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Physics in Perspective

Nordström, Ehrenfest, and the Role of Dimensionality in Physics

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In the summer of 1916, Finnish physicist Gunnar Nordström (1881–1923) arrived in Leiden to carry out research with Paul Ehrenfest (1880–1933), Hendrik A. Lorentz's successor in the chair of theoretical physics. Nordström had recently published the first five-dimensional unified model of the universe, a theory that went virtually unnoticed by the physics community. Ehrenfest's personal journals reveal that Nordström's visit coincided with a flowering of Ehrenfest's own interest in dimensionality, which resulted in his well-known paper on the connection between the fundamental laws of physics and the three-dimensionality of space. I examine Nordström's and Ehrenfest's collaboration and explore the relationship between their ideas and the Kaluza-Klein model of five-dimensional unification.

Key words: Gunnar Nordström; Paul Ehrenfest; Theodor Kaluza; Oskar Klein; Kaluza-Klein theory; extra dimensions; compactification.

Introduction

The recent revival of Kaluza-Klein theory has prompted the realization that Theodor Kaluza (1885-1954) was not the first to propose a five-dimensional unified theory of gravitation and electromagnetism in 1921.[1] Kaluza-Klein theory, based on Kaluza's proposal and subsequent papers by Oskar Klein (1894–1977) in 1926,[2] postulates an extension of the general relativistic metric to accommodate the vector potential for the electromagnetic force. In Klein's version, the scale of the extra-dimension is small and compact and is related to the quantization of electric charge. Variations of the theory were developed by Albert Einstein (1879–1955), Peter Bergmann (1915–2002), Wolfgang Pauli (1900–1958), Pascual Jordan (1902–1980), and others. In recent decades, higher-dimensional supergravity, superstring, and M-theory models have incorporated some of its elements.

Abraham Pais (1918–2000) pointed out in 1982 that the Finnish physicist Gunnar Nordström (1881–1923, figure 1) had "proposed to use a five-dimensional space for the unification of electromagnetism with a *scalar* gravitational field" in 1914,[3] fully seven years before Kaluza published his proposal. Pais's comment and the subsequent repub-

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Fig. 1. Gunnar Nordström (1881-1923). Courtesy of the Department of Physical Sciences, University of Helsinki, Finland.

lication of Nordström's paper in English and its republication have spurred a growing recognition of the significance of Nordström's contribution to higher-dimensional unification.

Nordström's prequel to Kaluza-Klein theory raises several historical questions. First, why is there no public record (for instance, citations or comments) by Nordström's contemporaries reacting to his proposal? Second, in the absence of such public recognition, were there private communications between Nordström and subsequent unifiers such as Kaluza, Klein, or Einstein? Third, even if such direct communications did not occur, did Nordström's work exert any indirect influence on that of his contemporaries?

Several factors come into play in attempting to answer these questions. One concerns the conceptual relationship between Nordström's work and Einsteinian general relativity. A second involves the personal relationship between Nordström and Einstein. A third is Nordström's isolation from the mainstream of the European community of physicists during various stages in his career. Finally, we must take into account Nordström's premature death and its effect on the reception of his theory.

Nordström's Five-Dimensional Theory

Nordström's five-dimensional theory likely received little attention at the time because of its misfortune in being published just as Einstein was completing his general theory of gravitation. Einstein's theory involved relationships between tensor quantities within the context of a Riemannian curved space-time manifold, while Nordström's was a simpler vector-scalar theory in a five-dimensional extension of Minkowski flat space-time. By combining Maxwell's equations of electromagnetism with a scalar, Lorentz-covariant description of gravity, Nordström's theory did not encompass the deep connection between geometry and matter that characterized general relativity, nor did it include the property of general covariance (in contrast to Kaluza's later five-dimensional tensor model, which was based upon a generalization of general relativity). Nor could Nordström's theory reproduce the predictions of the bending of light and the precession of the perihelion of Mercury's orbit that eventually would confirm Einstein's general theory of relativity. As John D. Norton has noted, Einstein's announcement of the latter prediction in November 1915 "set new standards of empirical accuracy for gravitational theories," and consequently Einstein's theory "within a few years … [would] eclipse all others."[4] Nordström's theory could not pass these new litmus tests and thus became archaic virtually as soon as it was published.

Nordström himself soon recognized the superiority of Einstein's general theory of relativity over his own five-dimensional theory, and he therefore published no further papers on it; instead, he subsequently worked exclusively within the context of Einstein's theory, making vital contributions to the characterization of the general relativistic conservation of energy and momentum.[5] Most famously, in 1918 he developed a stationary solution to Einstein's equations with an electric field independently of that of the German physicist Hans Reissner (1874–1967);[6] their description of a charged black hole is now known as the Reissner-Nordström solution. Nordström gained recognition among general relativists for this and other contributions, and not for his five-dimensional theory.

