# HOW CAN ATOMS RADIATE? [1]

When, some months ago, the secretary had the kindness to ask me to deliver this lecture, I was glad, for various reasons, to accept the invitation. In the first place, and this would have been sufficient, because the great name of BENJAMIN FRANKLIN, for whom I have always felt a deep veneration, is connected with this place. Secondly, because your Institute once did me the honour to award me the FRANKLIN medal, which, to every man of science, is a very great distinction. I was not, at the time, able to come to Philadelphia to receive the medal but it would have been very unkind on my part, if I had not availed myself of this opportunity for expressing my thankfulness and appreciation.

Now, I suppose that you desire me to lay before you some questions taken from the newer development of physics and with which I am more or less familiar. So I chose as my subject the way in which the atoms of luminous bodies emit their radiations. Of course, this problem is intimately interwoven with the question: What is the nature of these radiations themselves? So long as, following NEWTON, physicists supposed light to consist of small particles or corpuscles moving along at high speed, it was natural and necessary to consider the atoms as something like small guns emitting these corpuscles. On the other hand the undulatory theory of light developed by HUYGENS, YOUNG and FRESNEL makes us think of vibrating particles more or less comparable with the vibrating bodies which produce sound.

You know that by a most remarkable change of views modern physics has been led to what one may call a revived corpuscular theory. Numerous phenomena, in the first place those of photo-electricity, can hardly be understood if the energy of light waves is supposed to spread out indefinitely over greater and greater

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[1]) Presented at the meeting of the Franklin Institute held March 31, 1927. Journal of the Franklin Institute. **205**, 449, 1928.

spaces. It seems beyond doubt, that by such a diffusion the energy would soon become too dilute to eject an electron from an atom, for which a definite amount of energy is required. So, one naturally came to the hypothesis of „light quanta", portions of energy concentrated in extremely small spaces and moving onward with the velocity of light.

There can be no doubt as to the amount of energy which must be associated with such a quantum. Phenomena and considerations into which I cannot now enter have shown that this amount must be proportional to the frequency, i.e. to the number of vibrations per second, of the light considered. It can therefore be represented by the product of the frequency $n$ and a certain constant coefficient which we shall denote, as is always done, by $h$. This is PLANCK's famous coefficient, which was introduced into science somewhat more than 25 years ago and which has come to play a most important part in many chapters of physics. PLANCK, indeed, was the first to realize that the continuity which observation first reveals to us in physical phenomena, may be found to be apparent only when we penetrate farther and farther into minute details. In order to explain the way in which the radiation of heat of different wavelenghts depends on temperature he supposed that in hot bodies there are small vibrating particles endowed with the curious property that they cannot take and lose energy in any arbitrarily small amount, but only in finite portions of definite magnitude, these portions, or elements of energy, as PLANCK originally called them, being proportional to the frequency. He therefore denoted their magnitude by $hn$.

The quantum theory, started in this way, rapidly developed. It may be said to have revolutionized physics; the idea of a definite frequency and a definite amount of energy belonging together in the way assumed by PLANCK has proved most fruitful. It has been applied to a wide range of phenomena and the value of PLANCK's constant has been deduced from many of them. To give you at least some idea of this magnitude, I may say that, if the frequency were a billion per second, the quantum would be equal to the work required to raise a weight of 0,0067 milligram to a height of a billionth of a centimetre.

That, notwithstanding the mystery that still hangs over its fundamental assumptions, the quantum theory contains something closely approaching reality, is shown by the striking agreement between the values found for the constant $h$ in different ways. Altogether the theory has been so successful that nowadays we cannot think of a picture of the physical world in which the constant $h$ would not appear, any more easily than we can imagine one in which there would be no question of the elementary electric charge, of AVOGADRO's constant or of the constant of gravitation.

To return to the light quanta, their energy $hn$ is so small that the number of quanta which our eye receives during a second, for instance, even when feeble light enters it, must be extremely great; they are to be counted by millions. All these quanta have to be emitted by the radiating atoms, and here we may remark that we can more easily conceive a continuous emission of them than one of corpuscles. These latter have always been regarded as substantial, whereas the light quanta are not material at all, being merely small amounts of energy; they may disappear by the transformation of this energy into some other form, into heat for instance, and similarly they may start into existence provided only that the ejecting atom has a sufficient amount of energy at its disposal. According to an extreme form of the quantum theory not only would there be light quanta but they would be the only constituents of a beam of light, there being nothing like waves or vibrations outside them. If this were so, we should have to seek a new interpretation of the phenomena of interference and diffraction that were so beautifully explained by the old theory of optics. One has indeed tried to do so and with some success. Yet, to my mind, the difficulties which one encounters are so serious that no choice is left us and that *this extreme form* of the theory must be discarded.

