electrodes was compared as in § 2 with that of the rheotan-wires of the compensative-current It was found that in a magnetic field of ± 5700 c. g s, the resistance of N°. 1 had increased 5,4 perct., that of N°. 2 7,4 perct. The specific resistance of these bars was ± 154000 in c. g. s. units[1]; 2 perct. of this is ± 3100 c. g. s The dissymmetry of the HALL-effect ($K_{11}-K_{22}$ of LEBRET) observed in one of the positions of the plate was ± 2700 c. g. s. The directions of the edges of the bars with respect to the piece out of which they had been cut did not coincide wholly with those in which the axes of symmetry of the plate had pointed with respect to the same piece; moreover in round plates $K_{11}-K_{22}$ is proportional indeed but not wholly equal to the dissymmetry; the agreement between the above mentioned numbers is satisfactory, if the unevitable errors of observation are taken into account.

4. Also with this method of observation sometimes different values for the resistance before and after the reversing of the field were obtained The inquiry into the cause of this phenomenon is related in the next communication. From the experiments mentioned in § 1 and § 3 we may however conclude, that a different increase of resistance in the magnetic field really exists for different directions in crystalline bismuth.

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[1]) In this determination a rather large error may occur, as the distance between the electrodes was only 6 m.M.

Dr. E. VAN EVERDINGEN Jr. *On the relation between the crystallographic directions and the resistance, the magnetic increase of resistance and the* HALL-*effect in bismuth.*

1. The researches published in the preceding communication induced me to put to myself the question, in what manner the coefficient of magnetic increase of resistance in a fixed plane is related to the position of this plane and the direction of the magnetic force with respect to the crystallographic axis of bismuth. During closer inquiry also the question rose, in what manner the HALL-coefficient is related to the same direction. The answer to these questions is given in § 8. Let me describe the course of experiments in close connection with the former communication.

2. The increase of resistance in the magnetic field being determined for the little bars, mentioned in the latter part of the former communication, the same experiments were repeated with a bar, cut from the same crystalline piece in a direction ⊥ the former two and ⊥ the principal cleavage-plane. It appeared, not only that this bar had a greater specific resistance, but also that it showed a much greater magnetic increase of resistance. Whereas for instance with N°. 2 a resistance was found of 154000 c. g. s. when not in the field, and a magnetic increase of resistance of 7,4

perct., here the corresponding numbers were 176000 and 12,2 perct. A difference in the resistance when not in the field was observed already in 1855 by MATTEUCCI [1]), who mentions that the conductivity of bismuth in the direction ⊥ the (principal) cleavage-plane is related to the conductivity // that plane as 1 : 1,16.

Hence it seemed a matter of importance to try to obtain plates in which this third direction, the direction of greatest resistance in a zero magnetic field, should be parallel to the plane sides; for such plates a large dissymmetry of the HALL-effect was expected. The examination of these plates ought then to be completed by that of three little bars, cut in the principal directions. As none of the crystalline pieces at hand was large enough to obtain a suitable plate from, ± 300 G. of bismuth were melted, cast in a porcelain shell and cooled slowly in the manner, described in the communication of 30 May 1896, p. 54 [2]) From this piece of bismuth were cut:

1⁰. A round plate with its sides vertical, i. e. ⊥ the horizontal surface of the congealed mass of bismuth (R 7).

2⁰. A round plate with its sides horizontal. (R 8).

3⁰. From the bismuth close to where the first plate had been cut two vertical and two horizontal bars. (1,2,I,II).

4⁰. A horizontal bar, ⊥ the last mentioned ones, taken from the bismuth close to where the second plate had been cut (3).

The examination of the plates gave results not answering the expectations. R 7 had its axes just in the

[1]) C R. T. XL p. 541, 914, 1855.
[2]) Communications N⁰. 26, p. 14.

directions of the bars, not so however R 8. The dissymmetry (K_{11}—K_{22} of LEBRET) was in neither particularly large, in R 7 for instance ± 5200 c. g. s. in a field of ± 7700 c. g. s. (For comparison we refer to the results of plate R 2 on p. 55 of the above mentioned communication [1]), where we may calculate for a field of 8600 c. g. s. a dissymmetry (K_{11}—K_{22}) of ± 14000 c. g. s.) It will appear hereafter (§ 8) that the anisotropy in resistance when not in the field has little to do with the magnetic increase of resistance. Moreover, the plates appeared to show a different HALL-effect; to this we shall revert in § 5.

3. Also the examination of the bars gave results that demanded closer inquiry. (See § 4 of the preceding communication). Remarkable was the very large difference occurring with some among them between the values, obained before and after reversing the magnetic field (Magn. A., Magn. B.), perpendicular to one of the oblong side-planes, for instance with the little bar N⁰. II in a field of 7700:

	Magn. A	Magn. B	Mean
Increase in %	41,2	23,6	32,4.

