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H. A. LORENTZ. — On the motion of electricity in a spherical shell placed in a magnetic field.

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H. A. LORENTZ. — *On the motion of electricity in a spherical shell placed in a magnetic field.*

Professor KAMERLINGH ONNES' experiment on the couple acting on a supraconducting spherical shell in which currents have been excited and which is placed in an extraneous magnetic field [1]), suggests the problem of the distribution of currents in such a shell and of the determination of the ponderomotive forces acting on it. These questions can be considered in the case also of an ordinary conductor and admit a simple solution if the extraneous field is supposed to be uniform. Then, at any instant, the electricity will be flowing along parallel circles around a certain axis OA, the intensity $\mathbf{C}$ of the current (which, in the case of a thin shell, can be considered as a surface current, to be measured by the quantity of electricity which, per unit of time and unit of length, is carried across a line perpendicular to the line of flow) being proportional to the sine of the angular distance from the pole A. A distribution of currents of this kind produces inside the sphere a uniform magnetic field h in the direction of OA, and at external points a field identical with that of a magnet of a certain moment $\mathbf{m}$ placed at the centre in the direction OA. The system of currents is fully characterized by the magnetic force $\mathbf{h}$, the vector $\mathbf{m}$, which may be called the magnetic moment of the sphere, being related to h according to the formula

$$\mathbf{m} = 2\pi a^3 \mathbf{h} ,$$

where a is the radius of the shell.

The way in which the currents in the sphere, and consequently the field $\mathbf{h}$, change in course of time, under the influence of a given extraneous field $\mathbf{H}$, is determined by the equation

$$-\frac{3c}{2\lambda}\mathbf{h} = \frac{a}{2c}(\dot{\mathbf{H}} + \dot{\mathbf{h}}) + \frac{3}{4}c\mu\,[\mathbf{H}\,.\,\mathbf{h}] \qquad (1)$$

[1]) Suppl. N°. 50a, § 2.

Here, the term on the left hand side, in which λ is the product of the conductivity and the thickness of the shell, replaces the current intensities divided by the conductivity, whereas the terms on the right hand side represent the different actions producing the currents, viz. the induction due to the change of **H**, the self induction determined by $\dot{\mathbf{h}}$ and the transverse forces by which one can account for the Hall effect. These latter lead to the term containing the vector-product $[\mathbf{H} \,.\, \mathbf{h}]$ multiplied by the coefficient μ of the Hall effect.

The sphere itself is supposed to be held at rest and the different quantities are expressed in rational units; this is why the factor c occurs in two of the terms.

It should also be remarked that the formula is a vector equation; both **H** and **h** can vary in direction as well as in magnitude.

Now if, as was the case in Professor KAMERLINGH ONNES' experiment, **H** is constant in the final state, (1) reduces to

$$\dot{\mathbf{h}} + p\mathbf{h} = r\,[\mathbf{H} \,.\, \mathbf{h}], \tag{2}$$

where

$$p = \frac{3c^2}{a\lambda}$$

and

$$r = -\frac{3}{2}\,\frac{c^2\mu}{a}.$$

Multiplying (2) by e^{pt} and defining a new vector **k** by

$$\mathbf{k} = e^{pt}\,\mathbf{h},$$

one finds

$$\dot{\mathbf{k}} = r\,[\mathbf{H} \,.\, \mathbf{k}], \tag{3}$$

showing that the vector **k** remains constant in magnitude, but has a precessional motion with the angular velocity $r\,\mathbf{H}$ about the line of force of the field **H** passing through the centre of the sphere.

Hence, the vector **h** also, and the system of currents which it characterizes, have this precessional motion; only, while it is performed, the currents decay proportionally to e^{-pt}, the vector **h** being given by

$$\mathbf{h} = e^{-pt}\mathbf{k}.$$

The ponderomotive action on the sphere is found at any instant to consist of a couple $[\mathbf{m}.\mathbf{H}]$, i. e. the couple with which the field **H** acts on the moment **m** of the sphere according to the ordinary rule.

If these considerations are applied to a supraconducting shell, for which we may put $p = 0$, the currents are found to persist, but, so long as there is a Hall constant μ differing from 0, they will show the precessional motion represented by (3), the velocity of which is independent of the conductivity. Hence, if the force **H** is horizontal and if **h** makes an angle with it, the moment **m**, rotating about the direction of **H** will change its direction and the couple which tends to turn the sphere about the wire by which it is suspended will change continually.

In KAMERLINGH ONNES' experiment this couple was shown not to alter appreciably in about six hours, so that we may safely conclude that the angle over which the precession took place during this lapse of time has been smaller than, say 20°. This means that the magnitude of $r\mathbf{H}$ has been less than $1.62 \,.\, 10^{-5}$. If the electrons moving in the currents were wholly free to obey the transverse force exerted by the field **H**, there would be a Hall effect of definite magnitude and sign; the Hall-constant would have the value

$$\mu = -\frac{1}{c\sigma},$$

where $-\sigma$ is the total charge of the free electrons contained in the shell per unit of surface. It may be inferred from this, in connexion with what we found about the magnitude of $r\mathbf{H}$ that, in the case of lead, the ratio between the number of free electrons and the number of atoms ought to be greater than

$$14\,\frac{H}{a\delta},$$

where H is the strength of the field expressed in gauss and δ the thickness of the shell. Now, this condition has certainly not been fulfilled, for H has been some tens gausses and the number of free electrons can be no more than a small fraction of that of the atoms.

The experiments therefore show that there is no question of a precession such as would exist if the electrons moving in the currents were wholly free. We must conclude that practically there is no Hall-effect and we must suppose, as Professor Onnes does, that the motion of the electrons is to a great extent insensible to the transverse forces exerted by the field. If, in the absence of a field, an electron moves in a certain path along a row of atoms, we must assume that the transverse forces are not able wholly to detach the path from that row.