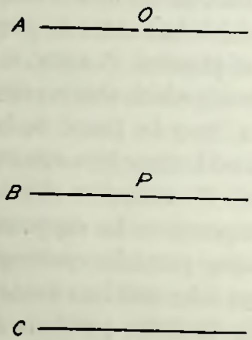

FIG. 1.

A single example will, I think, suffice to justify this conclusion.

Let us observe a diffraction phenomenon produced by means of *very feeble* light. A small opening $O$ (fig. 1) in an opaque screen $A$ may serve as source of light and a second hole $P$ in a screen $B$ as the diffracting opening. We observe the diffraction image or pattern on a third screen *C* at some distance behind *B*.

The image shows a distribution of light and darkness which depends on the shape of the opening in *B* and which can be perfectly calculated by means of the formulae given by FRESNEL. These enable us exactly to determine in all its details the delicate structure of the beam of light behind screen *B*, of which we see different sections when we displace screen *C* parallel to itself.

Now, suppose that the effect, the light which we *see*, is due to quanta. Then, either they must move in such a way that they can reach only the bright places on screen *C* or, if they can come to all points, it must be only at the bright places that they are able to illuminate the screen. The first alternative would require that the quanta be properly guided in their course, and if there were nothing else, this ought to be done by some mutual action. The possibility of this, however, must be excluded because one can easily realize circumstances under which the number of quanta to be found at any instant between the planes *B* and *C* is very small so that they are too far apart to act upon each other as would be required.

I had a case, for instance, in which from the intensity of the light and the known magnitude of the quanta I deduced that, per second, about seventy millions of them passed through the opening in the screen *B*. If we take into account that they move with the speed of light and that therefore those that pass in a second are distributed over a length of three hundred millions of metres, one sees that in the case considered the mean distance of successive quanta along the beam was about 400 cm. As the distance from *B* to *C* in the experiment which I have in view was no more than 16 cm, the number of quanta present in the beam between *B* and *C* was on an average 0,04. This means that most times there was in this space no quantum at all; at some instants there will have been one quantum between the two planes and sometimes there may have been two or more of them, but this must have been a very rare occurrence.

We may conclude from this that there can be no question of the motion of the quanta being controlled by some mutual action. There must be something of another kind which determines their course. Similarly, if we choose the other alternative just mentioned, there must be something else besides the quanta, on which it depends whether or not they shall be able to illuminate the screen.

Now this „something" must account for all the details of the diffraction phenomena which we can calculate with our old wave-theory and so it seems to me very natural to assume that, if there are quanta, which I shall not deny, there must be besides them something like the ordinary radiation field with which we have long been familiar. For this reason and for the sake of briefness, I shall now dismiss the light quanta altogether and shall simply speak in what follows of the emission of vibrations.

I shall put before you four different theories of this emission, the one that was universally accepted until thirteen years ago, the most remarkable theory developed by BOHR in 1913, the „dynamics of matrices", as it is called, which we owe to HEISENBERG, BORN, JORDAN and DIRAC, and, finally, a theory that was put forward by LOUIS DE BROGLIE and in the evolution of which SCHRÖDINGER had a great part.

In the oldest of these theories, the „classical" one as it is often named now, the atoms were supposed to contain small particles which have definite positions of equilibrium and can vibrate about them. Suitable assumptions were made concerning their masses and the forces acting on them, but as to their nature little progress was made, until one came to realize, and this was an important step, that they must be electrically charged. I may, perhaps, briefly review the grounds on which this latter assumption is based.

In the first place one learned to know by MAXWELL's theory, that the waves of light are of the same nature as the waves observed by HERTZ or as those that are used in wireless telegraphy. The wavelength of light is much smaller, but the general laws for its propagation are identical with those of the electromagnetic waves. One might suppose therefore that the light waves have their origin in something comparable with the alternating electric currents in a HERTZ vibrator, or in the aerial of a wireless installation.