It looked unacceptable, that such differences might be caused by HALL-currents, received in degrees differing for the two electrodes, though these touched the same side-plane, as these differences represented a considerable part of the total HALL-effect to be expected if the electrodes were placed on *different* sides of the bar.

4. In order to detect the character of this phenome-

[1]) Communications N⁰. 26, p. 16.

non and to ascertain that no errors were occasioned by the method of observation, it appeared desirable to take experiments:

a. With primary currents of various strength.

b. In four positions (1, 2, 3, 4), obtained by turning the bars about the line joining the primary electrodes, each time through an angle of 90°.

c. In four corresponding positions, but with front and back interchanged (viewed from the magnet-poles).

d. In the same positions, after reducing the dimensions of the cross-section to about one half of their original value.

e. With a plate deposed by electrolysis.

These experiments gave the following results:

a. The results are wholly independent of the strength and direction of the current.

b. With the little bar N°. II.

Position.	Magn. A	Magn. B	Mean	Difference
1	41,2 %	23,6 %	32,4 %	+ 17,6
2	12,2	32,1	22,2	— 19,9
3	37,1	25,0	31,0	+ 12,1
4	11,5	39,8	25,7	— 28,3
c. With the same little bar				
1	20,7	34,0	27,4	— 13,3
2	31,8	16,6	24,2	+ 15,2
3	18,6	34,8	26,7	— 16,2
4	41,7	10,8	26,2	+ 30,9
d. [1]) The same little bar.				
1	28,6	35,8	32,2	— 7,2

[1]) In the original communication by mistake a slightly different series was published.

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2	28,7	20,8	24,7	+ 7,9
3	27,6	31,0	29,3	— 3,4
4	32,0	19,7	25,8	+ 12,3

e. For both directions of magnetic force (A and B) quite the same increase of resistance.

Here we observe:

1°. Agreement between the positions 1 and 3 or 2 and 4; difference between 1 and 2 or 3 and 4 In the positions 2 and 4 the mean is always smaller.

2°. Interchanging front and back reverses the sign of the difference. Remarkably high or low values occur now with opposite magnetisation.

3°. In *d* the differences have been reduced on an average to less than half their value.

5. A tolerably probable explanation of these particulars we may derive from the observation of the HALL-effect in the plates R 7 and R 8. For whereas R 7 showed in a field of 7700 c. g. s. a HALL-constant of 3,36 c. g. s, R 8 gave 6,39 c. g. s., that is to say almost twice as much. The differences between the various bars show clearly enough, that the bismuth had not crystallised perfectly regularly. If we suppose the crystals at one end of the bar to be placed otherwise than those at the other end, then also at one end a HALL-effect may occur of double strength, and the full HALL effect of plate R 7 may appear as an error in the determination of the resistance. After the bar turning 90° about ist longest axis, the stronger HALL-effect appears there, where first the weaker effect was found, at least if the crystals in one cross-section are nearly

parallel: thence the reserve of sign in the difference. Front and back having been interchanged, at the electrode where first the strong HALL-effect was found, now the weak effect appears (the resistance electrodes touch the lower side); once more the sign of the difference is reversed. If the dimensions of the cross section are reduced to one half of their former value, the resistance is multiplied by *four*, the HALL-effect only by *two*, the relative error is devided by *two*.

This explanation was confirmed by the same regularities being generally observed also with the other bars.

In order to test it more closely, as for the moment indeed no certainty had yet been obtained that the HALL-effect depended solely on the position of a plane in the crystal with regard to the magnetic force, the HALL-effect of these little bars was determined directly in a frame of ebonite, composed expressly for this purpose, in the positions 1, 2, 3 and 4. (See § 3). Quite in accordance with the expectation, the result was that in many cases very different HALL-coefficients were obtained.

So we may put the differences to the account of the HALL-effect and henceforth take the mean of the values, obtained before and after reversing the magnetic field, and also of the positions 2 and 4 or 1 and 3, in order to obtain the value of the resistance. In the same manner we will proceed with regard to the experiments, made after the above mentioned ones with a set of three little bars, cut at right angles to each other out of one of the cristalline pieces, presented by the

„*Königliches Blaufarbenwerk Oberschlema*" [1]), which showed on experiment the same phenomena, but for the greater part much more regularly; and with the three little bars from the crystalline piece, mentioned in the preceding communication (obtained from MERCK), which were now subjected to a renewed and more extensive investigation, and showed still greater regularity.

6. In order to get a survey of the experiments we should direct our attention to what follows:

We know that bismuth crystallises in the hexagonal system, in rhomboids differing but little from cubes. The principal axis ends in the most acute solid angles. This principal axis coincides with FARADAY's magnecrystal-axis. According to MATTEUCCI it is moreover the axis of greatest resistance; also my experiments lead to this result. The most important result of these experiments is, that the increase of resistance for the directions of a plane perpendicular to the direction of the magnetic field is smallest in bismuth, when the principal axis coïncides with the lines of force, and that also the HALL-effect is much weaker, when the axis is placed in this position.