Since Nordström himself abandoned his five-dimensional theory, it probably is not surprising that Einstein never cited it. In 1919, for example, when Kaluza first contacted Einstein to inform him about his work, Einstein made no reference to Nordström's earlier theory in his response, but instead compared Kaluza's unified model to the one espoused by Hermann Weyl (1885–1955), who attempted to achieve unification by redefining the relativistic concept of length. Einstein took issue with both Kaluza's and Weyl's theories,[7] but recognized their mathematical elegance; he considered them flawed but noteworthy attempts to extend general relativity beyond a theory of gravitation by incorporating electromagnetism into it. Later, Einstein often referred to their theories, as well as to a related one proposed by Arthur S. Eddington (1882–1944).[8] For example, Einstein wrote to Kaluza in 1925 that, "I am still of the opinion that your idea to construct a relation between electricity and gravitation is of great originality and merits the serious interest of academic colleagues. Besides the Weyl-Eddington idea it is the only attempt to be taken seriously in that direction."[9] If Einstein was aware of Nordström's earlier theory, he ignored it completely, preferring instead to focus on tensor approaches to unification.

Einstein also may have had personal reasons to ignore Nordström's five-dimensional theory, since by 1919 the two theorists were on less than cordial terms with each other.[10] In that case, Einstein would have had little personal as well as little scientific motivation to examine Nordström's earlier publication, let alone to pass information on it to Kaluza, who apparently never became aware of Nordström's primitive five-dimensional theory. Nor did Klein ever refer to it. It went largely unnoticed until its rediscovery in the 1980s.

The Geographical Factor

The geographical factor played a critical role in Nordström's career. Born in Helsinki (Helsingfors), Finland, on March 12, 1881, Nordström received his higher education at the University of Helsinki at a time when theoretical physics was unheard of in his home country, so he began his studies in engineering. Then, aspiring to become a physical chemist, he left in 1906 to go to the University of Göttingen to work with Walther Nernst (1864–1941). As fate would have it, however, he attended the lectures of Hermann Minkowski (1864–1909) just when Minkowski was beginning to explore Einstein's special theory of relativity in terms of four-dimensional space-time. Nordström became fascinated with Minkowski's formulation and decided to switch his career path once again. Returning to the University of Helsinki in 1907, he became increasingly frustrated by the absence of others there who shared his commitment to theoretical physics, so he sought opportunities for collaboration with theoretical physicists in other European cities. He had to struggle hard to obtain travel grants. In response to one of his requests, he was bluntly informed, "One can study the fourth dimension at home, without any trips abroad."[11]

Nordström nevertheless managed to travel abroad on several occasions. In the summer of 1913, he traveled to Zurich under the sponsorship of Einstein. Einstein then was exploring various approaches to a relativistic theory of gravitation and was open to considering the scalar gravitational theory that Nordström was working on (and which was a precursor to his five-dimensional theory). Einstein suggested substantial improvements, and Nordström reworked his theory.[12] That September, at a meeting of the Society of German Natural Scientists and Physicians (Gesellschaft Deutscher Naturforscher und Ärtze) in Vienna, Einstein even presented a modified version of Nordström's work as a reasonable alternative to his own ideas,[13] which he had developed in conjunction with Marcel Grossmann (1878–1936). Experimental evidence, Einstein felt, would be the arbiter of the two designs.The following year, Einstein and the Dutch physicist Adriaan Fokker (1887–1972) published an article in which they pointed out that Nordström's flat, scalar gravitational theory could be better understood within the context of curved space-time.[14] Einstein apparently lost interest in Nordström's contributions only later, after he had fully developed his general theory of relativity.

Nordström's next opportunity to venture into the heart of Europe came in 1916, two years after he had published his five-dimensional proposal. This time he journeyed to Leiden where Paul Ehrenfest (1880–1933, figure 2) had established a leading center of theoretical physics. Ehrenfest relished inviting physicists from far and wide to Leiden to share ideas in daily discussions and weekly seminars. Many of them would stay in his house, a yellow stucco mansion just beyond the city center with a shady garden and pleasant brook running behind it (figure 3), where Ehrenfest could pamper his guests

Fig. 2. Paul Ehrenfest (1880–1933). *Credit*: American Institute of Physics Emilio Segrè Visual Archives, Margrethe Bohr Collection.

while probing their knowledge of various questions in theoretical physics. Also shaping the intellectual climate in Leiden was the esteemed Dutch theoretical physicist Hendrik A. Lorentz (1853–1928), who actively encouraged critical analysis of gravitation.[15] Nordström found an ideal opportunity in this nurturing atmosphere to discuss pressing issues in theoretical physics.