In the second place, a celebrated experiment made by ROWLAND has shown that the same effects that are produced by conduction currents in a metal can be brought about by what is called a convection current, namely by the motion of a charged body. A charged hard rubber disc, rapidly rotating in its plane, deflected a magnetic needle suspended at some distance in exactly the same way as could be done by a current flowing in a circular coil. The experiment has been repeated in many different forms and there can be no doubt that, when a charged sphere, for instance, is moved to and fro along a straight line it will produce waves of the same kind as can be obtained by an alternating current along that line. Something like this sphere, on a much smaller scale, a minute particle carrying an electric charge and vibrating in the interior of an atom, might well be the origin of the light which the atom radiates.

There are many other grounds for the assumption of small charged particles, ions or electrons as they have been called. The hypothesis makes it possible also to understand the phenomenon of the absorption of light, which is the inverse of the radiation and which we shall also have to consider. It is immediately clear that now part of the motion existing in the incident waves is communicated to the body. Thus, in this case, particles must be set vibrating *by the light* and this is what can be expected when they carry electric charges. According to MAXWELL's theory the beam of light is the seat of rapidly alternating electric forces and by definition such a force is one by which a charged body can be set into motion.

I think you now see the main features of the picture which the old theory gave us of optical phenomena. The charged particles in a luminous body are drawn towards their positions of equilibrium by forces, proportional to the distances over which they have been displaced from these positions. So each of them can vibrate with a definite frequency, just as an ordinary pendulum or a tuning fork, and by this the frequency of the emitted radiation is likewise determined. If now this radiation falls on the matter of a ponderable body, different things may happen, but at all events the beginning will be that the electrons or ions in the body are set vibrating. If the body is found to be not wholly transparent, we may conclude that there is some kind of resistance opposing the regular motion and converting it more or less into the irregular agitation which manifests itself as heat. In a transparent body like air or glass there is no such resistance and then it is found theoretically that the optical properties, namely the velocity of propagation *in* the body and the index of refraction are determined by the amount to which the electricity in the body is displaced by the periodically changing electric force in the beam to which it is exposed. If, under the action of a given alternating electric force, there is a considerable displacement of electricity in the direction corresponding to that of the force, we shall have a small velocity of propagation, showing itself in a high index of refraction.

This old theory certainly had a great beauty though it must be owned that its success was largely due to the fact that, not knowing very much about the structure of atoms, physicists felt free to make, concerning the particles and the forces that act upon them, just the hypotheses that best suited their purpose. Let me give you one or two examples of what could be done with the theory and then point out to you its great failures.

I may mention as a success the explanation of different phenomena by means of the principle of resonance. What I want to say is best illustrated by a simple experiment. Let $ns$ (fig. 2) be a small compass needle and let the south pole of a bar magnet be moved to and fro along the line $AB$ situated in the horizontal plane in which the needle can move and at right angles to its direction of equilibrium $ns$. The motion imparted to the needle will then depend on the relation between the frequency of its free or natural vibrations, i.e. those which it can perform under the sole action of terrestrial magnetism, and that of the oscillations of the pole $P$. If, first, $P$ is kept fixed in any position on $AB$, the needle will take a new position of equilibrium, the end $n$ being deflected towards the right when $P$ is on the side $B$, and towards the left when $P$ is on the side $A$ of the middle point $O$. When, next, the pole $P$ is slowly moved to and fro, the needle obediently follows it, being at any moment nearly in the position of equilibrium corresponding to the place of $P$ at that moment. The amplitude of these „forced" vibrations is found to increase, when, by raising the frequency of $P$, we make it approach that of the free vibrations of the needle. This goes so far that, when the two

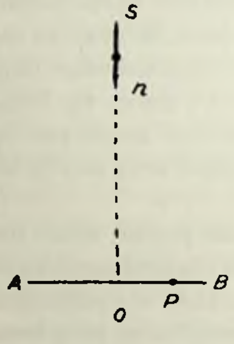

FIG. 2.

