literature.

Appendix A: Asymptotic calculation of $S_{\tau_o}(\omega;t)$

In this appendix the fluctuation spectrum (1.21) is calculated from the magnetic fluctuations (5.4). The size τ_o of the time window is assumed to be large compared to the elementary spin flip time but small compared to t, the time-lapse since the temperature jump at $t = 0$. Furthermore the time window comprises a number n_o of full periods of the frequency ω, hence $\tau_o = 2\pi n_o/\omega$. To summarize these assumptions (see Eq. (5.5)):

$$1 \ll \omega^{-1} < \tau_o \ll t, \xi_{eq}^2(T). \tag{A.1}$$

Also it is assumed that T is low so that $\xi_{eq}^2(T) \gg 1$ and that $t \gg 1$.

First in (1.21) the integral over t_1 is split into two parts, $t_1 \geq t_2$ and $t_1 \leq t_2$. Then (5.4) is substituted and after some changes of variables one obtains

$$
\begin{aligned}
S_{\tau_o}(\omega;t) &= \frac{1}{\pi \tau_o} \int\limits_0^{\tau_o} dt' \left[\int\limits_0^{t'} dt'' e^{-t''} \cos \omega t'' \sum_{n=-\infty}^{\infty} c_n(t + \frac{\tau_o}{2} - t' + t'') I_n(\gamma t'') \right. \\
&\quad + \left. \int\limits_0^{t'} dt'' e^{-t''} \cos \omega t'' \sum_{n=-\infty}^{\infty} c_n(t - \frac{\tau_o}{2} + t') I_n(\gamma t'') \right].
\end{aligned}
\tag{A.2}
$$

The arguments of the c_n's in (A.2) differ from t by an amount of order $\tau_o \ll t$. Therefore the arguments of the c_n's are replaced by t neglecting corrections of order τ_o/t. This leads to

$$S_{\tau_o}(\omega;t) \simeq \frac{2}{\pi \tau_o} \int\limits_0^{\tau_o} dt' \int\limits_0^{t'} dt'' e^{-t''} \cos \omega t'' \sum_{n=-\infty}^{\infty} c_n(t) I_n(\gamma t''). \tag{A.3}$$

From (3.18) and (3.21) it follows that

$$c_n(t) \simeq 1 - \frac{n}{\xi(t;T)} \simeq c_1^n(t) \quad (n > 0) \tag{A.4}$$

with corrections of order n^2/t. For the higher values of n in the sum of (A.3) the product $e^{-t''}I_n(\gamma t'')$ is sufficiently small to surpress further contributions:

$$S_{\tau_o}(\omega;t) \simeq \frac{2}{\pi\tau_o} \int\limits_0^{\tau_o} dt' \int\limits_0^{t'} dt'' e^{-t''} \cos\omega t'' \sum_{n=-\infty}^{\infty} c_1^{|n|}(t) I_n(\gamma t'')$$

$$= \frac{2}{\pi^2\tau_o} \operatorname{Re} \int\limits_0^{\pi} dk \int\limits_0^{\tau_o} dt' \int\limits_0^{t'} dt'' e^{-(1-i\omega-\gamma\cos k)t''} \sum_{n=-\infty}^{\infty} c_1^{|n|}(t) \cos(nk). \tag{A.5}$$

In the last step the integral representation (3.11) is invoked (and the $\cos\omega t''$ is replaced by $\operatorname{Re} e^{i\omega t''}$). The sum in (A.5) can be carried out and one finds

$$S_{\tau_o}(\omega;t) \simeq \frac{2(1-\gamma_c c)}{\pi^2\tau_o} \operatorname{Re} \int\limits_0^{\pi} dk \int\limits_0^{\tau_o} dt' \int\limits_0^{t'} dt'' \frac{e^{-(1-i\omega-\gamma\cos k)t''}}{1-\gamma_c\cos k} \tag{A.6}$$

where the shorthands $c \equiv c_1(t)$ and $\gamma_c = 2c/(1+c^2)$ are used. The integrals over t'' and t' can now be performed and one finds

$$\begin{aligned}
S_{\tau_o}(\omega;t) &\simeq \frac{2(1-\gamma_c c)}{\pi^2} \operatorname{Re} \int\limits_0^{\pi} dk \frac{1}{(1-\gamma_c\cos k)(1-\gamma\cos k - i\omega)} \\
&\quad - \frac{2(1-\gamma_c c)}{\pi^2\tau_o} \operatorname{Re} \int\limits_0^{\pi} dk \frac{1-e^{-(1-\gamma\cos k)\tau_o}}{(1-\gamma\cos k - i\omega)^2(1-\gamma_c\cos k)}
\end{aligned} \tag{A.7}$$

where $\omega\tau_o = 2\pi n_o$ is used.

The second term in (A.7) demonstrates the effect of a finite time window. In the limit of $\tau_o \to \infty$ only the first term remains and this term is proportional to the equilibrium spectrum. The evaluation of (A.7) will be done in two steps, starting with the first term:

$$S_1 = \frac{1-\gamma_c c}{\pi^2} \operatorname{Re} \int\limits_0^{2\pi} dk \frac{1}{(1-\gamma_c\cos k)(1-\gamma_c\cos k - i\omega)}. \tag{A.8}$$

Set $u = e^{ik}$. The integral in (A.8) then becomes a contour integral along the unit circle. Inside the unit circle two poles are located. One at z and one at

$$u_\omega = \frac{1}{\gamma}\left\{1 - i\omega - \sqrt{(1-i\omega)^2 - \gamma^2}\right\}. \tag{A.9}$$

88

After some algebraic manipulations one finds

$$
S_1 = \frac{2}{\pi} \left\{ -\frac{\frac{\gamma}{\gamma_c} - 1}{(\frac{\gamma}{\gamma_c} - 1)^2 + \omega^2} \right.
$$
$$
+ \frac{\gamma(1 - \gamma_c c)}{\gamma_c} \frac{\frac{\gamma}{\gamma_c} - 1}{(\frac{\gamma}{\gamma_c} - 1)^2 + \omega^2} \, \text{Re} \, \frac{1}{\sqrt{(1 - i\omega)^2 - \gamma^2}}
$$
$$
\left. + \frac{\gamma(1 - \gamma_c c)}{\gamma_c} \frac{\omega}{(\frac{\gamma}{\gamma_c} - 1)^2 + \omega^2} \, \text{Im} \, \frac{1}{\sqrt{(1 - i\omega)^2 - \gamma^2}} \right\} \tag{A.10}
$$

The equilibrium spectrum is attained in the limit $t \to \infty$, where $c \to z$ and $\gamma_c \to \gamma$, so

$$
S^{eq}(\omega) = \frac{2(1 - \gamma z)}{\pi \omega} \, \text{Im} \, \frac{1}{\sqrt{(1 - i\omega)^2 - \gamma^2}} \tag{A.11}
$$

Using $\gamma \simeq 1 - \frac{1}{2}\xi_{eq}^2(T)$, $\gamma_c \simeq 1 - \frac{1}{2}\xi^2(t;T)$ and $c \simeq 1 - \xi^{-1}(t;T)$ in (A.10) and using the assumptions (A.1) finally yields

$$
S_1 \simeq \frac{\xi_{eq}(T)}{\xi(t;T)} S^{eq}(\omega) \tag{A.12}
$$

and $S^{eq}(\omega)$ given by (5.7).

Then the second term of (A.7) is evaluated. First concentrate on the integral

$$
I = \int_0^\pi dk \, \frac{1 - e^{-(1 - \gamma \cos k)\tau_o}}{(1 - \gamma \cos k - i\omega)^2(1 - \gamma_c \cos k)}. \tag{A.13}
$$

Since $\tau_o \gg 1$ this integral will be analyzed asymptotically for large τ_o. To this end substitute $k = k'/\sqrt{\tau_o}$ and then drop the primes again. The leading term of the expansion for large τ_o then is

$$
I \simeq \frac{1}{\sqrt{\tau_o}} \int_0^\infty dk \, \frac{1 - e^{-(1 - \gamma)\tau_o - \gamma k^2/2}}{(1 - \gamma + \frac{\gamma k^2}{2\tau_o} - i\omega)^2(1 - \gamma_c + \frac{\gamma_c k^2}{2\tau_o})}
$$
$$
= 2\tau_o^{5/2} \int_0^\infty dk \, \frac{1 - e^{-k^2/2}}{k^2(k^2/2 - i\omega\tau_o)^2}. \tag{A.14}
$$

In the second step terms of order $(1 - \gamma)\tau_o \simeq \tau_o/\xi_{eq}^2(T)$ have been neglected. The real part of I equals

$$
\text{Re} \, I \simeq 2\tau_o^{5/2} \int_0^\infty dk \, \frac{(k^4/4 - \omega^2\tau_o^2)(1 - e^{-k^2/2})}{k^2(k^4/4 + \omega^2\tau_o^2)^2}
$$
$$
\simeq -\frac{\sqrt{2\tau_o}}{\omega^2} \int_0^\infty dx \, \frac{1 - e^{-x^2}}{x^2} = -\frac{\sqrt{2\pi\tau_o}}{\omega^2} \tag{A.15}
$$

89

where we assumed that $\omega \tau_o = 2\pi n_o \gg 1$. With this result the second term of (A.7) gives

$$
\begin{aligned}
S_2 &= \frac{2(1 - \gamma_c c)}{\pi^2 \tau_o} \operatorname{Re} \int\limits_0^\pi dk \, \frac{1 - e^{-(1-\gamma \cos k)\tau_o}}{(1 - \gamma \cos k - i\omega)^2 (1 - \gamma_c \cos k)} \\
&\simeq \frac{\xi_{eq}(T)}{\xi(t;T)} \frac{2}{\pi \sqrt{n_o}} S^{eq}(\omega)
\end{aligned}
\tag{A.16}
$$

Summing up (A.12) and (A.16) finally gives the result (5.6).

