Dr. L. H. SIERTSEMA. *On thermal coefficients of aneroids of Naudet.*

§ 1. On the occasion of the determination of the thermal coefficients and index-errors of some aneroids, which I undertook for the sake of observations of altitudes by Prof. MARTIN in the Moluccas, and by Prof. MOLENGRAAFF and others in Borneo, I consulted the extensive literature on this subject. Little more than indications are found there of the causes to which the usually rather great thermal coefficients can be attributed. Three causes are enumerated:

1°. Rise of temperature will cause expansion of the different parts of the instrument. Among others the surface of the vacuum-box will increase, and thus the atmospheric pressure will act more strongly.

2°. The coefficients of elasticity of the metals, of which the box and the spring are made, will decrease with rising temperature. The deflection therefore increases, and will cause a higher reading.

3°. The air, which still remains in the box, will expand by heating, and more counterbalance the exterior air-pressure by which an apparent decrease of the barometric pressure is caused.

A closer examination of the consequences of 1° will show us, that this cause explains but a small part of the thermal coefficient. For this purpose we will examine more in detail the construction of the instrument, in doing which we shall make use of the subjoined sketch of the aneroid. In this sketch PQ represents the vacuum-box, NB the spring, $BGHIK$ the system

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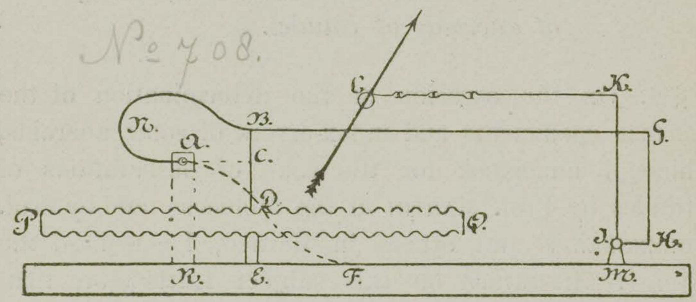

of levers for the transmission of the deflexion of the spring to the index, KL the chain and L the axis of the index. We will now investigate the influence, which the expansion of the different parts of the instrument alone will have on the readings. The changes caused by 2° and 3° will not be taken into account here.

Let us begin with the box. When the rise of temperature is t, the surface of the box will increase in the proportion $1 + 2\,a\,t$, where a is the coefficient of expansion for the box (argentan). The total pressure on the box therefore increases in the same proportion. The force which acts in B in consequence of the air-pressure, will also increase in the proportion $1 + 2\,a\,t$. This force will be balanced by the elastic forces of the spring and the box. If for the sake of simplicity we neglect those of the box, which probably are small compared with those of the spring, and when we besides suppose the

spring wholly unchanged, the deflection s will increase in the proportion $1 + 2\,a\,t$. But the spring does expand, and in order to find the exact deflection, we should have to know the exact relation between the deflection, the elastic force and the dimensions of the spring. By analogy with other phenomena of deflection we may assume, that when all dimensions increase in the same proportion, the deflection will decrease in this proportion. If now b is the coëfficient of expansion for the spring (steel), the total deflection after the rise of temperature will be

$$s\,\frac{1 + 2\,a\,t}{1 + b\,t} = s\,\{1 + (2\,a - b)\,t\}$$

and the increase of the deflection

$$s\,(2\,a - b)\,t.$$

With this amount then is increased the deflection of the point B from the position which it would have if no force were applied. But this last position (zero) will change by the expansion in a way which can be found in the following manner. The brass piece AR will by the expansion rise $A\,R\,c\,t = C\,E\,c\,t$, where c is the coefficient of expansion for brass. For the same reason the point B will rise $C\,E\,c\,t + B\,C\,b\,t$, and we find for the total rise of B

$$-\,s\,(2\,a - b)\,t + C\,E\,c\,t + B\,C\,b\,t.$$

When we take N as the fixed point of the lever NBG, which point rises $C\,E\,c\,t + B\,C\,b\,t$, we find for the rise of G

$$-\,\frac{N\,G}{N\,B}\,s\,(2\,a - b)\,t + (C\,E\,c + B\,C\,b)\,t$$

Yet there is another cause by which G will move. Fixed to the spring, which can turn in A, we find a curved steel rod, connected with the bottom-plate in F by a screw. Now AF being made of steel, AR of brass, which metals expand in a different degree, the spring will turn in A. The sinking of G by this reason is easily calculated to be $CE\,(c-b)\,t\frac{AG}{RF}$. Adding this quantity to the rise found above, we obtain

$$-\frac{NG}{NB}s\,(2a-b)\,t-CE\,(c-b)\frac{AG}{RF}t+(CEc+BCb)\,t.$$

The expansion of BG has no influence on the reading, because the levers in G and H are connected with turning joints. Of course we leave out of consideration the case of the lever BG bending on expanding, which arrangement is used for the compensation of the thermal coefficient. GH is made of steel, and therefore the rise of H will be