frequencies have become equal, the amplitude would become infinitely great in the ideal case of no resistance. But most curious and at first sight somewhat strange is what happens when we make the pole $P$ oscillate more rapidly than corresponds to the natural frequency of $ns$. The forced vibrations then take a phase opposite to what we had first, so that at a definite moment the needle is deflected in a direction opposite to that in which it is drawn by the pole $P$. The end $n$ will be on the right-hand side when $P$ is near $A$, and on the left-hand side when $P$ is near $B$. Again the amplitude of the forced vibrations will be greatest when the difference between the two frequencies is small. Let us now apply this to the propagation of light in a body, replacing the force due to the magnetic pole $P$ by the alternating electric force existing in a beam of light and the magnetic needle by the movable particles in the body. We shall suppose that there are various kinds of such particles, with different natural frequencies, and that the displacement of electricity by the motion of all these kinds together is in the direction corresponding to that of the electric force. Let us now, however, see what contribution to this general displacement is due to the particles of one particular kind. These will have, just like our magnetic needle, a definite frequency of their own, say $n_0$, and if the frequency of the incident light had exactly this value, we should have the maximum of resonance. The resistance to the vibrations, some kind of which we can imagine always to exist, will give rise to an absorption which now will be greater than for any other frequency of the incident light. Thus, if light falls on the substance, we shall have in the spectrum of the transmitted rays an absorption band, the middle of which has a position corresponding to the natural frequency of the group of particles considered.

Suppose, now, that the frequency of the incident light is somewhat smaller than $n_0$. Remembering what we saw in the case of the magnetic needle we can easily foresee that the particles in question will be displaced in the direction in which they are driven by the electric force. Hence, owing to this group of particles, the displacement of electricity in the direction of the electric force will be greater than it would be without them. The reverse will occur when the frequency of the incident light is greater than $n_0$. Then, at any moment, the particles now considered will be displaced in a direction opposite to that in which they are driven by the electric force and this will tend to diminish the total displacement which we have supposed to be in the direction corresponding to the electric force. Finally, if we take into account the relation between the displacement of electricity and the index of refraction, we see that the presence of particles with the natural frequency $n_0$ will increase the index of refraction for light whose frequency is below $n_0$ and diminish it for rays whose frequency is above that value, both effects diminishing when $n$ recedes from $n_0$ because by this the amplitude of the vibrations set up by the electric force becomes smaller.

These conclusions have been amply confirmed by experiment and all the phenomena of dispersion, i.e. those in which we are concerned with the way in which the velocity of propagation depends on the frequency, can be very satisfactorily explained on this basis.

The second problem in which the old theory has had, one may say, a brilliant success, is that of the scattering of light by the molecules of a body, if they are irregularly distributed over space, as is the case in liquids and gases. The movable charged particles contained in the molecules are set in motion by the incident light and thereby become themselves centres of emission. Consequently, the light does not remain confined to the direction of the incident beam; part of it is thrown sideways in different directions, and this of course implies a diminution of the intensity of the light that continues its course in the original path. The two phenomena, the „extinction", as the beam proceeds, and the scattering, are so closely connected that the degree to which one of them takes place follows directly from the numerical value that measures the other. Both are determined by what is known as the „extinction coefficient", the definition of which is as follows.

Let the intensity of a beam be measured by the amount of energy that is carried across a section per unit of time, and consider its diminution when the beam goes forward over a small distance $l$. Dividing this diminution by the length of $l$ one finds the extinction per unit of length, and division of this by the intensity such as it is at the beginning of the distance $l$ will give one the extinction coefficient $k$. If, for instance, $k = 0{,}001$ the intensity will decrease by one-thousandth part of its amount, when the rays go forward over a distance of a centimeter.

The coefficient of extinction is given by a celebrated theoretical formula which we owe to the late Lord RAYLEIGH, namely

$$k = \frac{32\pi^3 (\mu - 1)^2}{3N\lambda^4},$$

where $\pi = 3{,}1416$; $N$ is the number of molecules per unit of volume, $\lambda$ the wavelength of the light and $\mu$ the index of refraction.

The equation has been verified by accurate measurement of the extinction in gaseous media, and the values of $N$ to which it leads are in good agreement with the results obtained in other ways. But I most now call your attention to the deficiencies of the old theory. One of its worst failures was that it could not account for the structure of spectra, I mean to say, for the regularity in the spectra which shows itself in the numerical relations between the frequencies of the lines. The analogy with the phenomena of sound, on which much stress was laid in the old theory, was rather misleading than helpful when one tried to understand these relations.