Appendix B: Asymptotic calculation of $\bar{\chi}_{\tau_o}(\omega; t)$

In this appendix the dynamic susceptibility (1.16) is calculated from the response function (4.14). Here τ_o is the size of the time window around the time instant t (when at time $t = 0$ the chain was subjected to a temperature jump). The time window comprises a number n_o of periods of the frequency ω and hence $\tau_o = 2\pi n_o / \omega$. It is assumed (see Eq. (5.5)) that the time window is much larger than the elementary spin flip time but much smaller than the "age" of the system:

$$
1 \ll \omega^{-1} < \tau_o \ll t, \xi_{eq}^2(T)
\tag{B.1}
$$

It is also assumed that the temperature T is low so that $\xi_{eq}^2(T) \gg 1$ and that $t \gg 1$.

First (1.16) is substituted in (1.16) and one notes that the integrand of the integral over t'' contains two factors. The factor $\exp(-t'') I_0(\gamma t'')$ is maximal for $t'' = 0$ and the second factor, $1 - \gamma c_1(t' - t'')$, is maximal for $t'' = t'$, the other end of the integration interval. Therefore the integration interval is split into two parts, each containing a boundary maximum. In the first term, where t'' ranges from 0 to $t'/2$, one sets $\tau = t''$ and in the second term one sets $\tau = t' - t''$:

$$
\begin{aligned}
\bar{\chi}_{\tau_o}(\omega; t) &= \frac{\beta}{\tau_o} \int\limits_{t-\tau_o/2}^{t+\tau_o/2} dt' \left[\int\limits_0^{t'/2} d\tau e^{-\tau} I_0(\gamma \tau) \{1 - \gamma c_1(t' - \tau)\} e^{i\omega \tau} \right. \\
&\quad + \left. e^{i\omega t'} \int\limits_0^{t'/2} d\tau e^{-t'+\tau} I_0(\gamma(t' - \tau)) \{1 - \gamma c_1(\tau)\} e^{-i\omega \tau} \right]
\end{aligned}
\tag{B.2}
$$

Since t' is of order t (because $\tau_o \ll t$) one may expect that the integral over τ in the second term hardly varies with t' so that the average with the phase factor $\exp(i\omega t')$ over an integer number of periods $2\pi/\omega$ vanishes. A detailed analysis shows that this term gives rise to contributions of order τ_o/t and of order $\tau_o/\xi_{eq}^2(T)$.

It remains to calculate the first term in (B.2). The argument of c_1 can be replaced by t thereby neglecting terms of order τ_o/t. Using the relation $1 - \gamma c_1(t) \simeq$

$\xi^{-1}(t;T)$, see (3.21), one finds

$$\tilde{\chi}_{\tau_o}(\omega;t) \simeq \frac{\beta}{\tau_o \xi(t;T)} \int_{t-\tau_o/2}^{t+\tau_o/2} dt' \int_0^{t'/2} d\tau e^{-\tau+i\omega\tau} I_0(\gamma\tau)$$

$$= \frac{\beta}{\pi\tau_o\xi(t;T)} \int_0^{\pi} dk \int_{t-\tau_o/2}^{t+\tau_o/2} dt' \int_0^{t'/2} d\tau e^{-(1-i\omega-\gamma\cos k)\tau}. \tag{B.3}$$

In the second step the integral representation (3.11) for the modified Bessel functions is invoked. The integrals over τ and t' can be carried out and one finds

$$\tilde{\chi}_{\tau_o}(\omega;t) \simeq \frac{\beta}{\pi\xi(t;T)} \int_0^{\pi} dk \left[\frac{1}{1 - i\omega - \gamma\cos k} \right.$$
$$\left. - \frac{e^{-(1-i\omega-\gamma\cos k)(t-\tau_o/2)/2} - e^{-(1-i\omega-\gamma\cos k)(t+\tau_o/2)/2}}{\frac{1}{2}(1 - i\omega - \gamma\cos k)^2} \right]. \tag{B.4}$$

The second term in (B.4) demonstrates the effect of a finite time window τ_o and it is clear that this contribution vanishes up to corrections of order τ_o/t. In the first term of (B.4) one can recognize the equilibrium dynamic susceptibility (obtained in the limit of $t \to \infty$) and one finds the result (5.8).

Finally the equilibrium dynamic susceptibility,

$$\tilde{\chi}^{eq}(\omega) = \frac{\beta}{2\pi\xi_{eq}(T)} \int_0^{2\pi} dk \frac{1}{1 - i\omega - \gamma\cos k}, \tag{B.5}$$

will be calculated. To this end one substitutes $u = e^{ik}$ in (B.5) and one obtains a contour integral around the unit circle. Inside the contour there is one pole, located at

$$u_\omega = \frac{1}{\gamma} \left\{ 1 - i\omega - \sqrt{(1 - i\omega)^2 - \gamma^2} \right\}, \tag{B.6}$$

which is located near z in the limit $\omega \to 0$. Consequently one finds

$$\tilde{\chi}^{eq}(\omega) = \frac{\beta}{\xi_{eq}(T)} \frac{1}{\sqrt{(1 - i\omega)^2 - \gamma^2}}. \tag{B.7}$$

When one uses $1 - \gamma^2 \simeq \frac{1}{2}\xi_{eq}^{-2}(T)$ and the fact that $1 \ll \omega^{-1} \ll \xi_{eq}^2(T)$ one finds (5.9).

References

[1] Fisher, D. S. and Huse, D. A., Phys. Rev. **B 38** (1988) 373.
[2] Edwards, S. F. and Anderson, P. W., J. Phys. **F 5** (1975) 89.
[3] Glauber, R. J., J. Math. Phys. 4 (1963) 294.

[4] Sompolinski, H. and Zippelius, A., Phys. Rev. **B 25** (1982) 6860.

[5] Ginzburg, S. L., Sov. Phys. JETP **63** (1986) 439, [Zh. Eksp. Teor. Fiz. **63** (1986) 754].

[6] Ocio, M., Bouchiat, H., and Monod, P., J. Phys. (Paris) Lett. **46** (1985) L647.

[7] Alba, M., Bouchiat, H., Hammann, J., Ocio, M., and Refregier, P., J. Appl. Phys. **61** (1987) 61.

[8] Svedlindh, P., Nordblad, P., and Lundgren, L., preprint (1987).

[9] Reim, W., Koch, R. H., Malozemoff, A. P., and Ketchen, M. B., Phys. Rev. Lett. **57** (1986) 905.

[10] Refregier, P., Ocio, M., and Bouchiat, H., Europhys. Lett. **3** (1987) 503.

[11] Refregier, P. and Ocio, M., Revue Phys. Appl. **22** (1987) 367.

[12] Refregier, P., PhD thesis Université de Paris-Sud, Centre d'Orsay (1987).

[13] Binder, K. and Young, A. P., Rev. Mod. Phys. **58** (1986) 801.

[14] Bray, A. J. and Moore, M. A., Phys. Rev. Lett. **58** (1987) 57.

[15] Bray, A. J. and Moore, M. A., in: *Heidelberg Colloquium on Glassy Dynamics*, eds. J. L. van Hemmen and I. Morgenstern (Springer, Heidelberg, 1987).

[16] Fisher, D. S. and Huse, D. A., Phys. Rev. Lett. **56** (1986) 1601.

[17] Mattis, D. C., Phys. Lett. **A 56** (1976) 421.

[18] Koper, G. J. M. and Hilhorst, H. J., J. Physique **49** (1988) 429.

[19] Abramowitz, M. and Stegun, I. A., *Handbook of mathematical functions* (National Bureau of Standards, Washington DC, 1964).

[20] Leitão, U. A. and Kleemann, W., Phys. Rev. **B 35** (1987) 8696.

[21] Leitão, U. A., Kleemann, W., and Ferreira, I. B., Phys. Rev. **B 38** (1988) 4765.

[22] Fischer, K. H., Phys. Stat. Sol. (b) **116** (1983) 357.

Chapter V

Acceleration of Spin Glass Dynamics by Temperature Variations *

In this chapter it is shown for the example of the one-dimensional model system that a spin glass subject to a linearly varying temperature reaches a quasistationary state in which the dynamics is much faster than in a constant temperature state.

---

*Apart from minor modifications, this chapter will appear in Physica **A**.

1 Introduction

In this chapter it is shown that, when a spin glass is subjected to a nonconstant temperature, a modification of its dynamics occurs. In particular it is demonstrated that the one-dimensional model, subject to a linearly varying temperature, quickly reaches a quasistationary state in which the dynamics is much faster than in a constant temperature state and of which the properties depend singularly on the temperature rate of change. This effect is completely due to the random temperature dependence of the correlations in the spin glass state (see [1,2]).

