Prof. Dr. E. COHN *and* Dr. P. ZEEMAN. *Observations concerning the propagation of electrical waves in water.*

The greater part of the following observations were made by us jointly in Strassburg in the summer of 1893. One of us (P. Z. in Leiden) has filled up a blank in the observations then made by some new series of observations. According to the original plan the investigations made were preliminary, being introductions to questions, relating to the behaviour of *conductors*, and as yet not treated. We infer however from some publications made of late that also the results obtained for *pure water* may be useful.

Method.

The method is closely related to the one exposed in the paper ,,on the propagation of electrical vibrations in water" [1]). It is necessary to refer to this paper for the disposition of the experiments. We desired to make the method as accurate as possible. Our attention was especially directed to three points, viz: In the first place it is assumed in the calculation of the

[1]) E. Cohn, *Berliner Berichte*, 3 December 1891; *Wied. Ann.* 45 p. 370 (1892).

refractive indices, that the waves are travelling through a medium unlimited on either side. However in the first place a lateral boundary is practically necessary. If this boundary has the form of two concentric cylinders, it is yet possible to solve the resulting mathematical problem: it appears, that for the required frequencies and geometrical relations, the electrical forces are almost normal to the axis of the cylinder, and that the velocity of the waves is sensibly equal to that of the wave in the infinite medium. One may infer, that the same holds good if the waves are propagated along two parallel metallic wires, and if the only lateral boundaries, besides these wires, of the traversed dielectric (the water) are *metallic* plates, parallel to the axes of the wire. If however for the latter is substituted a different insulator (the material of the basin and then the air), only the experiment can decide, to what degree the case of the infinite wave is realized. Our first experiments were undertaken with the view to decide this question. They related to waves, half a wavelength being $l_0 = 188$ cm. The water was contained [1]) in a basin, the length of which was 66 cm. and the breadth 39 cm., filled in different sets of experiments to the height

$$h = 18,\ 22,\ 28.5 \text{ cm.}$$

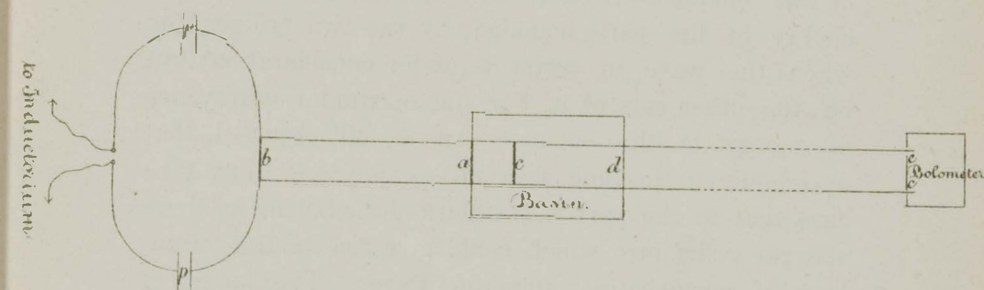
the wires being always 10 cm. above the bottom. A

[1]) The following numbers refer to *all* the series of observations, series 1 of the table given below excepted; in the last the dimensions were: length 51 cm., breath 35.5 cm., height of the water 28 cm., height of the wires 15 cm.

systematic variation of the apparent refractive index dependent on the hulk of the mass of water was not found. Accordingly we desisted from the use of a metallic enclosure, which would have complicated the method of observation. However we must confess, that the totality of the data now given seems to evince a slight variation of the apparent refractive index in the expected direction [1]).

Secondly it was questionable whether by the method of observation used before one was sure of the existence in water of *one pure* vibration, determined by the wirelength *ab* in air [2]).

Accordingly [3]) we made the first part *p b p* of the "secondary conductor" congruent to the "primary", *fastened the bridge b*, and determined *the position of a* by resonance.



[1]) Vide the end!
[2]) Cf. figure and substance of the cited paper.
[3]) Cf. for the following the subjoined figure.

Nevertheless it appeared sometimes from an exploration of the vibrations on the other side of *a* in air, (before the basin was in its place), that here several waves were superposed. We did not succeed in every instance to find the cause of the disturbance; we have however made measurements in water only for these waves, which gave a pure vibration with a sharply defined maximum.