Every one knows that a stretched string can give a series of tones, the fundamental tone and the upper harmonics, each of which is produced by a particular mode of vibration. The frequencies of these tones are proportional to the natural numbers 1, 2, 3, ..., a law which was deduced long ago from a mathematical theory which also made it possible to calculate the pitch of each tone, when one knew the tension of the string and its mass per unit of length. By means of a similar theory, only of somewhat greater mathematical complexity, we can determine the modes of vibrations of other systems, of stretched membranes, for instance, and of rods or spheres of elastic substances; in these cases also the ratios between the frequencies of the different tones, though in general less simple than in the case of the string, can be completely accounted for. Now, the existence, in the spectrum of a gaseous body which is a chemical element, of a certain number of lines, clearly shows that, just as vibrating bodies of the kind I have mentioned, atoms of a definite constitution can send forth waves of different frequencies. It was natural to expect, here also, some numerical relation between the frequencies, and such a relation has really been brought to light; its form, however, is such that it has baffled all attempts to deduce it by considerations like those used in the theory of elasticity.

It will suffice for our purpose to discuss the simplest case, the ordinary spectrum of hydrogen glowing in a GEISSLER tube. It consists of a series of lines, which are ordinarily distinguished by the letters $\alpha$, $\beta$, $\gamma$, .... The line $\alpha$ is seen in the red; $\beta$ and $\gamma$ are likewise in the visible spectrum and the remaining ones have their places in the ultraviolet. The law which governs their positions, was discovered, half a century ago, by the Swiss physicist BALMER. It has been found to be in perfect agreement with the observations; in fact, it is equalled in this respect by few other physical laws.

We can express BALMER's law by saying that the frequencies of the lines $\alpha$, $\beta$, $\gamma$, ... are proportional to the quantities

$$\frac{1}{4}-\frac{1}{9}, \quad \frac{1}{4}-\frac{1}{16}, \quad \frac{1}{4}-\frac{1}{25}, \quad \ldots\ldots,$$

the series of which you can easily continue, the formation of the successive terms being at once apparent.

It is seen immediately that, as we proceed in the series, the terms, though continually increasing, can never go beyond the value $\frac{1}{4}$, which, of course, implies that the differences between successive frequencies become continually smaller and tend towards zero. These differences are proportional to the quantities

$$\frac{1}{9}-\frac{1}{16}, \quad \frac{1}{16}-\frac{1}{25}, \quad \frac{1}{25}-\frac{1}{36}, \quad \ldots\ldots$$

This means that as we pass through the spectrum from the side of the red to that of the violet the lines are more and more crowded together (fig. 3) and that there is a limiting position $\rho$ to which they come very closely but which they cannot wholly reach. Series of lines presenting these same features are found in the spectra of other elements and we are undoubtedly concerned here with something quite essential and fundamental. Now, this remarkable structure is wholly beyond the powers of the classical theory; it is impossible to explain it by any assumption about particles vibrating about positions of equilibrium.

I should now like to say some words of the wonderful theory by which BOHR has explained the hydrogen spectrum. He did so without having to imagine a structure of the atom specially invented for the purpose. The starting point was the idea, already put forward by RUTHERFORD, that the hydrogen atom consists of a positively charged nucleus and a negatively charged eleotron circulating around it. The mass of the nucleus is about 1850 times that of the electron and therefore, in a first approximation, the nucleus may be supposed to remain at rest. The electron is attracted by it with the ordinary electrostatic force, the law of which is the same in form as that of gravitation, the intensity of the force being inversely proportional to the square of the distance. Thus, the case is much like that of a planet moving around the sun; the electron will in general move in an elliptical orbit. Eventually the orbit may be a circle, and, again for the sake of brevity, we shall confine ourselves to orbits of this kind. The consideration of elliptical orbits would give us the same result.

Now, how can we, with this simple model of the atom, explain a law so complicated as BALMER's? BOHR has performed this by making two bold assumptions, the one referring to the state of motion of the electron and the other to the emission of radiation.

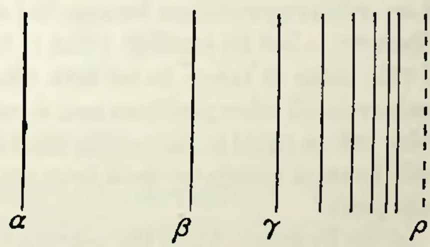


FIG. 3.

In PLANCK's theory of the radiation of heat it had been assumed that each of the vibrators which give rise to this phenomenon can only have one of certain definite amounts of energy, namely, one of the amounts $hn$, $2hn$, $3hn$, etc., where $h$ is the constant which we know already and $n$ the frequency of the vibrator. Similarly, among all the circular motions of his electron of which one can think, BOHR singles out certain specified ones, assuming that these are the only ones that really occur; he calls them the „stationary motions". The selection is made by the introduction of a condition very similar to the one to which we just now subjected the motion of the vibrator; in a sense, it is the same condition adapted to the circumstances of the new case.