The remainder of this section is devoted to a brief introduction of the one-dimensional model. In Section 2 an expression is derived for the decay of the magnetization of the model subject to a linearly varying temperature and here it is shown that the dynamics is accelerated. Section 3 deals with the magnetic response function which involves the calculation of the spin correlation function. The latter calculation provides the evidence for a quasistationary state.

Finally, in Section 4, the results are discussed within a more general framework.

1.1 The simple spin glass model

Our simple spin glass model consists of a 1-dimensional Ising spin chain with Hamiltonian

$$\mathcal{H}(\{s\}; \{J(T)\}) = - \sum_k J_k(T) s_k s_{k+1} \tag{1.1}$$

in which the bonds are written as

$$J_k(T) = \eta_k(T) \mathcal{J} \tag{1.2}$$

with $\mathcal{J}$ the typical bond strength and $\eta_j(T)$ *randomly temperature dependent* coefficients. We stipulate that the coefficients $\eta_j(T)$ and $\eta_k(T)$ are independent for $j \neq k$ and that furthermore

(i) for $T = 0$ the coefficient $\eta_k(0)$ is either $+1$ or -1 with equal probability,

(ii) for a temperature increment ΔT the coefficient $\eta_k(T)$ changes sign with probability $\Gamma \Delta T / 2$, where Γ is a constant; formally, $\eta_k(T)$ is a Markov process (as a function of T) with a transition probability per temperature increment ΔT given by

$$\mathcal{G}_{\Delta T}(\eta_k' | \eta_k) = \begin{cases} \frac{1}{2} \Gamma \Delta T & \text{if } \eta_k' \neq \eta_k \\ 1 - \frac{1}{2} \Gamma \Delta T & \text{if } \eta_k' = \eta_k \end{cases} \tag{1.3}$$

As a result the spin correlations vary randomly with temperature. In this model frustration effects as are present in more elaborate spin glass models manifest themselves via the temperature dependence of the couplings $J_k(T)$. We have argued in

94

Chapter IV (see also Ref. [3]) how such a model may arise if one renormalizes a d-dimensional bar of size $L^{d-1} \times \infty$.

At a *fixed* temperature T, this model is equivalent to a Mattis spin glass [4] as can be seen after one transforms the spin variables according to

$$\tilde{s}_k(T) \equiv \begin{cases} \eta_0(T)\eta_1(T)\ldots\eta_{k-1}(T)s_k & (k>0) \\ s_0 & (k=0) \\ \eta_k(T)\eta_{k+1}\ldots\eta_{-1}(T)s_k & (k<0). \end{cases} \tag{1.4}$$

This converts the Hamiltonian (1.1) into a Hamiltonian for a ferromagnetic chain of Ising spin variables $\tilde{s}_k(T)$. Its correlation length is

$$\xi_{eq}(T) = -1/\log \tanh \beta \mathcal{J}$$

$$\simeq \tfrac{1}{2}e^{2\beta\mathcal{J}} \qquad \text{as} \quad T \to 0, \tag{1.5}$$

where $\beta = 1/T$ since Boltzmann's constant $k_B \equiv 1$.

The similarity of the correlations of two thermodynamic states at temperatures T and T' is expressed by the *overlap length*, which is twice the characteristic length of the *overlap function*

$$C_n^{eq}(T,T') \equiv \overline{\langle s_j s_{j+n}\rangle_T^{eq}\langle s_j s_{j+n}\rangle_{T'}^{eq}}$$

$$= \tanh^n \beta\mathcal{J} \tanh^n \beta'\mathcal{J} e^{-n\Gamma|T-T'|} \qquad (n>0), \tag{1.6}$$

where the angular brackets denote a thermal average and the overbar denotes an average over disorder, i.e. over the random η_k's. The overlap length approaches $\xi_{eq}(T)$ when T and T' become equal, and vanishes when T and T' are very different. This is exactly the type of behaviour that has been found for more realistic spin glass models by Bray and Moore [1,2].

We choose the dynamics for this model to be of the Glauber type [5]: a master equation, with specific "Glauber" transition rates, is postulated for the probability of finding a particular realization of the spin chain at a particular instant of time. From this master equation one can derive evolution equations for the spin correlation functions, in particular for the time dependent spin average $\langle s_k\rangle_t$ and for the time dependent correlation function $\langle s_k s_{k+n}\rangle_t$. We shall use these evolution equations in the next sections. Glauber dynamics for an Ising chain leads to a longest relaxation time τ_{eq} for the pair correlations given by

$$\tau_{eq} = \tfrac{1}{2}(1-\gamma)^{-1}$$

$$\simeq \tfrac{1}{4}e^{4\beta\mathcal{J}}$$

$$\simeq \xi_{eq}^2 \qquad \text{as} \quad T \to 0, \tag{1.7}$$

95

where $\gamma \equiv \tanh 2\beta\mathcal{J}$.

The model (1.1) obviously does not undergo a spin glass transition at a finite temperature. Our point of view is, however, that the aging effects of interest will appear on spatial scales less than the correlation length ξ_{eq} and on time scales less then the largest relaxation time τ_{eq}. Throughout this work we shall therefore restrict ourselves to temperatures so small that

(i) ξ_{eq} is much larger than any other length in the problem, and

(ii) τ_{eq} is much larger than the largest time τ_{exp} attainable in a laboratory experiment, i.e.

$$(1-\gamma)t \ll 1 \tag{1.8}$$

for all $t \lesssim \tau_{exp}$.

2 Decay of the magnetization

In this section we shall derive an expression for the decay of the magnetization of the system while the temperature T_t varies linearly with time t according to

$$T_t = T_0 + \dot{T}t \qquad\qquad (t \geq 0). \tag{2.1}$$

For simplicity we stipulate that at $t = 0$ the chain is fully magnetized, $\langle s_k \rangle_0 = 1$. Other initial conditions can also be dealt with.

2.1 An equation for the magnetization

For a chain with Hamiltonian (1.1) Glauber dynamics [5] yields the following time evolution equation [3] for the spin averages:

$$\frac{d\langle s_k \rangle_t}{dt} = -\langle s_k \rangle_t + \tfrac{1}{2}\gamma\eta_{k-1}(T_t)\langle s_{k-1} \rangle_t + \tfrac{1}{2}\gamma\eta_k(T_t)\langle s_{k+1} \rangle_t. \tag{2.2}$$

The constant $\dot{T}$ will be taken small enough and the temperature T_t low enough so that we may take γ to be constant during the time interval over which we consider $\langle s_k \rangle_t$.

Eq. (2.2) constitutes a complete set of equations that can be solved in principle but whose solution still contains the randomly temperature dependent and hence randomly time dependent coefficients $\eta_k(T_t)$. Since we are only interested in the disorder averaged quantity $\overline{\langle s_k \rangle_t}$ we shall construct another complete set of equations that does not have this drawback. We introduce

$$\sigma_n(t) \equiv \begin{cases} \overline{\eta_k(T_t)\eta_{k+1}(T_t)\ldots\eta_{k+n-1}(T_t)\langle s_{k+n} \rangle_t} & \text{for} \quad n > 0 \\[2mm] \overline{\langle s_k \rangle_t} & \text{for} \quad n = 0 \\[2mm] \overline{\eta_{k-1}(T_t)\eta_{k-2}(T_t)\ldots\eta_{k+n}(T_t)\langle s_{k+n} \rangle_t} & \text{for} \quad n < 0. \end{cases} \tag{2.3}$$

The disorder average restores the translational invariance so that σ_n does not depend on the spin index k. From (2.2) and (2.3) it immediately follows that

$$\dot{\sigma}_0(t) = -\sigma_0(t) + \tfrac{1}{2}\gamma\sigma_{-1}(t) + \tfrac{1}{2}\gamma\sigma_1(t). \tag{2.4}$$

Then let us consider $\sigma_1(t)$. Using the product rule for differentiation we obtain

$$\dot{\sigma}_1(t) = \overline{\eta_k(T_t)\frac{d\langle s_{k+1}\rangle_t}{dt}} + \overline{\frac{d\eta_k(T_t)}{dt}\langle s_{k+1}\rangle_t}. \tag{2.5}$$

For the first term we can invoke (2.2) to eliminate the time derivative and for the second term we write

$$\overline{\frac{d\eta_k(T_t)}{dt}\langle s_{k+1}\rangle_t} = \dot{T}\lim_{\Delta T \downarrow 0}\overline{\frac{\eta_k(T_t + \Delta T) - \eta_k(T_t)}{\Delta T}\langle s_{k+1}\rangle_t}. \tag{2.6}$$

Here and in the following we take the constant $\dot{T}$ to be positive. For negative $\dot{T}$ the absolute value should be taken at the appropriate places.

