before (7) and the sodium lines remained visible continuously\(^1\). One now can wait till the density of the sodium vapour is the same at various heights.

By rotating the tube continuously round its axis, I have still further advanced this. The absorption lines now are equally broad from the top to the bottom. If the electromagnet was put on, the absorption lines immediately widened along their whole length. Now the explanation in the manner of (8) fails.

10. I should like to have studied the influence of magnetism on the spectrum of a solid. Oxide of erbium has, as was found by BUNSEN and BAHR the remarkable property of giving by incandescence a spectrum with bright lines. With the dispersion used however the edges of these lines were too indistinct to serve my purpose.

11. The different experiments from 3 to 9, make it more and more probable, that the absorption- and hence also the emission-lines of an incandescent vapour, are widened by the action of magnetism. Hence, if this is really the case, then by the action of magnetism in addition to the free vibrations of the atoms, which are the cause of the line spectrum, other vibrations of changed period appear.

I hope to decide by future investigation whether it is really inevitable to admit this specific action of magnetism.

\(^1\) Pringsheim l. c. p. 456.

Dr. P. ZEEMAN. On the influence of magnetism on the nature of the light emitted by a substance. (Part II.)

12. From the representation I had formed to myself of the nature of the forces, acting in the magnetic field on the atoms, it seemed me to follow that with a band-spectrum and with external magnetic forces the phenomenon I had found with a line-spectrum would not occur.

It is however very probable that the difference between a band- and a line-spectrum is not of a quantitative but of a qualitative kind\(^1\). In the case of a band-spectrum the molecules are complicated, in the case of a line-spectrum the widely dissociated molecules contain but a few atoms. Further investigation has shown that the representation I had formed of the cause of the widening in the case of a line-spectrum, in the main was really true.

13. A glass tube closed at both ends by glass plates with parallel faces, was placed between the poles of the RÜHMKÖRFF electromagnet in the same manner as the tube of porcelain in § 7. A small flame under the tube vapourized the iodine, the violet vapour filling the tube.

\(^1\) Kayser in Winkelmann’s Handbuch II. l. p. 421.
By means of electric light the absorption spectrum could be examined. As the temperature is low this is the band-spectrum. With the high dispersion used, one sees in the bands a very great number of fine dark lines. If the current round the magnet is being closed, one observes, contrary to what the experiments with sodium vapour teach, that no change in the dark lines is observed.

The absence of the phenomenon in this case supports the explanation, that even in the first experiment, with sodium vapour (§ 7) the convection currents have had no influence. For in the case now considered, the convection currents originated by magnetism, which I believed to be possible in that case, apparently are insufficient to cause a change of the spectrum, and though I could not see it in the appearance of the absorption lines (cf. § 7) also the band-spectrum is like the line-spectrum very sensible to changes of density and of temperature.

14. Although the means at my disposal did not enable me to execute more than a preliminary approximate measurement, I yet thought it of importance to determine approximately the value of the magnetic change of the period.

The widening of the sodium lines to both sides amounted to about $\frac{1}{40}$ of the distance of the said lines, the intensity of the magnetic field being about $10^4$. Hence follows a positive and negative magnetic change of $\frac{1}{1000}$ of the period.

15. The train of reasoning, mentioned in (1) and by which I was induced to search after an influence of magnetism was at first the following. If the consideration is true that in a magnetic field a rotatory motion of the ether is going on, the axis of rotation being the direction of the magnetic forces (Kelvin and Maxwell) and if the radiation of light may be represented as caused by the motion of the atoms, relative to the centre of mass of the molecule, revolving in all kinds of orbits, suppose for simplicity circles, then the period or what comes to the same, the time of describing the circumference of these circles will be determined by the forces acting between the atoms and then deviations of the period to both sides will occur by the influence of the perturbing forces between ether and atoms. The sign of the deviation of course will be determined by the direction of motion, as seen from out the lines of force. The deviation will be the greater the more the plane of the circle approximates a position perpendicular to the lines of force.

16. Somewhat later I elucidated the subject by representing to myself the influence exercised on the period of a vibrating system if this is linked together with another in rapid rotatory motion. Lord Kelvin, now 40 years ago 1), gave the solution of the following problem. Let the two ends of a cord of any length be attached to two points at the ends of a horizontal arm made to rotate round a vertical axis through its middle point at a constant angular velocity, and let a second cord bearing a material point be attached to the middle of the first cord. The motion now is investigated in the case, the point is infinitely little disturbed from its position of equilibrium. With great angular velocity the solution becomes rather simple. Circular vibrations of the point in contrary directions have slightly different

periods. If for the double pendulum we substitute a luminiferous atom and for the rotating arm the rotational motion about the magnetic lines of force, the relation of the mechanical problem to our case will be clear.