A last objection relates to the disturbances, which the regular shape of the waves must necessarily undergo through the outer coats and the leads of the little "Leiden jars" which collect the energy of the vibrations and transmit it to the bolometer. It is possible to avoid the jars and *to remove entirely the measuring apparatus from the wave to be measured,* by arranging the parrallel wires, along which the wave is propagated, and their prolongation so, that they terminate directly in the bolometer. Now, one does not measure the energy in the part bounded by the two bridges, (*ac* when the wave in water is under consideration) but on the other side of it. For the maximum conveyance of energy to the bolometer it is still required, that *ac* schould be in tune with *ba*, -- only one must take care that in the part between *c* and the bolometer there are no reflexions, which could give rise to the formation of perturbating standing waves. The danger of such reflections exists in two places: First at the bolometer itself. If it is used in the manner generally adopted, according to the statements in the literature of the subject, it collects energy from the vibrations travelling through space, entirely independent of the

leads connected with the wires. We did not obtain suitable results before we had put the bolometer in a metallic enclosure; however at *e*, where the (insulated) leads pass through this enclosure, there now occur strong reflections of the incident waves. The reflected wave is again reflected at *c*. The delivery of energy in the bolometer is then generally determined not only by the length *ac*, but also by *ce*. These reflexions are however rendered harmless according to BJERKNES' principle by the introduction between *c* and the bolometer of sufficiently long pieces of wire (varying according to the wavelength to 100 M.), so that on account of the powerful damping of the vibrations the resonance of the end *c e* is of no consequence.

Secondly there is on account of the high refractive index of water a powerful reflexion at *d* where the wave passes from the water into the air. This reflexion may, as with the former method of observation, prevent us from finding the maximum sought for, if to the length *cd* corresponds *nearly* the same period, as to *ac* and *ab* (or a multiple), h. e. if the oscillation period for the whole length traversed in water *ad* is *nearly* a multiple of the oscillation period for the chosen air length *a b*. However there is reversal of the phase, if the reflection takes place at the bridge *a*, if the reflection takes place in the water from air there is no reversal of the phase of the electric intensity; hence follows: *full* resonance exists between *ab* and *cd*, if *ad* is $^1/_2$, $^3/_2$, $^5/_2$. . . . . . half wavelengths.

Consequently half wavelengths *ab*, which would suffice *approximately* to these conditions are to be avoided.

(This had escaped us in one of the series of observations in 1893, $l_o = 188$ cm.; this series has been replaced by the new series 1 and 2).

If the measurements without the jars were undertaken with the here mentioned precautions, and if on the other hand coats of the jars of sufficient smallness, were chosen, it was impossible to establish a measurable difference in the results of the two methods of observation.

Of the comparative measurements, relating as well to the waves in air, as to the waves in water, one of the latter has been entered in the table under n⁰. 3. The jars were glass tubes 0,1 cm. in thickness, closely surrounding the parallel wires, thick 0,2 cm., and distant 7 cm.; the coats were $1\,^3/_4$ turns *at most* of 0,05 cm. thick copper wire.

SUMMARY OF THE RESULTS OF THE OBSERVATIONS [1].

Series N⁰.	Place a. Time.	l_o	l_w	n	h	θ	with or without jars.
1	Leiden 1895	155.5 (3)	17.4 (0.4) 17.5 (1.2) 17.3 (1.5)	8.98	v. pag. 2	19.2	with
		155.2 (3)	17.7 (0.6) 17.4 (0.8) 17.4 (1.0)	8.90	18	18.7	with.
2	d⁰.	»	17.4 (0.6) 17.4 (0.8) 17.4 (1.0)	8.95	22	18.7	with.
		»	17.3 (0.6) 17.3 (0.8) 17.3 (1.0)	8.99	28.5	18.2	with.
3	Strassburg 1893	341.5 (4)	38.7 (0.6)	8.89	22	20.2	without.
		»	39.0 (0.6)	8.86	»	22.0	with.
4	d⁰.	376.0 (4)	42.9 (0.6)	8.85	18	21.2	without.
		»	42.7 (0.6)	8.89	22	21.3	without.
		»	42.5 (0.6)	8.93	28.5	21.1	without.
5	d⁰.	562.0 (4)	63.9 (0.6)	8.89	18	22.0	without.
		»	63.6 (0.6)	8.94	22	22.3	without.
		»	63.4 (0.6)	8.97	28.5	22.3	without.

The table has been calculated in the following manner: Let b denote the position of the bridge in air, a the boundary of the water and the bridge in that place; c, c' c'' the bridges in water (only *one* being present at a time); then the parts $b\,a$, $a\,c$, $a\,c'$, $a\,c''$

[1]) All lenghts in cm.

are directly observed, the exact position of the bridge being at every turn found in this manner: one compares the deflections of the bolometer for three aequidistant positions, and varies these positions, always making the distances as small as possible, until the deflections for the two exterior positions are equal inter se and yet distinctly smaller than the deflection for the medium position. In the columns l_o and l_w are in parenthesi ( ) the values of the used lateral displacements.