For any quantity $A(\eta)$ that depends on the $\eta_k(T)$ for T in the interval $[T_0, T_t]$ we have

$$\overline{A(\eta)} = \prod_\ell \int \mathcal{D}[\eta_\ell(T)]\mathcal{P}([\eta_\ell(T)], T_0, T_t)A(\eta), \tag{2.7}$$

where $\mathcal{P}([\eta_\ell(T)], T_0, T_t)$ is the probability of having the ℓth bond realize the function $\eta_\ell(T)$ in the interval $[T_0, T_t]$. We shall discretize the T-axis in small intervals of size ΔT, small enough so that the probability for more than one change of sign of $\eta_k(T)$ per interval is negligible. Because of the Markov property of the $\eta_k(T)$ we have to leading order in ΔT

$$\mathcal{P}([\eta_k(T)], T_0, T_t + \Delta T) = \\ \mathcal{G}_{\Delta T}(\eta_k(T_t + \Delta T)|\eta_k(T_t))\mathcal{P}([\eta_k(T)], T_0, T_t). \tag{2.8}$$

Using (2.7) and (2.8) in the right hand side of (2.6) yields

$$\begin{aligned}
&\lim_{\Delta T \downarrow 0}\overline{\frac{\eta_k(T_t + \Delta T) - \eta_k(T_t)}{\Delta T}\langle s_{k+1}\rangle_t} \\
&= \lim_{\Delta T \downarrow 0}\sum_{\eta'_k = \pm 1}\prod_\ell \int \mathcal{D}[\eta_\ell(T)] \\
&\quad \times \mathcal{G}_{\Delta T}(\eta'_k|\eta_k(T_t))\mathcal{P}([\eta_\ell(T)], T_0, T_t)\frac{\eta'_k - \eta_k(T_t)}{\Delta T}\langle s_{k+1}\rangle_t \\
&= -\Gamma\prod_\ell \int \mathcal{D}[\eta_\ell(T)]\mathcal{P}([\eta_\ell(T)], T_0, T_t)\eta_k(T_t)\langle s_{k+1}\rangle_t \\
&= -\Gamma\sigma_1(t), \tag{2.9}
\end{aligned}$$

where in the second last step we used (1.3). By combining (2.5), (2.2), (2.6), and (2.9) we find that

$$\dot{\sigma}_1(t) = -\sigma_1(t) + \tfrac{1}{2}\gamma\sigma_0(t) + \tfrac{1}{2}\gamma\sigma_2(t) - \Gamma\dot{T}\sigma_1(t). \tag{2.10}$$

In the same way one can derive

$$\dot{\sigma}_n(t) = -\sigma_n(t) + \tfrac{1}{2}\gamma\sigma_{n-1}(t) + \tfrac{1}{2}\gamma\sigma_{n+1}(t) - |n|\Gamma\dot{T}\sigma_n(t) \quad (n = 0, \pm 1, \ldots) \tag{2.11}$$

which constitutes the announced set of nonrandom equations. The initial condition becomes $\sigma_n(0) = \delta_{n,0}$. From the solution we only need $\sigma_0(t)$ which by the definition (2.3) is equal to the magnetization $\overline{\langle s_k \rangle_t}$.

2.2 Solution of the equation for the magnetization

We shall solve the above set of equations (2.11) in the continuum limit $\sigma_n(t) \to \sigma(x,t)$, where $\sigma_n(t)$ can be regarded as corresponding to the density $\sigma(x,t)$ integrated over the interval $(n - \tfrac{1}{2}, n + \tfrac{1}{2})$. In terms of the density $\sigma(x,t)$ Eq. (2.11) becomes

$$\{\partial_t - \mathcal{L}\}\,\sigma(x,t) = 0 \tag{2.12}$$

with

$$\mathcal{L} = \tfrac{1}{2}\gamma\partial_{xx} - 1 + \gamma - \Gamma\dot{T}|x|. \tag{2.13}$$

The initial condition becomes $\sigma(x,0) = \delta(x)$, and $\sigma(x,t)$ should vanish for $|x| \to \infty$. Because the initial condition is symmetric we can restrict ourselves to the subset of symmetric solutions. The operator $\mathcal{L}$ is hermitian and the eigenfunctions $\phi_m(x)$ satisfy

$$\mathcal{L}\phi_m(x) = -\lambda_m \phi_m(x) \tag{2.14}$$

for eigenvalues λ_m $(m = 1, 2, \ldots)$. Eq. (2.12) with Eq. (2.13) is also the Schrödinger equation for a particle in a linear potential well. Its symmetric eigenfunctions are the Airy functions [6]

$$\phi_m(x) = \sqrt{\alpha_m}\left(\frac{2\Gamma\dot{T}}{\gamma}\right)^{\frac{1}{6}} \mathrm{Ai}\left(\left(\frac{2\Gamma\dot{T}}{\gamma}\right)^{\frac{1}{3}}|x| - a'_m\right) \quad (m = 1, 2, \ldots) \tag{2.15}$$

with the normalization constant

$$\alpha_m^{-1} = 2\int_0^\infty dy \;\mathrm{Ai}^2\left(y - a'_m\right) \quad (m = 1, 2, \ldots) \tag{2.16}$$

where a'_m is positive and the mth root of $\mathrm{Ai}'(-a'_m) = 0$. The eigenvalues are given by

$$\lambda_m = 1 - \gamma + \tfrac{1}{2}\gamma a'_m \left(\frac{2\Gamma\dot{T}}{\gamma}\right)^{\frac{2}{3}} \quad (m = 1, 2, \ldots). \tag{2.17}$$

98

With (2.15) and (2.17) we find for the Green function G of problem (2.12)

$$G(x, x'; t) \equiv \sum_{m=1}^{\infty} e^{-\lambda_m t} \phi_m(x) \phi_m^*(x')$$

$$= \sum_{m=1}^{\infty} \alpha_m \left(\frac{2\Gamma \dot{T}}{\gamma} \right)^{\frac{1}{3}} \mathrm{Ai} \left(\left(\frac{2\Gamma \dot{T}}{\gamma} \right)^{\frac{1}{3}} |x| - a_m' \right) \mathrm{Ai} \left(\left(\frac{2\Gamma \dot{T}}{\gamma} \right)^{\frac{1}{3}} |x'| - a_m' \right)$$

$$\times \exp \left[- \left\{ 1 - \gamma + \tfrac{1}{2} \gamma a_m' \left(\frac{2\Gamma \dot{T}}{\gamma} \right)^{\frac{2}{3}} \right\} t \right]. \tag{2.18}$$

For not too short times ($t \gg 1$) the density $\sigma(x, t)$ does not vary much over the interval $(-\frac{1}{2}, \frac{1}{2})$ so that as a good approximation, and because of the initial condition $\sigma(x, 0) = \delta(x)$,

$$\overline{(s_k)_t} \approx \sigma(0, t) = G(0, 0; t)$$

$$= \frac{e^{-(1-\gamma)t}}{\sqrt{2\pi \gamma t}} \mathcal{F} \left(\left(\frac{\Gamma \dot{T}}{\gamma} \right)^{\frac{2}{3}} t \right), \tag{2.19}$$

with the scaling function $\mathcal{F}$ given by

$$\mathcal{F}(u) = 2^{\frac{5}{6}} \sqrt{\pi \gamma u} \sum_{m=1}^{\infty} \alpha_m \, \mathrm{Ai}^2(-a_m') \exp \left\{ -2^{-\frac{1}{3}} \gamma a_m' u \right\}. \tag{2.20}$$

This function is such that

$$\mathcal{F}(0) = 1, \tag{2.21a}$$

$$\mathcal{F}(u) \simeq 2^{\frac{5}{6}} \sqrt{\pi \gamma u} \alpha_1 \, \mathrm{Ai}^2(-a_1') \exp \left\{ -2^{-\frac{1}{3}} \gamma a_1' u \right\} \qquad \text{as} \quad u \to \infty. \tag{2.21b}$$

The property (2.21a) is proved in the appendix.

2.3 Discussion of the result

We recall that the times t that are of interest satisfy

$$t \lesssim \tau_{exp} \ll \tau_{eq} = \tfrac{1}{2}(1 - \gamma)^{-1}. \tag{2.22}$$

Therefore Eq. (2.19) shows that at a fixed constant temperature, i.e. for $\dot{T} = 0$, power law decay of the magnetization is observed. This decay and its consequences were discussed in the previous chapter. A basic result of the present study, brought out by Eq. (2.19), is that for nonconstant temperatures a new time scale

$$\tau_{qs}(\dot{T}) = \left(\frac{\Gamma \dot{T}}{\gamma} \right)^{-\frac{2}{3}} \tag{2.23}$$

comes into play. For $\tau_{qs} \lesssim \tau_{exp}$, this time scale becomes experimentally accessible. Eq. (2.19) then describes the crossover from the power law relaxation for times $t \lesssim \tau_{qs}$ to exponential relaxation, with a time constant τ_{qs}, for larger times. For $\tau_{exp} \lesssim \tau_{qs}$ this crossover no longer occurs on experimentally accessible time scales.

3 The time dependent magnetic response function

In this section we derive an expression for the time dependent magnetic response function for the situation where the temperature varies linearly according to (2.1). We shall do so by calculating the magnetization due to an infinitesimally small magnetic field $H(t)$.