Nordström in Leiden

Ehrenfest kept a scientific notebook, now preserved in the Museum Boerhaave in Leiden, in which he recorded his thoughts and impressions on research topics that piqued his interest. These often took the form of questions that he then would try to answer. As his biographer Martin J. Klein has remarked, “He was the world’s champion questioner in physics.”[16]

On July 6, 1916, Ehrenfest recorded in his notebook: “Nordström has been with us for two weeks.”[17] Three days later, he made an entry that indicated the likely nature of their initial discussions: “In three-dimensional space, Newtonian planetary orbits are closed. Are they also closed in non-Euclidean three-dimensional space?”[18] This entry, followed by another one regarding the wave equation in non-Euclidean space, offers the first indication of Ehrenfest’s interest in a critical question that would lead to his

Fig. 3. Paul Ehrenfest's house in Leiden, The Netherlands. Photograph by the author.

pioneering paper, "In what way does it become manifest in the fundamental laws of physics that space has three dimensions?"[19] Nordström's interest in higher-dimensional theories strongly suggests his influence on the timing of Ehrenfest's first inquiries into this topic.

The nature of Nordström's and Ehrenfest's collaboration is further indicated by a subsequent notebook entry. On August 11, 1916, Ehrenfest wrote:

> On a warm evening in the garden, I spoke on and off with Nordström about:
> Generalizing the electrodynamic equation to *N*-dimensional space....
> The connection [in] gravitational theory between electrical charge and gravitational mass....
> Hyper-Maxwell-Theory in R_4 [and] different generalizations in R_n.[20]

We have here solid indications of the detailed discussions between Nordström and Ehrenfest on their attempts to develop a unified theory of gravitational and electricity in higher-dimensional space-time. Nordström apparently conveyed his interest in such a theory to Ehrenfest, who eagerly began to pursue its implications, as we shall see.

During his visit to Leiden, Nordström met and married Cornelia van Leeuwen, one of Lorentz's students. In 1918 he returned to Finland with his wife, where he was appointed to a professorship at the Helsinki University of Technology. He thus was again isolated scientifically from the leading centers of research in Europe, just when Kaluza in Königsberg reawakened interest in the fifth dimension. Nordström taught general relativity in Helsinki and was a leading advocate of Einstein's work, nominating him twice for the Nobel Prize.[21] Nordström died prematurely in 1923, "probably as a result of his earlier careless experimental work with radioactive substances."[22]

Ehrenfest and the Dimensionalifty of Physical Laws

Ehrenfest's fascination with the topic of dimensionality also can be traced to earlier sources, including the lectures of Felix Klein (1849–1925) and David Hilbert (1862–1943) that he heard at the University of Göttingen during 1902–1904, and the discussions he had there with Tatyana Alexeyevna Afanassjewa, a Russian geometrist, whom he met and married in Göttingen in 1904. One indication of the strong interest of the two Ehrenfests in higher dimensions is that their oldest daughter Tatyana, home-schooled by her parents, kept her dolls in a cardboard model of a hyperboloid.[23]

Nevertheless, Nordström's visit to Leiden apparently provided the stimulus for Ehrenfest to attempt to resolve a long-standing mystery, one that Aristotle, Ptolemy, Immanuel Kant (1724–1804) and others had addressed: Why is the observed universe three-dimensional? Kant, in particular, had examined this question in his first treatise, "Thoughts on the True Estimation of Living Forces," in which he uncovered a relationship between the form of Newtonian gravitation and the dimensionality of space.[24] Now it was Ehrenfest's turn to explore this question.

Ehrenfest mentioned in his notebook various forays he made into the mathematics of hyperspace, including an unusual modification of the Pythagorean theorem with the squares of the symbols replaced by cubes. He noted a revolution in St. Petersburg, Russia, in 1916 – curiously lodged among his mathematical speculations. Then, on May 3, 1917, he recorded his conclusions on the dimensionality of physical laws. He found certain inevitable outcomes when generalizing Riemannian three-dimensional space (R_3) to higher dimensions. One, related to Kant's "Thoughts," concerned planetary motion, gravitation, and three-dimensional space. He wrote: "Only in R_3 are planetary orbits ultimately possible."[25] In other words, for spaces other than three-dimensional (or space-times other than four-dimensional), planets cannot move around the Sun in closed Keplerian elliptical orbits; instead, they must follow unstable, spiral trajectories.