From the measured lengths follows.

$$l_o = ba + \delta \quad l_w = ac + \delta = cc' = c'\ c''$$

δ being the wire length 'equivalent' to the bridge. This can be determined for waves in water from each of the sets of observations in brackets } { of the series 1 and 2. The values entered under l_w in these sets are always calculated successively as

$$ac + 4.5,\ cc',\ c'c''.$$

In the same manner δ has been determined for a great many air waves, l_o between 200 and 600 cm., from

$$l_o = ba + \delta = aa' = a'a'',$$

$a,\ a',\ a''$ being the several positions of the bridge, giving resonance. Always δ was found between 4 and 5 cm. Hence for *all* wavelengths has been accepted:

$$\delta = 4.5$$

Further follows the refractive index at the temperature ϑ of the water

$$n' = {}^{l_o}\!/_{l_w}$$

Thence we have calculated the refractive index n *at* 17° C by means of the temperature coefficient determined by Heerwagen [1])

$$n = n' + 0.0201\ (\vartheta - 17).$$

The conductivity of the water was in all series of observations 5 to 10.10^{-10} (mercury = 1); the single values being of no consequence [2]) have not been entered in the table.

Discussion.

From the results of the observation an answer to both the following queries can be abstracted:

1. is the refractive index in the domain of the used frequencies a constant? and if this is the case,

2. is this constant equal to the square root of the specific inductive capacity, resulting from experiments with *stationary* electric fields?

The third of the series of observations 1 to 5 of the table must serve as an example, that the two methods of measurement, with and without jars, give identical results. In fact the difference of the measured wavelengths, the difference of temperature being taken into

[1]) *Wied. Ann.* 49. p. 279. 1893.
[2]) Vide E. Cohn. l. c.

account, only amounts to 0.15 cm. Besides the series is in full harmony with the middle set of series 4, to which belongs the same h and nearly the same l_o, though it was made at another time and with another primary vibration. — The series 1, differing from the others by different dimensions of the bulk of the water, (approximately the same as the *last* sets of the series 2, 4, 5) must support the result of series 2, and especially the value of the bridge correction $\delta = 4.5$.

For answering the above questions then remain the series 2, 4, 5, of each of which are made three sets under otherwise equal circumstances for the frequencies. $\nu = 97, 40, 27$ millions of entire vibrations. The three sets differ by the depths of the water:

$$h = 18,\ 22,\ 28.5 \text{ cM.}$$

Summarising the values of n are for

$\nu/10^6$ \ h	18	22	28.5
97	8.90	8.95	8.99
40	8.85	8.89	8.93
27	8.89	8.94	8.95

Allowing first the possibility, in accordance with the remark on pag. 2, that the velocity of propagation depends on the value of h, we have for answering the first question to regard the numbers of the several columns separately.

Taking $h = 18, 22, 28.5$ and for each ν the mean value: $n = 8.87, 8.92, 8.94$, all errors in the observed l_w appear to be inferior to 0.15 cM. Accidental errors of this value cannot be considered as non-existing; hence it follows:

1°. In the region of frequencies between 27 and 97 millions *dispersion cannot* be demonstrated. The greatest difference between the observed refractive indices becomes 0.06, equal to $^2/_3$ per cent.

Seeking to represent *all* observations by *one* value of n, the most favourable value is found to be

$$n = 8.91 \text{ at } 17^\circ\ C,$$

with this the errors in the observed l_w are calculated:

+ 0.0	— 0.1	— 0.2
+ 0.3	+ 0.1	— 0.1
+ 0.1	— 0.2	— 0.3.

Also these deviations *may* be accidental. However in all the three lines the values of the n's rise with ascending h, and this suggests the suspicion, that the systematic error mentioned on pag. 2 has become effectual. If it exists, and if it is the only one, the real value of n must lie *above* the observed one. Then however this error is not inherent to our special method of observation, but according to the geometrical dimensions in a yet *higher* degree to *all* other methods in which refractive indices and specific inductive capacities are determined from lengths of electrical waves. According to the statical method and with an accuracy unattainable in the measurement of velocities of propagation, HEERWAGEN [1]) has found the square root of the specific inductive capacity

$$n = 8{,}99 \text{ at } 17^\circ\ C.$$

We believe, that it is allowed to conclude:

2°. the refractive index of water for vibrations the frequency of which is less than 100 millions per second, is equal to the square root of the specific inductive capacity — as may be stated with the highest degree of certainty, hitherto attained. If this certainty seems insufficient, the method may be improved according to what has been said on pag. 2; in the other case we should consider as the most accurate value of the refractive index, as well as of the spec. induct. capacity, the one obtained by the statical method.

---

[1]) l. c.

22

COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES.

N°. 22.

EDUARD IJDO — PRINTER — LEIDEN.