3.1 An expression for the response function

When a small magnetic field $H(t)$ is applied, the corresponding Zeeman energy adds to the Hamiltonian (1.1) of the system so that the evolution equation (2.2) is modified by terms proportional to the magnetic field (see [3]),

$$\frac{d\langle s_k \rangle_t}{dt} = -\langle s_k \rangle_t + \tfrac{1}{2}\gamma\eta_{k-1}(T_t)\langle s_{k-1}\rangle_t + \tfrac{1}{2}\gamma\eta_k(T_t)\langle s_{k+1}\rangle_t$$
$$+ \beta H(t)\left\{1 - \tfrac{1}{2}\gamma\eta_{k-1}(T_t)\langle s_{k-1}s_k\rangle_t - \tfrac{1}{2}\gamma\eta_k(T_t)\langle s_k s_{k+1}\rangle_t\right\}. \quad (3.1)$$

We shall be interested in $\overline{\langle s_k \rangle_t}$ to linear order in the magnetic field and therefore it suffices to consider the spin correlation function to zeroeth order in the magnetic field. For $j \neq k$ Glauber dynamics [5] leads to the set of equations [3]

$$\frac{d\langle s_j s_k \rangle_t}{dt} = -2\langle s_j s_k \rangle_t + \tfrac{1}{2}\gamma\eta_{j-1}(T_t)\langle s_{j-1}s_k\rangle_t + \tfrac{1}{2}\gamma\eta_j(T_t)\langle s_{j+1}s_k\rangle_t$$
$$+ \tfrac{1}{2}\gamma\eta_{k-1}(T_t)\langle s_j s_{k-1}\rangle_t + \tfrac{1}{2}\gamma\eta_k(T_t)\langle s_j s_{k+1}\rangle_t. \quad (3.2)$$

We shall proceed as in Section 2 where we defined $\sigma_n(t)$ by Eq. (2.3). Furthermore we define

$$c_n(t) \equiv \overline{\eta_k(T_t)\eta_{k+1}(T_t)\ldots\eta_{k+n-1}(T_t)\langle s_k s_{k+n}\rangle_t} \qquad (n > 0) \quad (3.3)$$

which is the disorder average of the Mattis transformed (cf. Eq. (1.4)) spin correlation function at time t (temperature T_t). From (3.1), (2.11), and (3.3) it immediately follows that

$$\dot{\sigma}_n(t) = -\sigma_n(t) + \tfrac{1}{2}\gamma\sigma_{n-1}(t) + \tfrac{1}{2}\gamma\sigma_{n+1}(t) - |n|\Gamma\dot{T}\sigma_n(t) + h_n(t) \quad (3.4)$$

where the "local magnetic field" is given by

$$h_n(t) = \beta H(t)\overline{\eta_k(T_t)\eta_{k+1}(T_t)\ldots\eta_{k+n-1}(T_t)}$$
$$\times \left\{1 - \tfrac{1}{2}\gamma\eta_{k-1}(T_t)\langle s_{k-1}s_k\rangle_t - \tfrac{1}{2}\gamma\eta_k(T_t)\langle s_k s_{k+1}\rangle_t\right\} \quad (n > 0). \quad (3.5)$$

When one uses the Mattis transformation (1.4) to transform to the spin variables $\tilde{s}_k(T_t)$ it becomes obvious that

$$h_n(t) = \delta_{n,0}\beta H(t)\{1 - \gamma c_1(t)\}. \tag{3.6}$$

Then we consider the spin correlation function. One has $c_0(t) \equiv 1$ and by the same procedure as outlined in Section 2 one finds from (3.2) and (3.3) that

$$\dot{c}_n(t) = -2c_n(t) + \gamma c_{n-1}(t) + \gamma c_{n+1}(t) - n\Gamma\dot{T}c_n(t) \qquad (n > 0). \tag{3.7}$$

As in Section 2.2 we study Eqs. (3.4), (3.6), and (3.7) in the continuum limit, $\sigma_n(t) \to \sigma(x,t)$, $h_n(t) \to h(x,t)$, and $c_n(t) \to c(x,t)$, and we shall start with the density $\sigma(x,t)$. Due to the local magnetic field $h(x,t)$ we have instead of the homogeneous problem (2.12) the inhomogeneous problem

$$\{\partial_t - \mathcal{L}\}\sigma(x,t) = h(x,t). \tag{3.8}$$

Its solution, in terms of the Green function defined by (2.18), is:

$$\sigma(x,t) = \int dx' \, G(x,x';t)\sigma(x',0) + \int_0^t dt' \int dx' \, G(x,x';t')h(x',t-t'). \tag{3.9}$$

Since for not too short times the density $\overline{\langle s_k \rangle_t}$ can be approximated by $\sigma(0,t)$ and since $h(x,t)$ has a delta peak at $x = 0$, we can write Eq. (3.9) as

$$\overline{\langle s_k \rangle_t} = \int_0^t dt' \, \chi(t,t-t')H(t-t') \tag{3.10}$$

with the dynamic susceptibility

$$\chi(t,t-t') = \beta G(0,0;t')\{1 - \gamma c(1,t-t')\}. \tag{3.11}$$

We took, for simplicity, the initial density $\sigma(x,0) = 0$.

3.2 The spin correlation function $c(1,t)$

According to Eq. (3.11) the response function $\chi(t,t-t')$ depends not only on the time difference t' between stimulus and response, but also on the absolute time $t-t'$ (measured with respect to the instant that the system was quenched to low temperatures). The $(t-t')$-dependence comes in via the correlation function $c(1,t-t')$, which shows that aging in the spin glass chain is a result of the nonstationarity of the spin correlations. In this subsection we shall see that a linearly varying temperature introduces a very particular situation where the spin correlations acquire stationary values. We shall call this quasistationarity.

Let us first formulate the problem for the spin correlation function:

$$\begin{cases} \partial_t c(x,t) = \gamma\partial_{xx}c(x,t) - 2(1-\gamma)c(x,t) - x\Gamma\dot{T}c(x,t) & (x > 0) \\[2mm] c(0,t) = 1, \quad c(x,t) \to 0 \text{ as } x \to \infty \end{cases} \tag{3.12}$$

The problem is very similar to (2.12) and the eigenfunctions are likewise Airy functions. The general time-dependent solution takes the form

$$c(x,t) = c^{qs}(x) + e^{-2(1-\gamma)t}$$
$$\times \sum_{m=1}^{\infty} g_m \, \mathrm{Ai} \left(\left(\frac{\Gamma \dot{T}}{\gamma} \right)^{\frac{1}{3}} x - a_m \right) \exp \left(-\gamma a_m \left(\frac{\Gamma \dot{T}}{\gamma} \right)^{\frac{2}{3}} t \right) \tag{3.13}$$

in which the coefficients g_m are dependent on the initial condition, and where a_m is positive and the mth root of $\mathrm{Ai}(-a_m) = 0$. The quasistationary solution c^{qs} is given explicitly by

$$c^{qs}(x) = \mathrm{Ai} \left(\left(\frac{\Gamma \dot{T}}{\gamma} \right)^{\frac{1}{3}} x + x_0 \right) / \mathrm{Ai}(x_0) \tag{3.14}$$

where x_0 is the constant

$$x_0 = \left(\frac{\gamma}{\Gamma \dot{T}} \right)^{\frac{2}{3}} \frac{2(1-\gamma)}{\gamma}. \tag{3.15}$$

We call this solution quasistationary because it is formally a time-independent solution of (3.12) but, as noted above, for a nonstationary situation.

The time-dependent part of the correlations decays, just as the magnetization, on a time scale τ_{qs} (see Eq. (2.23)) which for sufficiently high temperature rates of change is much smaller than the experimental time scale τ_{exp}. In that case $c(x,t)$ decays rapidly towards its stationary value and aging is absent. We shall therefore first consider $c^{qs}(x)$.

We define a quasistationary correlation length

$$\xi_{qs}(\dot{T}) \equiv \left(\frac{\Gamma \dot{T}}{\gamma} \right)^{-\frac{1}{3}} \tag{3.16}$$

(note that $\tau_{qs} = \xi_{qs}^2$) and express the quasistationary correlation function (3.14) in terms of this length and ξ_{eq}:

$$c^{qs}(x) \simeq \mathrm{Ai} \left(\frac{x}{\xi_{qs}} + \frac{\xi_{qs}^2}{\xi_{eq}^2} \right) / \mathrm{Ai} \left(\frac{\xi_{qs}^2}{\xi_{eq}^2} \right). \tag{3.17}$$

We first analyze the limit $\xi_{qs} \to \infty$ which occurs when $\dot{T} \to 0$. From the behaviour of the Airy function for large argument [6] it then follows that

$$c^{qs}(x) \simeq e^{-x/\xi_{eq}} \tag{3.18}$$

as it should (see [3]). For the case where $\dot{T}$ is sufficiently large so that $\xi_{qs} \ll \xi_{eq}$ we find for $x \gg \xi_{qs}$

$$c^{qs}(x) \simeq 3^{\frac{1}{6}} \Gamma(\tfrac{1}{3}) \sqrt{\pi} \left(\frac{x}{\xi_{qs}} \right)^{-\frac{1}{4}} \exp \left\{ -\frac{2}{3} \left(\frac{x}{\xi_{qs}} \right)^{\frac{3}{2}} \right\}. \tag{3.19}$$

The latter decay is nonexponential with characteristic length $\xi_{qs}(\dot{T})$.