$$-\frac{NG}{NB}s\,(2a-b)\,t-CE\,(c-b)\frac{AG}{RF}t+(CEc+BCb)\,t-GHb\,t.$$

The axis I, connected with the bottomplate by brass supports, will rise $MIct$, so that the angular motion at I amounts to

$$\frac{1}{HI}\left(-\frac{NG}{NB}s\,(2a-b)-CE\,(c-b)\frac{AG}{RF}+\right.$$
$$\left.+CEc+BCb-GHb-MIc\right)t$$

and the displacement of K to the left becomes

$$\frac{KI}{HI}\left(-\frac{NG}{NB}s\,(2a-b)-CE\,(c-b)\frac{AG}{RF}+\right.$$
$$\left.+(CE-MI)\,c+(BC-GH)\,b\right)t.$$

The expansion of the steel chain, and the brass arm, by which the taps of the axis L are connected with I, will cause another alteration in the displacement, which for a point of the chain L reaches the value

$$\frac{KI}{HI}\left(-\frac{NG}{NB}s\,(2a-b)-CE\,(c-b)\frac{AG}{RF}+\right.$$
$$\left.+(CE-MI)\,c+(BC-GH)\,b\right)t-KL\,(c-b)\,t.$$

In order to find from this the variation of the reading, we will determine the consequences of a variation δA of the barometric pressure A. The deflection s will increase with $\frac{s}{A}\,\delta A$, which consequently will be the displacement of B. For that of KL we find $\frac{s}{A}\cdot\frac{NG}{NB}\cdot\frac{KI}{HI}\,\delta A$. From this we deduce, that a rise of temperature t corresponds to an apparent fall of barometric pressure

$$\frac{\frac{KI}{HI}\left(-\frac{NG}{NB}s(2a-b)-CE(c-b)\frac{AG}{FR}+(CE-MI)c+(BC-GH)b\right)-LK(c-b)}{\frac{s}{A}\cdot\frac{NG}{NB}\cdot\frac{KI}{HI}}\,t$$

or

$$\left(-A\,(2a-b)+\frac{A}{s}\frac{NB}{NG}\left\{-CE\frac{AG}{RF}(c-b)+\right.\right.$$
$$\left.\left.+(CE-MI)\,c+(BC-GH)\,b-KL.\frac{HI}{KI}(c-b)\right\}\right)t.$$

The coefficient of t represents the thermal coefficient.

In order to obtain an estimate of the value of this coefficient, a rough measuring is performed on an aneroid. When we assume in connection with this measuring

$a=0.000018$, $b=0.000011$, $c=0.000019$, $A=760$,

$\frac{A}{s} = 200$ [1]), $CE = 16$. $\frac{AG}{BF} = \frac{60}{28} = 2.14$, $BC = 6$, $GH = 19$,

$$MI = 6.5 \; \frac{NB}{NG} = \frac{23}{83} = 0.28, \; KL = 43, \quad \frac{HI}{KI} = \frac{6}{23} = 0.26,$$

we find for the thermal coefficient — 0.037.

Ordinarily the thermal coefficients are much larger in absolute value. This result therefore confirms the assertion that the expansion of the instrument is not one of the principal causes of the thermal coefficient.

§ 2. The two other above named causes for variations by heating are better adapted to explain the thermal coefficient. Often has attention been directed to both of them; the last has been used to compensate the thermal coefficient.

From these causes we can easily deduce an approximate value for the thermal coefficient. In doing this we shall, for the sake of simplicity, and on account of the preceding results, neglect the expansion of the different metallic parts.

Let A be the barometric pressure,

p the pressure of the air, which still remains in the box, measured at 0°,

α the coefficient of expansion for air.

The effect of the atmospheric pressure on the box at $t°$ will be that of a force, applied in the centre, amounting to

$$\{A - p(1 + \alpha t)\} c$$

1) See REINHERTZ, Zeitschr. f. Instrumentenk. VII, p. 157 (1887), where for the deflection of the box is given in the mean 0.005 mM. for a variation of barometric pressure of 1 mM.

where c depends on the shape and the dimensions of the box.

With this force balance the elastic forces, which are excited by the deflection f at the centre of the box, and by an equal deflection of the spring.

E_1 being the coefficient of elasticity for the metal of the box at 0°, we can represent the force excited by a deflection f at 0° by $f E_1 k_1$, where k_1 again is a quantity depending on the shape and dimensions of the box. When now we suppose that at $t°$ E_1 is changed into $E_1(1-\eta_1 t)$ we find for our force at $t°$

$$f E_1 (1 - \eta_1 t) k_1.$$

If we call E_2, η_2, k_2 the corresponding quantities for the spring, the force produced by the deflection of the spring will be

$$f E_2 (1 - \eta_2 t) k_2$$

and the equation of equilibrium becomes

$$f[k_1 E_1 (1 - \eta_1 t) + k_2 E_2 (1 - \eta_2 t)] = c[A - p(1 + \alpha t)].$$

If at a first approximation we neglect the expansion of the metal (see above), we can consider c, k_1 and k_2 as constants, and unite them with E_1 and E_2 to a new constant

$$k = \frac{c}{k_1 E_1 + k_2 E_2}$$

so that we find

$$f\left(1 - \frac{k_1 E_1 \eta_1 + k_2 E_2 \eta_2}{k_1 E_1 + k_2 E_2} t\right) = k\left(A - p(1 + \alpha t)\right).$$