Both the vibrator and the atom of hydrogen have, in each of their states of motion, a definite amount of energy and for both we can distinguish between the kinetic energy and the potential energy. Let us compare these parts of the energy in the two cases, not forgetting that in the expression for the potential energy there is always an arbitrary constant; this is unavoidable because we are free to choose the position in which the potential energy is said to be zero and with which all other positions are compared. The indefiniteness will not affect our results, because these will depend on the differences only between the values of the energy in different states.

The kinetic energy of a moving mass is always given by the product of its magnitude and half the square of the velocity. It is constant for the electron moving in the circle, whereas for the vibrator it oscillates between zero and a certain maximum value; we may in this case speak of its mean value during a full period.

The potential energy of the vibrator — we may think here of an ordinary pendulum because the same is true for all kinds of vibrators — has its smallest value in the position of equilibrium. If this value is taken to be zero, the potential energy will be positive in all other positions and it may be shown that its mean value will be equal to that of the kinetic energy. Hence, if we take half the total energy we shall have the mean value of each of the two parts.

In the hydrogen atom the potential energy increases when the electron gets farther away from the nucleus. It has its greatest value at infinite distance and if we denote this by $C$ we have, for any finite distance, to diminish $C$ by a certain amount, which we may call the wanting potential energy and which turns out to be inversely proportional to the distance. According to a theorem well known in dynamics the kinetic energy is now equal to half this wanting potential energy, so that the total energy is obtained when we diminish $C$ by half the wanting potential energy. Thus, the total energy increases with the radius of the orbit. If the atom is given in one of its modes of motion, it will require some work to remove the electron to a great distance, even if we are satisfied with having it there without any kinetic energy.

Now, BOHR's assumption may be expressed in the same form as PLANCK's if we fix our attention upon the kinetic energy. Instead of saying that the total energy of the vibrator must be a multiple of $hn$, we may just as well require that the mean kinetic energy be a multiple of $\frac{1}{2}hn$. In the case of the hydrogen atom, we can understand by frequency the number of revolutions per unit of time and now, if $n$ is given this meaning, BOHR's hypothesis amounts to this, that the circular motion must be such that the kinetic energy is just a multiple of $\frac{1}{2}hn$. We shall write for it $k \cdot \frac{1}{2}hn$, $k$ being an integral number.

By this, the stationary states are completely defined, and the total energy will be known for each of them because the kinetic energy has been determined. If, after all, the formulae become somewhat less simple than they are for the vibrator, it is only because now the frequency, $n$, is not a constant, but depends, according to KEPLER's third law, on the radius of the orbit.

Working out the necessary calculations one finds for the energy of a stationary state

$$C - \frac{A}{k^2},$$

where the constant $A$ has the value

$$A = \frac{2\pi^2 me^4}{h^2}.$$

In this equation $m$ is the mass of the electron and $e$ its charge, whose magnitude is equal to that of the charge of the nucleus [1]). The different stationary modes of motion are characterized by the values 1, 2, 3, ... of the quantum number „$k$". I may add that the radii of the orbits are in the ratios of 1, 4, 9, ....

[1]) Let $r$ be the radius of the orbit and $v$ the velocity of the electron. The force with which the electron is attracted by the nucleus is

$$\frac{e^2}{r^2},$$

so that by the law of circular motion, one has

$$\frac{mv^2}{r} = \frac{e^2}{r^2}.$$

The frequency is

$$n = \frac{v}{2\pi r}$$

and the quantum condition requires that

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}khn.$$

From these equations one finds

$$r = \frac{k^2h^2}{4\pi^2me^2}$$

and

$$\tfrac{1}{2}mv^2 = \frac{2\pi^2me^4}{k^2h^2}.$$

This is also the value of $A/k^2$.