In Eq. (3.11) only the nearest neighbour correlation $c(1,t)$ occurs. Since we shall assume that always $\xi_{qs}(\dot{T}) \gg 1$, we may expand Eq. (3.17) in this quantity:

$$c^{qs}(1) \simeq 1 - \frac{1}{\xi_{qs}} f\left(\frac{\xi_{qs}}{\xi_{eq}}\right) \tag{3.20}$$

with the scaling function f given by

$$f(u) = -\operatorname{Ai}'(u^2)/\operatorname{Ai}(u^2). \tag{3.21}$$

This function describes the crossover of $c^{qs}(1)$ from the quasistationary to the equilibrium regime that occurs when $\dot{T}$ tends to zero. It has the properties (see [6])

$$f(0) = 3^{\frac{1}{3}}\Gamma\left(\tfrac{2}{3}\right)/\Gamma\left(\tfrac{1}{3}\right) = 1.05\ldots \tag{3.22a}$$

$$f(u) \simeq u \quad \text{as} \quad u \to \infty. \tag{3.22b}$$

In the limit $\dot{T} \to 0$ the aging effects will reappear. If we wish to describe the crossover to this limit, we also have to consider the deviation of the correlation function from its quasistationary value. From (3.13) and (3.20) we have to first order in ξ_{qs}^{-1} with t/τ_{qs} and ξ_{qs}/ξ_{eq} fixed:

$$c(1,t) \simeq 1 - \frac{1}{\xi_{qs}} f\left(\frac{\xi_{qs}}{\xi_{eq}}\right) + \frac{e^{-t/\tau_{eq}}}{2\sqrt{\pi\gamma t}} g\left(\frac{t}{\tau_{qs}}\right) \tag{3.23}$$

Here the scaling function g is given by

$$g(u) = 2\sqrt{\pi\gamma u} \sum_{m=1}^{\infty} g_m \operatorname{Ai}'(-a_m) e^{-\gamma a_m u} \tag{3.24}$$

with history dependent coefficients $g_1, g_2, \ldots$. From (3.24) we see that for fixed $\dot{T}$ one can make a large time expansion and that the first correction term to quasistationarity follows from

$$g(u) = 2\bar{g}_1 \sqrt{\pi\gamma u}\, e^{-\gamma a_1 u} \left\{ 1 + \mathcal{O}\left(e^{-\gamma(a_2 - a_1)u}\right) \right\} \quad \text{for} \quad u \to \infty \tag{3.25}$$

which contains only a single undetermined coefficient $\bar{g}_1 = g_1 \operatorname{Ai}'(-a_1)$. The higher order terms in the expansion are negligible for $u \gg 1$, i.e. for $t \gg \tau_{qs}$. Hence, taking into account only the first correction term, we have from (3.23) and (3.25)

$$c(1,t) \simeq 1 - \frac{1}{\xi_{qs}} \left\{ f(0) + \bar{g}_1 e^{-\gamma a_1 t/\tau_{qs}} \right\} \tag{3.26}$$

where we used $\tau_{qs} \ll \tau_{eq}$, $\xi_{qs} \ll \xi_{eq}$ and $\tau_{qs} = \xi_{qs}^2$. This describes, within this model, the crossover from quasistationarity to aging behaviour.

Then let us finally turn to the magnetic response function. The time scales are ordered as $1 \ll t' \ll t \lesssim \tau_{exp} \ll \tau_{eq}$ and from (3.11) with (2.19) and (3.23) we obtain

$$\chi(t, t - t') \simeq \frac{\beta}{\sqrt{2\pi\gamma t'}} \mathcal{F}\left(\frac{t'}{\tau_{qs}}\right) \left[\frac{1}{\xi_{qs}} f\left(\frac{\xi_{qs}}{\xi_{eq}}\right) + \frac{1}{2\sqrt{\pi\gamma t}} g\left(\frac{t}{\tau_{qs}}\right)\right]. \tag{3.27}$$

There are, according to the value of τ_{qs}, three regimes to consider. First $\tau_{qs} \gtrsim \tau_{eq}$, and since $t \ll \tau_{eq}$ also $t \ll \tau_{qs}$, resulting from an extremely small temperature rate of change. The behaviour of $\chi(t, t - t')$ is then dominated by aging effects.

For larger $\dot{T}$, when $\tau_{qs} \ll \tau_{exp}$, terms dependent on the initial conditions (aging effects) decay on the time scale τ_{qs} (cf. Eq. (3.26)). One then has

$$\chi(t, t - t') \simeq \begin{cases} \dfrac{\beta f(0)}{\xi_{qs}\sqrt{2\pi\gamma t'}} & \text{for } t' \ll \tau_{qs} \ll t \\[3mm] \dfrac{2^{\frac{1}{3}}\beta f(0)\alpha_1 \, \mathrm{Ai}^2(-a'_1)}{\tau_{qs}} e^{-2^{\frac{1}{3}}\gamma a'_1 t'/\tau_{qs}} & \text{for } \tau_{qs} \ll t' \ll t. \end{cases} \tag{3.28}$$

For the dynamic susceptibility $\bar{\chi}(\omega; t)$, which is the Fourier transform of the magnetic response function $\chi(t, t - t')$ with respect to t', one can derive similar results. Considering frequencies ω such that $1 \ll \omega^{-1} \ll t \lesssim \tau_{exp}$ we find, from Eq. (3.27), that the dynamic susceptibility decays rapidly with time t (on a time scale τ_{qs}) to the quasistationary value

$$\bar{\chi}(\omega) \simeq \begin{cases} \dfrac{2^{\frac{2}{3}}\beta f(0)\alpha_1 \, \mathrm{Ai}^2(-a'_1)}{\gamma a'_1} \left(1 - 2^{-\frac{1}{3}}\mathrm{i}\gamma a'_1 \omega \tau_{qs}\right) & \text{for } \omega\tau_{qs} \ll 1 \\[3mm] \dfrac{\beta f(0)(1 + \mathrm{i})}{2\sqrt{\omega\tau_{qs}}} = f(0)\dfrac{\xi_{eq}}{\xi_{qs}} \bar{\chi}^{eq}(\omega) & \text{for } \omega\tau_{qs} \gg 1, \end{cases} \tag{3.29}$$

where $\bar{\chi}^{eq}$ is the equilibrium dynamic susceptibility for this model, see [3]. Eq. (3.29) shows that for not too low frequencies, $\omega\tau_{qs} \gg 1$, the dynamic susceptibility is increased by a factor $\xi_{eq}(T)/\xi_{qs}(\dot{T})$ with respect to its value in equilibrium.

3.3 Qualitative understanding of the results

The above described behaviour can be understood from a simple physical picture of the spin glass chain. At every instant of time t there is a fraction $\rho(t)$ of bonds that are not satisfied by their neighbouring spins. The temperature T is very low so that it is very unlikely that a spin inside a correlated region flips. During a small time increment Δt the temperature changes by an amount $\Delta T = \dot{T}\Delta t$. A fraction $\Gamma\dot{T}\Delta t/2$ of the bonds changes sign and this changes the fraction of frustrated bonds by $(1 - 2\rho)\Gamma\dot{T}\Delta t/2$. Each frustrated bonds performs a random walk until it meets another one: then the two annihilate one another. Their average spacing grows as $\sqrt{t}$ (see Ref. [3]) so the fraction of random walkers decreases in a time lapse Δt by $\rho^3(t)\Delta t$ due to the annihilations. Hence for small $\rho(t)$ it is clear that $\rho(t)$ remains stationary when the two contributions cancel. This leads to $\rho^{qs} \sim (\Gamma\dot{T})^{\frac{1}{3}}$ which corresponds to the result (3.16) because $\rho^{qs} \sim \xi_{qs}^{-1}$. The dependence of the dynamic susceptibility on the fraction of frustrated bonds is understood (see Ref. [3]) from the fact that only the boundary spins of the frustrated bonds respond

to an infinitesimally small field. This is because their local field is zero whereas the local field for spins neighbouring satisfied bonds is $2J$ Finally, the maximum relaxation time of the system is reduced from the value $\tau_{eq} = \xi_{eq}^2(T) = 1/2(1 - \gamma)$ for a system at a fixed temperature T to the value $\tau_{qs} = \xi_{qs}^2 \sim \dot{T}^{-\frac{2}{3}}$ due to the smaller (quasistationary) correlation length.

4 Discussion

We have shown that, for a simple spin glass model, the dynamics is drastically modified when the system is subjected to a linearly varying temperature. The system then quickly reaches a quasistationary state of which the properties are dictated by the magnitude of the temperature rate of change $\dot{T}$. With the quasistationary state a length $\xi_{qs}(\dot{T})$ is associated that diverges as $\dot{T}^{-\frac{1}{3}}$ for $\dot{T} \to 0$. In this state the dynamic susceptibility is increased by a factor $\xi_{eq}(T)/\xi_{qs}(\dot{T})$ with respect to its value in equilibrium. A similar enhancement of the dynamic susceptibility should be observable if not the temperature but a background magnetic field is varied sufficiently slowly. This effect is currently being investigated [7].

Within the framework of a domain theory for aging in spin glasses [8,9] one can arrive at qualitatively the same conclusions. The basic assumption of the theory is that the spin glass equilibrates through the growth of domains. When the temperature is continually changed at no matter how small a rate the growing domain structure is continually disrupted. The phenomenon of interest here is that a situation arises where the growing and the breaking of the domains balance each other such that the characteristic domain size is stationary. In this situation there is no longer a growing length scale in the system and hence there is no aging. The characteristic domain size remains finite (and dependent on the temperature rate of change) so that there is also a finite and time-independent maximum relaxation time. This maximum relaxation time depends, through the characteristic domain size, on the temperature rate of change and can presumably be made small compared to experimental time scales.