It needs not to be proved that the above mentioned considerations are at most of any value as indications of somewhat analogous cases. I however communicate them because they were the first motive of my experiments.

17. A real explanation of the magnetic change of the period seemed me to follow from Prof. Lorentz’s theory 1).

In this theory it is accepted that in all bodies, small electrically charged ponderable particles are present, that all electric phenomena are dependent upon the configuration and motion of these "ions" and that the light vibrations are vibrations of these ions. Then the charge, configuration and motion of the ions completely determine the state of the ether. The said ion, moving in a magnetic field, experiences mechanical forces of the kind as above mentioned and which must explain the variation of the period. Prof. Lorentz to whom I communicated these considerations, at once kindly informed me of the manner in which according to his theory the motion of an ion in a magnetic field is to be calculated, and pointed out to me that, if the explanation following from his theory was true, the edges of the lines of the spectrum ought to be circularly polarized. The amount of widening might then be used to determine the ratio of charge and mass to be attributed in this theory to a particle giving out the vibrations of light.

The above mentioned extremely remarkable conclusion of Prof. Lorentz relating to the state of polarization in the magnetically widened line, I have found to be fully confirmed by experiment (§ 20).

18. We shall now proceed to establish the equations of motion of a vibrating ion, when it is moving in the plane of \((x, y)\) in a uniform magnetic field in which the magnetic force is everywhere parallel to the axis of \(z\) and equal to \(H\). The axes are chosen so that if \(x\) is drawn to the east, \(y\) to the north, \(z\) is upwards. Let \(e\) be the charge (in electromagnetic measure) of the positively charged ion, \(m\) its mass. The equation of motions then are:

\[
\begin{align*}
\frac{d^2x}{dt^2} &= - k \frac{\partial^2 x}{\partial t^2} + eH \frac{dy}{dt} \\
\frac{d^2y}{dt^2} &= - k \frac{\partial^2 y}{\partial t^2} - eH \frac{dx}{dt}
\end{align*}
\]  \(1) \)

The first term of the second member expresses the elastic force, drawing back the ion to its position of equilibrium, the second term gives the mechanical force due to the magnetic field.

This is satisfied by:

\[
\begin{align*}
x &= x e^{-\frac{t}{\beta}} \\
y &= \beta e^{-\frac{t}{\beta}}
\end{align*}
\]

provided

\[
\begin{align*}
m s^2 x &= - k \frac{\partial^2 x}{\partial t^2} + eH s \beta \\
m s^2 \beta &= - k \frac{\partial^2 \beta}{\partial t^2} - eH s x
\end{align*}
\]

\(1)\) The equations of relative motion.

Now \( m, k, e, \) are to be regarded as known quantities.

For us the period \( T \) is particularly interesting. If \( H = 0 \), it follows from (3).

\[
s = i \frac{k}{\sqrt{m}} = i \frac{2\pi}{T}
\]

or

\[
T = \frac{2\pi \sqrt{m}}{k}.
\]

(4)

If \( H \) is not 0, it follows from (3) approximately

\[
s = i \frac{k}{\sqrt{m}} (1 \pm \frac{eH}{2k\sqrt{m}})
\]

Putting \( T' \) for the period in this case, we have:

\[
T = \frac{k}{2\pi \sqrt{m}} \left(1 \pm \frac{eH}{2k\sqrt{m}}\right).
\]

(5)

Hence the ratio of the change of the period to the original period becomes:

\[
\frac{eH}{2k\sqrt{m}} = \frac{eH}{m} \frac{T}{4\pi}.
\]

(6)

A particular solution of (1) is that representing the motion of the ions in circles. If revolving in the positive direction (viz. in the direction of the hands of a watch for an observer standing at the side towards which the lines of force are running) the period is somewhat less then if revolving in the negative direction. The period in the first case is determined by the value of (5) with the minus sign, in the second with the plus.

The general solution of (1) proves that by the ions are described besides circles, also elliptical slowly rotating orbits. In the general case, the original motion of the ion having an arbitrary position in space, it is perfectly clear that the projection of the motion in the plane of \((x, y)\) has the same character. The motion decomposed in the direction of the axis of \(z\) is a simple harmonic motion, independent of and not disturbing the one in the plane of \((x,y)\) and hence not influenced by the magnetic forces. Of course the consideration of the motion of an ion now given, is only to be regarded as the very first sketch of a theory of the luminiferous motions.

19. Imagine an observer looking at a flame placed in a magnetic field in a direction such, that the lines of force run towards or from him.