I now come to BOHR's second assumption. The atom is supposed not to radiate so long as it is in one of its stationary states. An emission of light takes place only when there is a transition from one of these states to another in which the energy has a smaller value. In such a transition or jump the atom loses a certain amount of energy and *this* energy is radiated. As to the frequency of the radiation, it is supposed to be the one that is associated with the amount of energy just mentioned. Thus, if $E$ is the energy of the atom before the transition, in the first stationary state, and $E'$ the energy after the transition, in the final state, so that $E - E'$ is the radiated energy, the frequency is

$$\frac{E - E'}{h}.$$

Following this assumption, we obtain the lines $\alpha$, $\beta$, $\gamma$, ... of the Balmer series, when we consider a transition from the third, or the fourth, or the fifth stationary state, and so on, always to the second state. Indeed, the energy in the third state is $C - A/9$ and that in the second $C - A/4$, by which the frequency in the case of the first jump becomes

$$\frac{A}{h}\left(\frac{1}{4} - \frac{1}{9}\right).$$

Similarly, we find for the transition from the fourth state to the second

$$\frac{A}{h}\left(\frac{1}{4} - \frac{1}{16}\right),$$

etc. So we really find the ratios between the frequencies of the lines, exactly as they are according to BALMER's law.

But, in addition to this, the absolute values of the frequencies, and not only their ratios, can be calculated. If, as is usually done in spectroscopy, we use, instead of the frequency $n$, the number of wavelengths in a centimeter, which is $n/c$, when $c$ is the velocity of light, we must replace in our formulae $A/h$ by

$$\frac{A'}{h} = \frac{2\pi^2 me^4}{ch^3}.$$

When, in this expression, we substitute for $c$, $m$, $e$ and $h$ their numerical [1]) values, we find 190 000 whereas the value deduced from the measured wavelengths of the lines is 109 700.

Thus, BOHR has really calculated from known quantities the frequencies of the light emitted by a hydrogen atom, just as we deduce the frequency of a string from its tension and its mass per unit of length. The agreement is so close that we can scarcely doubt that his formula is the right one.

This theory of the hydrogen spectrum is certainly one of the greatest achievements in modern physics. It is also one of the most fruitful for it has enabled BOHR and those who worked along the same lines to give us an insight into the nature of spectra much more intricate than that of hydrogen. All over the world, spectroscopists are now using BOHR's ideas for disentangling the multiplicity of spectral lines which otherwise would be most bewildering.

Yet, there are many points which we do not understand so well as we might wish. According to the old theory the electron ought already to radiate while it performs its stationary motion in a circle or an ellipse; in BOHR's theory, however, it is not allowed to do so, just because one wants the motion to be stationary. Then we should like to see how it is that only the selected stationary states, and no others, can exist in reality. And, finally, the frequency is simply deduced from the amount of energy that is available for radiation. We should certainly be more satisfied if in the picture there were something like a vibrator, as we formerly imagined it. In connection with this I must remark that the orbital motions themselves are periodic but their frequencies are wholly different from those of the emitted radiations. The red hydrogen line, for instance, is due to a jump from the third stationary state to the second. In these two states the frequency is 2/27 of $A/h$ and 2/8 of $A/h$ respectively, whereas the frequency of the emitted light is

$$\frac{5}{36}\frac{A}{h}.$$

The three numbers are proportional to 8, 27 and 15.

The difficulties which I pointed out to you now are so serious

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1) $c = 3.10^{10}$; $m = 8{,}98 \,.\, 10^{-28}$; $e = 4{,}77.10^{-10}$; $h = 6{,}55 \,.\, 10^{-27}$, all in C.G.S. units.

that many physicists have been led to the idea that some radical change in our fundamental dynamical conceptions will have to be made. I shall conclude with some remarks about two new and remarkable theories which have this tendency.

The first of these theories has been developed by HEISENBERG, BORN, JORDAN and DIRAC; it is usually called the dynamics of matrices. To my regret I can scarcely give you an adequate outline of it; I cannot even explain to you in a short time what mathematicians understand by a „matrix". I must confine myself to the general idea underlying the theory.

When, following BOHR, we want to calculate the frequency of the radiation emitted by hydrogen, we proceed in three steps. First, we determine the motions, in elliptical or circular orbits, that are possible according to ordinary mechanics. Then, we select among all these motions those which satisfy certain quantum conditions; these are the stationary states. Finally, we fix our attention on one of the transitions of which I have spoken. The difference between the values of the energy, before and after the transition, gives us the frequency of one of the spectral lines. The formula, which we find for it, contains the charge $e$ of the electron, its mass $m$ and PLANCK's constant $h$.

Now, the fundamental idea of the physicists just mentioned is this. We are not primarily concerned with the motion of the electron in the atom; what we want to account for, first of all, is the radiation that goes out from it. Therefore, now that we encounter so many difficulties, had we not better refrain entirely from examining the motion of the electron; could we not try, by some direct method of calculation, to deduce the emitted frequencies from $e$, $m$ and $h$? This is what HEISENBERG, BORN, JORDAN, DIRAC and others attempt to do and in which they have established rules of calculation which really lead to the Balmer lines and which can be usefully applied to many other problems. Moreover, these rules teach us something about the relative intensities of the lines, a point about which it would be very difficult to draw some information from, for instance, the old theory of vibrating particles.