Now taking

$$\eta = \frac{k_1 E_1 \eta_1 + k_2 E_2 \eta_2}{k_1 E_1 + k_2 E_2}$$

a quantity that lies between η_1 and η_2, this equation becomes

$$f(1-\eta t) = k[A - p(1+\alpha t)],$$

or, neglecting terms with t^2

$$f = k[A - p - \{p\alpha - (A-p)\eta\}t].$$

Instead of the barometric reading A we shall therefore find (neglecting index-errors)

$$A - p - \{p\alpha - (A-p)\eta\}t.$$

The correction becomes

$$p + \{p\alpha - (A-p)\eta\}t,$$

and the thermal coefficient

$$\lambda = p\alpha - (A-p)\eta = p(\alpha+\eta) - A\eta.$$

We find here that λ depends on the pressure A, agreeing to what has often been observed [1]).

§ 3. For a numeric comparison of the deduced expression of λ with experiments we should have to know the values of η_1 and η_2 and the proportion $\frac{E_1 k_1}{E_2 k_2}$, which is equal to that of the elastic forces caused at 0° in the box and in the spring, and moreover the air-pressure p, which would require a special experimental research. In default of such complete data, we can try a rough comparison by means of what is known about these quantities.

If $p = 0$, that is to say, if the box is completely exhausted, λ reaches its greatest negative value $-A\eta$. Regarding η we may suppose that the spring produces a much greater elastic force than the box for the same deflection. We may therefore thake $\eta = \eta_2$, and using the results of MAYER [2]) for steel, assume for η a value between 0.000224 and 0.000309. The greatest possible negative value of λ would

[1]) See e. g. WIEBE, Zeitschr. f. Instrumentenk. X, p. 429 p. (1890).
[2]) MAYER, Americ. J. of Sc. (4) I, p. 81 (1896).

then be $-A\eta = -760 \times 0.000309 = -0.235$. This agrees pretty well with the results of JELINEK [1]), who finds among 108 aneroids only 3 which have $-\lambda$ greater than 0.235. Also HARTL [2]), testing 81 instruments, finds as highest limit -0.24.

Ordinarily a much smaller value is found, as is apparent from the above mentioned investigation of JELINEK. He finds λ [3])

for	9	instruments	between	+ 0.23	and	0.00
»	9	»	»	0.00	»	— 0.07
»	82	»	»	— 0.07	»	— 0.17
»	8	»	»	— 0.17	»	— 0.37

The positive values are for the most part found with instruments of smaller size (pocket-size) with which a less accurate workmanship, and greater air-pressure are not impossible.

To λ between -0.07 and -0.17, when $A = 760$, $\eta = 0.0003$, $\alpha + \eta = 0.004$, would correspond a pressure in the box of 39 to 14 mM. When $\eta = 0.000224$ these limits would be 25 and 0 mM. Such values of p are quite possible.

The relation of λ to the pressure has been investigated a. o. by WIEBE. He expresses the thermal coefficient in the form

$$\lambda = a + b\,(760 - A)$$

and finds for b values, which mostly (8 out of 9) lie between 0.0002 and 0.0003. When we compare this form with our expression

[1]) JELINEK, Carl's Repert. XIII, p. 72 (1877).
[2]) HARTL, Zeitschr. f. Vermessungsw. 1882, p. 458.
[3]) See JORDAN, Handbuch der Vermessungsk. II, p. 499, 3 ed.

$$\lambda = p(\alpha + \eta) - A\eta$$

we see that b must be equal to η, which is sufficiently confirmed by the observed values.

HARTL [1]) has determined thermal coefficients at different pressures of aneroids after removing the box, and charging the spring with weights. The thermal coefficient in this case is expressed by $\lambda' = -A\eta$, and so is proportional to the pressure. HARTL finds with four aneroids:

Aneroid	pressure A	therm. coeff. spring. $-\lambda'$	$-\frac{\lambda'}{A}$
I	735	0.262	0.00036
II	759	0.365	0.00048
	682	0.350	51
	600	0.289	48
III	793	0.428	0.00054
	748	0.400	53
	644	0.397	62
IV	759	0.335	0.00044

The proportionality with the pressure is confirmed by II. By III only when we exclude the last number. The value found for $-\frac{\lambda'}{A}$, which should be equal to η, is however much greater than the values found by MAYER.

As far as is to be concluded from these data, the observed thermal coefficients are on the whole not in contradiction with the expression found above. For a more accurate treatment better data would be required. More researches such as promised by HARTL, are therefore much to be desired.

1) HARTL, Zeitschr. f. Instrumentenk. VI (1886).

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