It is certainly interesting to know whether the predicted behaviour occurs in nature. In the experiment one would have to look for the value of the temperature rate of change that is required to provoke the effect. The total amount of variation of the temperature over the measuring period, namely $\dot{T}t$, should remain so small that there is no appreciable change in the rate of the thermally activated processes that play a role. Experiments indicate that such is the case, for typical spin glasses like CuMn [10] and CdCr$_{1.7}$In$_{0.3}$S$_4$ [11], when $\dot{T}t \lesssim 0.2$K.

Appendix

In this appendix we prove Eq. (2.21a). From (2.19) one has

$$\mathcal{F}(u) = 2^{\frac{1}{6}} \sqrt{\pi \gamma u} \sum_{m=1}^{\infty} \alpha_m \, \text{Ai}^2(-a'_m) \exp\{2^{-\frac{1}{3}} \gamma a'_m u\} \qquad (A.1)$$

For small u the sum in (A.1) will be dominated by the larger terms (m large). These are the contributions that will be calculated.

The normalization constant α_m, given by Eq. (2.16), is written as

$$\alpha_m^{-1} = 2 \int_{-a_1'}^{\infty} dx \; \mathrm{Ai}^2(x) + 2 \sum_{k=1}^{m-1} \int_{a_k'}^{a_{k+1}'} dx \; \mathrm{Ai}^2(-x). \tag{A.2}$$

With the asymptotic expansion [6]

$$\mathrm{Ai}(-x) \simeq \pi^{-\frac{1}{2}} x^{-\frac{1}{4}} \sin\left(\tfrac{2}{3}x^{\frac{3}{2}} + \tfrac{\pi}{4}\right) \tag{A.3}$$

one has for large values of k

$$\int_{a_k'}^{a_{k+1}'} dx \; \mathrm{Ai}^2(-x) \simeq \frac{1}{\pi} \int_{a_k'}^{a_{k+1}'} dx \; x^{-\frac{1}{2}} \sin^2\left(\tfrac{2}{3}x^{\frac{3}{2}} + \tfrac{\pi}{4}\right). \tag{A.4}$$

Also one as for larger k values [6]

$$a_k' \simeq \left(\frac{3\pi(4k-3)}{8}\right)^{\frac{2}{3}} \tag{A.5}$$

with which (A.4) can be written as

$$\int_{a_k'}^{a_{k+1}'} dx \; \mathrm{Ai}^2(-x) \simeq \frac{1}{\pi}\left(\frac{3}{2}\right)^{-\frac{2}{3}} \int_{0}^{\pi} d\theta \; \left(\tfrac{2}{3}(a_k')^{\frac{3}{2}} + \theta\right)^{-\frac{2}{3}} \sin^2\left(k\pi + \theta - \tfrac{\pi}{2}\right)$$

$$\simeq \frac{1}{\pi a_k'} \int_{0}^{\pi} d\theta \; \cos^2\theta = \frac{1}{2a_k'}. \tag{A.6}$$

With (A.6) and (A.5) in (A.2) one finds for large values of m

$$\alpha_m^{-1} \simeq \sum_{k=1}^{m-1} \left(\frac{3\pi(4k-3)}{8}\right)^{-\frac{2}{3}}$$

$$\simeq \int_{0}^{m} dk \; \left(\frac{3\pi(4k-3)}{8}\right)^{-\frac{2}{3}}$$

$$\simeq \frac{2\sqrt{a_m'}}{\pi}. \tag{A.7}$$

From Ref. [6] one has

$$\mathrm{Ai}(-a_m') \simeq (-1)^{n-1}\pi^{-\frac{1}{2}}(a_m')^{-\frac{1}{4}}. \tag{A.8}$$

The property (2.21a) is proved by substituting Eqs. (A.7) and (A.8) in (A.1) and by replacing the sum by an integral:

$$
\begin{aligned}
\mathcal{F}(u) &\simeq 2^{-\frac{1}{6}}\sqrt{\pi\gamma u}\sum_{m=1}^{\infty}\frac{1}{a'_m}\exp\{-2^{-\frac{1}{3}}\gamma a'_m u\} \\
&\simeq 2^{-\frac{1}{6}}\sqrt{\frac{\gamma u}{\pi}}\int_{a'_1}^{\infty}da\ a^{-\frac{1}{2}}\exp\{-2^{-\frac{1}{3}}\gamma a u\} \\
&\rightarrow 1 \quad \text{as} \quad u\rightarrow 0.
\end{aligned}
\tag{A.9}
$$

References

[1] Bray, A. J. and Moore, M. A., Phys. Rev. Lett. **58** (1987) 57.

[2] Bray, A. J. and Moore, M. A., in: *Heidelberg Colloquium on Glassy Dynamics*, eds. J. L. van Hemmen and I. Morgenstern (Springer, Heidelberg, 1987).

[3] Koper, G. J. M. and Hilhorst, H. J., Physica **A 155** (1989) 431.

[4] Mattis, D. C., Phys. Lett. **A 56** (1976) 421.

[5] Glauber, R. J., J. Math. Phys. **4** (1963) 294.

[6] Abramowitz, M. and Stegun, I. A., *Handbook of mathematical functions* (National Bureau of Standards, Washington DC, 1964).

[7] Nifle, M. and Hilhorst, H. J., private communication.

[8] Koper, G. J. M. and Hilhorst, H. J., J. Physique **49** (1988) 429.

[9] Fisher, D. S. and Huse, D. A., Phys. Rev. **B 38** (1988) 373.

[10] Nordblad, P., Svedlindh, P., Sandlund, L., and Lundgren, L., Phys. Lett. **A 120** (1987) 475.

[11] Refregier, P., Vincent, E., Hammann, J., and Ocio, M., J. Physique **48** (1987) 1533.

Samenvatting

Dit proefschrift handelt over de dynamische eigenschappen van een bijzondere klasse van magnetische systemen, die bekend staat onder de naam *spinglazen*. Het betreft hier systemen, waarvan de wisselwerkingen tussen de magnetische momenten (spins) op ongeordende wijze met elkaar in competitie zijn. Het belangrijkste kenmerk van spinglazen is dat beneden een redelijk scherp bepaalde overgangstemperatuur T_f de spins "invriezen" in *willekeurige* richtingen op een zodanige manier dat er geen globale ferro- of antiferromagnetische ordening ontstaat. Ook de tijdschaal, waarop de relaxatieprocessen in spinglazen zich afspelen, neemt extreem snel toe met afnemende temperatuur. De dynamica van deze systemen vertoont dus sterke overeenkomsten met die van gewone glazen (zoals het bekende vensterglas); vandaar de naam spinglas.

Een gangbare manier om meer begrip te krijgen van een onbekend systeem is door gebruik te maken van een gemiddelde-veld benadering. Voor een ferromagnetisch systeem levert dit een eenvoudige theorie op die kwalitatief het gedrag van het systeem goed beschrijft. Voor spinglazen is de theorie, zelfs in gemiddelde-veld benadering, uitermate ingewikkeld en alhoewel men er al rond 1975 mee is begonnen heeft het tot 1983 geduurd voordat men met name de lage-temperatuur fase goed beschrijven kon. Het beeld dat men nu heeft van een spinglas vertoont enige gelijkenis met dat van een ferromagneet. De gemiddelde-veld benadering voor een spinglas geeft, net als bij de ferromagneet, een welgedefinieerde kritische temperatuur T_c waaronder het systeem geen twee, zoals bij de Ising ferromagneet, maar oneindig veel fasen kent. De ordeparameter is niet, zoals bij een ferromagneet, de magnetisatie maar een ingewikkelder functie van de gemiddelden van alle spins. Alle fasen van een spinglas hebben dezelfde thermodynamische eigenschappen: zij verschillen slechts met een niet-extensief bedrag in vrije energie. De niet-extensieve bijdragen aan de vrije energie kunnen worden beschouwd als onderling onafhankelijke stochastische variabelen getrokken uit een exponentiële verdeling met een breedte die afhangt van de temperatuur.

Omgekeerd kan het lage-temperatuur spinglas beschreven worden met een eenvoudig model dat bestaat uit een verzameling energienivo's, waarvan de (vrije) energiewaarden aan de hierboven gegeven specificaties voldoen. Vervolgens kan het model worden uitgebreid door, met behulp van een "master"-vergelijking, voor te schrijven hoe het systeem van het ene energienivo naar het andere kan springen. Het idee is dan dat daarmee de extreem langzaam verlopende dynamica van een spinglas verklaard kan worden.

In dit proefschrift wordt het asymptotisch gedrag, voor lange tijden, berekend van een autocorrelatiefunctie in het hierboven beschreven "kinetic random energy model". De studie is beperkt tot een aantal overgangswaarschijnlijkheden waarvoor analytische resultaten konden worden verkregen. In alle gevallen tonen de berekeningen aan dat de autocorrelatiefunctie asymptotisch vervalt als een negatieve macht van de tijd t (zoals $t^{-\alpha}$) of als een gerekt exponentiële functie (zoals $\exp(-t^\beta)$ of $t^{-\gamma}\exp(-t^\beta)$ met $0 \leq \beta < 1$). Alle exponenten (α, β en γ) zijn temperatuurafhankelijk. Deze resultaten stemmen kwalitatief overeen met wat men

109

experimenteel vindt.