Let us suppose that the said observer could see the very ions of § 16, as they are revolving, then the following will be remarked. There are some ions moving in circles and hence emitting circularly polarized light, if the motion is round in the positive direction the period will for instance be longer than with no magnetic field, if in the negative direction shorter. There will also be ions seemingly stationary and really moving parallel to the lines of force with unaltered period. In the third place there are ions which seem to move in rotating elliptical orbits.

If one desires to know the state of the ether originated by the moving ions, one may use the following rule, deduced by Prof. Lorentz from the general theory. Let us suppose that in a molecule an ion \( P \)—of which the position of equilibrium be \( P_0 \)—has two or more motions at the same time, viz. let the vector \( P_0P \) always be obtained by adding the vectors \( P_0P \) which should occur in each of the composing motions at that moment, then the state in the ether, at a very great distance, in comparison with \( P_0P \), will be obtained by superposing the states, which would occur in the mentioned distinct cases.
Hence it follows in the first place that a circular motion of an ion, gives circularly polarized light in points on the axis of the circle.

Further one may choose instead of the above considered elliptical orbits a resolution more fit to our purpose. One may resolve the motion of the ion, existing before the putting on of the magnetic force, in a rectilinear harmonic motion parallel to the axis of $z$ and two circular (right-hand and left-handed) motions in the plane of $(x,y)$.

The first remains unchanged under the influence of the magnetic force, the periods of the last are changed.

By the action of the grating the vibrations originated by the motion of the ions are sorted according to the period and hence the complete motion is broken up into three groups. The line will be a triplet. At any rate one may expect that the line of the spectrum will be wider than in the absence of the magnetic field and that the edges will give out circularly-polarized light.

20. A confirmation of the last conclusion may be certainly taken as a confirmation of the guiding idea of Prof. Lorentz's theory. To decide this point by experiment, the electromagnet of § 2, but now with pierced poles was placed so that the axes of the holes are in the same straight line with the centrum of the grating. The sodium lines were observed with an eye-

---

1) I saw afterwards that Stonex, Trans. Dublin, IV, endeavours to explain the existence of doublets and triplets in a spectrum by the rotation of the elliptical orbits of the "electrons" under the influence of perturbing forces.

piece with a vertical cross wire. Between the grating and the eye-piece were placed, the quarter undulation plate and Nicol, which I formerly used in my investigation of the light normally reflected from a polarly magnetized iron mirror.

The plate and the Nicol were placed relatively in such a manner, that right-handed circularly-polarized light was quenched. Now according to the preceding the widened line must at one edge be right-handed circularly-polarized, at the other edge left-handed. By a rotation of the analyser over $90^\circ$ the light that was first extinguished must be admitted and vice versa. Or, if first the right edge of the line is visible in the apparatus, a reversal of the direction of the current makes the left edge visible. The cross wire of the eye-piece was set in the bright line. At the reversal of the current the visible line removed! This experiment could be repeated any number of times.

21. A small variation of the preceding experiment is the following. With unchanged position of the $\lambda = \frac{1}{k}$ plate the analyser is turned round. The widened line is then by one revolution twice wide and twice fine.

22. The electromagnet was turned $90^\circ$ in a horizontal plane from the position of § 20, the lines of force now being perpendicular to the line joining the slit with the grating. The edges of the widened line now appeared to be plane polarized, at least so far as the present apparatus permitted to see, the plane of polar-

---

5) Zeeman. Communications Nr. 15.
isation being perpendicular to the line of the spectrum. This phenomenon is at once evident from the consideration of § 19. The circular orbits of the ions being perpendicular to the lines of force are now seen on their edges.

23. The experiments 20 to 22 one may consider as a proof that the light vibrations are caused by the motion of ions, as introduced by Prof. Lorentz in his theory of electricity. From the measured widening (§ 14) by means of relation (6), now the ratio $\frac{e}{m}$ may be deduced. It then appears that $\frac{e}{m}$ is of the order of magnitude $10^7$. Of course this result from theory is only to be considered as a first approximation.

24. It may be deduced from the experiment of § 20 whether the positive or the negative ion revolves. If the lines of force were running towards the grating, the right-handed circularly-polarized rays appeared to have a greater \(^1\) period. Hence in connection with § 17 it follows that the negative \(^1\) ions revolve or at least describe the greater orbit.

25. Especially now the magnetisation of the lines of a spectrum can be interpreted in the theory of Prof. Lorentz, the further inquiry of it becomes very attractive. A series of further questions already present themselves. It seems very promising to investigate the

\(^1\) In this reprint we have corrected the erroneous statement in the first edition of this paper. This correction has been previously mentioned Phil. Mag. July '97 p. 59 and Archives Néerl. d. Sc. exactes et nat. Sér. II, T. I, p. 367, 1898.