So this matrix dynamics well deserves the attention that is now given it on many sides, the more so because in some cases, in which it does not lead to the same consequences as BOHR's theory, observation appears to be in favour of it. It can, however, not be denied that, with regard to the question *how* atoms radiate, we are almost farther from a solution than we were with BOHR's theory; from the outset the question is purposely avoided, because it is considered as lying beyond what we can know, or want to know.

On the other hand, the last of the theories on my list, tries again to give us a picture of the mechanism of radiation. It was originated by LOUIS DE BROGLIE, who, some years ago, made an ingenious attempt somewhat better to understand BOHR's quantum condition. According to his views the motion of the electron in a circle is accompanied by a progression of some kind of waves along that line. The frequency of these waves is the one that corresponds to the energy which we attribute to the electron, and the waves are supposed to have a certain velocity, not equal to that of the electron, but closely connected with it, the relation between the two velocities being as could be reasonably expected. Dividing the velocity of the waves by their frequency one finds the wavelength, i.e. the distance over which one must go forward in the direction of propagation to come back to the same phase of vibration. Now, it is clear that, when the waves are propagated along a circle, it is necessary that, at a chosen point of that line, one should find but one phase and not two different ones. This gives us the condition that the circumference of the circle must be just a wavelength or a certain number of full wavelengths. When this is worked out one finds exactly the quantum condition of BOHR's theory.

Stimulated by DE BROGLIE's ideas, SCHRÖDINGER has further developed this „wave mechanics". The electron in the hydrogen atom now disappears from the stage; it is replaced by something that is distributed all over the space surrounding the nucleus, though its density, if I may so call it, rapidly diminishes as the distance from the nucleus increases. This „atmosphere" can be the seat of certain changes comparable with wave motions which SCHRÖDINGER determines by means of a properly chosen equation; he calls this the wave equation. From it he deduces numbers equal to the values of the energy in BOHR's stationary states. Furthermore he has been able to assign to any point of the atmosphere a certain quantity which may be considered as the density of an electric charge. Its values are such that the total charge is equal to that which formerly was attributed to the electron, so that now, at distant points, the action of the nucleus is neutralized by that of the atmosphere. Finally, the wave equation shows that there can be cases in which the distribution of electric charge is not invariable but fluctuates periodically with a frequency just equal to that of the radiations emitted by the atom. These fluctuations can give rise to electromagnetic waves much like those which we produced by the motion of charged bodies. If all this is true we really have here, in a formerly unexpected form, what we wanted to attain in the classical theory, a radiation that is the direct consequence of real periodic changes in the atom.

SCHRÖDINGER's theory has a great beauty and I fear I did but poor justice to it in my attempt to present part of it in plain words. By a curious coincidence it is in many respects mathematically equivalent to the dynamics of matrices so that in their further development the two theories have many times gone side by side. SCHRÖDINGER has developed his ideas to a considerable extent and has applied them to many interesting questions. Yet, with all due appreciation, I must lay stress upon some outstanding difficulties, which, I must not forget to add this, SCHRÖDINGER himself perfectly realizes. The first is this. The wave equation expresses the influence which, in virtue of its electric charge, the nucleus has on the changes going on in the atmosphere. Now, since the charge distributed over the atmosphere is of the same nature as that of the nucleus, only with the opposite sign, and as similar actions go out from it, because, at a distant point, it counterbalances the force exerted by the nucleus, we should expect some mutual action between the different parts of the atmosphere, one part having, by its charge, an influence on what takes place in the other part. These mutual actions, however, have been formally excluded. If we took them into account, we should no longer find BALMER's series.

Doubts may also be raised as to the spreading out of the charge of the electron over a space of the dimensions of the atom. A distribution of this kind could, if it existed, be hardly limited to the hydrogen atom. Now, there are many cases in which a charge can be removed from an atom and in which after its expulsion it behaves as the ordinary free electrons. It would therefore become necessary to assume that, according to circumstances, the charge may be either concentrated in the shape of an electron, or much more widely diffused.

I need scarcely add that in making these remarks I do not in the least mean to disparage the value of the new theories from whose further development certainly much may be hoped.