Vanwege de extreem trage dynamica beneden de overgangstemperatuur is het te verwachten dat ook de equilibratie van een spinglas zeer langzaam verloopt en het is dan ook zeer de vraag of een spinglas, dat ter experimentele bestudering (snel) wordt afgekoeld tot een temperatuur onder de overgangstemperatuur, binnen een experimenteel tijdsbestek thermodynamisch evenwicht bereikt. Niettemin heeft het theoretisch onderzoek zich lange tijd (noodgedwongen) geconcentreerd op de evenwichtseigenschappen van het spinglas in de lage-temperatuur fase.

Vooral na 1982 kwamen er meer en meer experimentele bewijzen van het niet-evenwichtskarakter van spinglazen. De eenvoudigste manier waarop dit kan worden aangetoond is met behulp van een veldsprong experiment: op het tijdstip $t = 0$ "schrikt" men het spinglas naar een temperatuur beneden de overgangstemperatuur T_f terwijl het magneetveld op een kleine constante waarde (bijvoorbeeld 10 Gauss) wordt gehouden. Vervolgens laat men het systeem gedurende een *wachttijd* t_w ongemoeid om op het tijdstip $t = t_w$ het magneetveld uit te schakelen. Daarna registreert men het verval van de magnetisatie. Als men dit experiment herhaalt voor verschillende waarden van de wachttijd (experimentele waarden lopen van een seconde tot enkele dagen) dan ziet men dat het verval van de magnetisatie trager verloopt naarmate de wachttijd groter is. Uit deze en andere experimenten blijkt dat de magnetische responsfunctie, in tegenstelling tot wat men verwacht in een systeem in evenwicht, afhangt van de *leeftijd* van het spinglas. Men spreekt in dit verband over *veroudering*.

Een belangrijk deel van dit proefschrift handelt over veroudering in spinglazen. Het begint met de formulering van een theorie die veroudering in spinglazen verklaart vanuit een beeld van groeiende (en krimpende) *domeinen*. Het hierboven beschreven experiment kan op eenvoudige manier worden uitgelegd met behulp van deze theorie: vanaf het moment $t = 0$ beginnen zich in het spinglas domeinen te vormen, waarbinnen de spincorrelaties zich gedragen als in evenwicht. Daardoor groeit het spectrum van relaxatietijden van het totale systeem naar langere en langere tijden zodat na een steeds grotere wachttijd de magnetische relaxatie zich op steeds grotere tijdschaal afspeelt.

De, leeftijdsafhankelijke, lineaire respons van een naar evenwicht strevend spinglas kan dus worden begrepen met behulp van een met de (leef)tijd toenemende lengteschaal. Bij grotere verstoringen, zoals grotere temperatuur- en magneetveldsprongen, komt een tweede lengteschaal, de *overlaplengte* in het spel. Deze overlaplengte geeft de typische afstand waarover de correlaties in twee copieën van hetzelfde systeem bij verschillende temperaturen en magneetvelden overeenkomen. Het bijzondere van spinglazen is, dat deze overlaplengte snel afneemt als functie van bijvoorbeeld het verschil in temperatuur van de twee copieën. Met behulp van deze tweede lengteschaal worden alle niet-lineaire verouderingseffecten verklaard.

In dit proefschrift wordt ook een één-dimensionaal spinglas model behandeld waarmee een aantal aannamen, die zijn gemaakt in de domeintheorie, worden gerechtvaardigd. Met name de leeftijdsafhankelijke lineaire respons wordt in verband gebracht met een in het model gedefinieerde lengteschaal die in de tijd varieert. In het bijzonder wordt ook de geldigheid van het fluctuatie-dissipatie-theorema voor

het niet-evenwichtsspinglas bediscussieerd.

Als laatste wordt in dit proefschrift beschreven hoe men verouderingseffecten in spinglazen kan onderdrukken. Het idee daarachter is dat de, bij constante temperatuur gebruikelijke, groei van domeinen wordt tegengewerkt door een *afbraak* tengevolge van een lineair met de tijd variërende temperatuur. De lengteschaal in het model groeit dan niet meer maar blijft op een door de mate van temperatuurvariatie gegeven waarde. De theorie voorspelt, voor niet te grote temperatuurveranderingen, dat de magnetische responsfunctie varieert op een tijdschaal die afhangt van de mate van de temperatuurverandering en die veel kleiner is dan bij een constante temperatuur.

Curriculum Vitae

In juni 1970 behaalde ik het diploma MULO A en B (richting wiskunde) in Amsterdam en in december 1975 behaalde ik, met lof, het diploma HTS-Elektrotechniek (specialisatie electronica) in Alkmaar. Nadat ik van januari 1976 tot mei 1977 mijn militaire dienstplicht vervulde ben ik bij de Sylvius Laboratoria (Faculteit Geneeskunde van de Rijksuniversiteit) te Leiden aangesteld als electronicus op de afdeling Histochemie en Cytochemie; vanaf augustus 1982 in deeltijd (voor 50%).

In juni 1982 heb ik het colloquium doctum aan de Rijksuniversiteit te Leiden behaald waarna ik onmiddellijk ben begonnen aan de opleiding natuurkunde van dezelfde universiteit. Het propaedeutisch examen is behaald in augustus 1983 en in januari 1986 behaalde ik het doctoraal examen in de theoretische natuurkunde. De doctoraalscriptie had als onderwerpen "Diffusie op percolatienetwerken" en "Een eenvoudig model voor spinglazen". De twee onderwerpen zijn bestudeerd onder leiding van respectievelijk prof. dr. P.W. Kasteleyn en prof. dr. H.J. Hilhorst.

Vanaf februari 1986 tot februari 1990 ben ik aangesteld als promovendus bij het Instituut Lorentz van de Rijksuniversiteit te Leiden in dienst van de Stichting Fundamenteel Onderzoek der Materie (werkgroep VS-th-L). Begeleid door prof. dr. H.J. Hilhorst nam ik deel aan het theoretisch onderzoek naar de dynamica van veeldeeltjessystemen, in het bijzonder van spinglazen. De resultaten van het onderzoek zijn in dit proefschrift beschreven.

Met ingang van februari 1990 word ik aangesteld als universitair docent bij de faculteit der Wiskunde en Natuurwetenschappen van de Rijksuniversiteit te Leiden om, onder leiding van prof. dr. D. Bedeaux, werkzaam te zijn op het gebied van de Fysische en Macromoleculaire Chemie.

Stellingen behorende bij het proefschrift

On the Dynamics of Spin Glasses

1. Struiks poging om voor verouderende systemen lineaire-respons relaties te formuleren door het tijdsverschil tussen respons en stimulus te schalen met de leeftijd van het systeem is overbodig en verduidelijkt bovendien niets omtrent het verouderingsproces.

    *L.C.E. Struik, Physical aging in amorphous polymers and
    other materials, Elsevier Scient. Publ., Amsterdam, 1978.*

2. Gegeven een stel "dronken wandelaars" in een zeer lange en uiterst smalle steeg. Zodra twee van die wandelaars elkaar tegenkomen slaan ze elkaar neer en is de weg vrij voor de overgeblevenen. Als de kans dat een wandelaar spontaan neervalt of weer opstaat verwaarloosd mag worden dan neemt de fractie na een tijd t nog overeindstaande wandelaars asymptotisch af als $t^{-\frac{1}{2}}$.

    Hoofdstuk IV van dit proefschrift.

3. Als in de onder stelling 2 geformuleerde situatie per tijdseenheid en per eenheid van lengte een klein aantal, α, nieuwe wandelaars wordt geboren dan ontstaat een quasistationaire toestand, waarbij gemiddeld per tijdseenheid evenveel wandelaars worden geveld als er geboren worden. Die toestand wordt gekarakteriseerd door een wandelaarsdichtheid die afneemt als $\alpha^{\frac{1}{3}}$ met $\alpha \to 0$.

    Hoofdstuk V van dit proefschrift.

4. De golffunctie, die men gebruikt bij de quantummechanische beschrijving van een fysisch proces, is afhankelijk van de kennis die men van dat proces heeft.

    N.G. van Kampen, Physica A 153(1988)97.

5. Het berekenen van het typische aantal metastabiele toestanden met gegeven magnetisatie en energie van een Ising spinglasketen is equivalent met het berekenen van de toestandssom van een Ising spinglasketen met stochastisch-ruimteafhankelijk magneetveld.

6. Er bestaat niet zoiets als een niet-lineaire Schrödinger vergelijking.

7. De bewering, dat de Yang-Baxter relatie een voldoende voorwaarde is voor de oplosbaarheid van een statistisch-mechanisch model, kan niet worden bewezen.

    M. Wadati et al., Phys. Rep. 180(1989)247.

8. De sterke concentratieafhankelijkheid van de fluorescentieopbrengst van gekleurde celkernen, gemeten met een microfotometer met hoge numerieke apertuur bij golflengten waarbij een sterke emissieabsorptie optreedt, is te verklaren door zowel de golflengte- als de concentratieafhankelijkheid van de brekingsindex in beschouwing te nemen.

    H.J. Tanke, Cytometry 2(1982)359.

9. De bijdrage van de synthetiserende cellen in een DNA-histogram kan beter op het oog worden bepaald dan met behulp van uitgebreide data-analyse.

10. Veroudering is een jonge en veelbelovende tak van de wetenschap.

G.J.M. Koper, januari 1990.