Ehrenfest came to a related conclusion in regard to the Balmer and other spectral lines predicted by the Bohr model of the atom. He noted that a finite array of atomic-energy levels can exist only if the electron moves in three spatial dimensions; in higher dimensions, the Bohr atom cannot have either a ground-state energy level or a finite range of excited levels, thus forbidding the observed spectral patterns. Ehrenfest also found that in other than three dimensions the wave equation always would entail dispersion, so that when traveling either in a vacuum or through matter the waves then cannot propagate in pulses or groups, which would essentially make the transmission of energy or information impossible.

In short, if the physical universe were other than three-dimensional, then many of its dynamical aspects would be unstable, and humans simply could not exist. Conversely, since we do live in a stable universe that is subject to physical laws, we know that it must be three-dimensional.

Although Ehrenfest's paper was published in the respected *Proceedings of the Royal Academy of Sciences in Amsterdam* (*Koninklijke Akademie van Wetenschappen te Amsterdam*), his findings were scarcely noticed by the physics community, perhaps because of the disruption of scholarly communications owing to the Great War, when many European libraries found it difficult or impossible to obtain foreign journals. Eventually, in 1920, his paper was republished in German in the *Annalen der Physik*.[26]

Ehrenfest's Research on Higher-Dimensions

Ehrenfest maintained a strong interest in the question of dimensionality throughout the 1920s. His student George Uhlenbeck (1900–1988) noted that whenever Ehrenfest explored a theory he immediately would press for its generalization to higher dimensions and attempt to determine the resulting change in its behavior. For example, after Ehrenfest and Harry Bateman (1882–1946) published their paper, "The Derivation of Electromagnetic Waves from a Basic Wave-Function,"[27] Ehrenfest wanted Uhlenbeck to help him generalize it. "Immediately," Uhlenbeck recalled, "he had to do it in n-dimensions because it was typical of Ehrenfest that he was always interested in the fact of dimensions."[28] Uhlenbeck also pointed out that Ehrenfest was interested in exploring differences between odd and even-dimensional spaces. The two worked together on this topic in 1925. "That was *very* important for him, you see," Uhlenbeck reported. "Why is Huygens' principle not valid in even dimensions?"[29]

Ehrenfest's wife Tatyana was similarly interested in the properties of higher- dimensional models. She apparently considered employing a fifth dimension to resolve dilemmas with Bohr's atomic model in early 1922, as Einstein revealed in a skeptical comment in a letter to Ehrenfest: "Her [Tatyana's] opinion that the quantum problems may be healed by help of a fifth dimension is not intelligible for me."[30]

Ehrenfest learned about Oskar Klein's five-dimensional work in 1926 and invited him to Leiden that June. Ehrenfest and Uhlenbeck then strongly supported Klein, discussing the topic virtually every day with him that summer. Klein honed his ideas and wrote his pioneering papers on five-dimensional unification and quantum physics.[31] Ehrenfest was particularly impressed with Klein's mechanism for explaining the smallness of the fifth dimension, a process now known as compactification. By reducing it to a tiny circle of undetectable (Planck-scale) proportions, Klein overcame the obstacles that Ehrenfest first addressed in his paper of 1917. It turned out that one could secure the benefits of an extra dimension without having to face the hurdles posed by altering accepted physical laws.

Ehrenfest's interest in higher dimensions continued, as can be seen in his correspondence with Einstein. In the late 1920s, Einstein became excited by the prospect of a five-dimensional theory that would encompass quantum mechanics within a fully deterministic framework, writing to Ehrenfest: "I think that Kaluza-Klein has correctly indicated the right way to proceed. Long live the fifth dimension."[32]

Conclusion

In the second and third decades of the 20th century, Nordström, Kaluza, and Klein advanced independent proposals for unification by means of a fifth dimension. I have not uncovered a direct causal connection between Nordström's proposal and Kaluza's and Klein's later work, but I have shown that Nordström's visit to Leiden in 1916-1918 evidently heightened Ehrenfest's interest in dimensionality. In particular, I have indicated how Nordström's visit coincided with Ehrenfest's examination of the consequences of extra dimensions on physical laws, how Ehrenfest continued to explore this topic, and how he lent crucial support to Klein's work on it in 1926. We thus see that there likely was an indirect connection between Nordström's theory and Ehrenfest's subsequent advocacy of Kaluza-Klein theory.

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MISSING LINK

Man's a kind
of Missing Link,
fondly thinking
he can think.

Piet Hein