For the cast bismuth of course we know nothing about the position of the principal axis. With regard to this bismuth we will compare the direction of greatest resistance with the directions perpendicular to it. For the sake of simplicity we will enter only a mean value for the two other directions under „second direction", and mention here only, that the differences between

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1) See Communications N⁰. 26, p. 14.

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these two are generally very small and never so large as those between one of them and the principal axis.

Cast bismuth (§ 2).

Direction:	Principal axis (⊥ plate R 7)	Second direction	N⁰.	Remarks
Specific resistance	147000		3	
		130000	1,2,I,II	
Increase of resistance in percts.	21,5	27,9	1	The figures in the first column represent not the increase of the resistance 147000, but the increase of the resistance 130000, when the princ. axis coincides with the lines of force.
	25,4	28,5	2	
	25,0	29,6	II	
	32,6	28,6	I	
		33,5	3	
HALL-constant.	1,77	7,77	1	The figures behind the accolades are mean values.
	3,36	8,73	2	
	5,12 } 3,87	4,78	II	
	5,22	5,65 } 6,53	I	
		9,17	3	
	3,36		R 7	
		6,39	R 8	

Though large differences remain, yet we clearly see that the direction adopted as „principal axis" possesses the enumerated qualities. The mean value of the HALL-constants for the four little bars (1, 2, I, II) in the

position, corresponding, with that of plate R 7 between the poles, 3,87, differs not much from the value obtained with R 7, 3,36; also the mean value 6,53 agrees with the value 6,39 of R 8.

The differences in the first column for increase of resistance should explain the dissymmetry observed in R 7. Combining them to two mean values, we find 23,5 (1,2) and 28,3 (I, II), hence difference = 4,8 %.

The original resistance is for both directions 130000, so we calculate for the dissymmetry 6240, whilst we observed ± 5200. Hence the difference of resistances found is more than sufficient for explaining the dissymmetry. Moreover it appeared on examination, that also the sign of the dissymmetry agreed with the observed differences.

Crystalline piece from OBERSCHLEMA.

Direction:	Principal axis	Second direction	N⁰.	Remarks
Specific resistance	146000		4	
		122000	5	
		156000 ?	6	6 had a burst.
Increase of resistance in percts	17,5		5	
	26,8		6	
		30,5	4, 5, 6	
HALL-constant.	0,96		5	
	4,43		6	
		7,59	4, 5, 6	

Very regular were especially the results for increase of resistance obtained with 5, partly because this was the longest of the three bars. Both the values, obtained before and after reversing the field, and the mean values for positions differing by 180° were always equal within a fraction of a percent. The HALL-coefficients observed were 0,96 and 7,37 [1]).

Crystalline piece from MERCK.

Direction:	Principal axis (⊥ plate R6)	Second direction	N°.	Remarks
Specific	172000		3	
resistance		151500	1,2	
Increase of	6,5		1	
resistance	7,7		2	
in percts.		16,4	1, 2, 3	
HALL-constant.	1,28		1	
	1,47		2	
		7,30	1, 2, 3	
	± 2,00		R 6	This value was deduced from an experiment in a weaker magnetic field.

With these bars almost always the same resistance is found before and after reversing the field. The

[1]) With this bar the experiments have been repeated at very low temperatures; the results will be published afterwards.

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greater resistance and smaller increase of resistance we may ascribe perhaps to a very slight impurity and rapid cooling.

7. Attempts have been made also to measure in these bars the dissymmetry of the HALL-effect. These however have not met with success, probably because, with these bars, it is not allowed to neglect the dimension in the direction of the magnetic force. Now in this direction the resistance is not or but little increased, and dissymmetry appears as soon as the HALL-electrodes are not placed one just over the other. In this manner we may perhaps explain why with the bars at hand dissymmetry was observed, though the above-given explanation supposes that no dissymmetry appears when one of the crystal-directions coincides with the magnetic force (in this case the increase of resistance depends indeed for all the directions of the plane of our plate on *one* vector).

In concluding we wish to point out, how the above-described phenomena may serve to explain a great many irregularities, appearing, in dissymmetry and mean HALL-effect, at the examination of the same plate in different positions [1]). As indeed the plates are generally not homogeneously crystalline, both observed quantities will to some extent depend on the position of the crystals happening to be placed at the electrodes, and we ought to expect a priori a different HALL-effect as soon as the plate is clamped in a different position. Only the regular differences between positions of sym-

[1]) See Communications N°. 26, p. 7.

metry and dissymmetry will not yet be cleared up in this way.

8. In order to explain the above mentioned phenomena we should connect them with the magnetisation. We might suppose the state of matters to be as follows:

In a zero magnetic field the resistances in different directions in a crystal of bismuth may be represented by the radii-vectores in an ellipsoid of revolution the greater axis of which coincides with the principal axis of the crystal.

A magnetic force, directed along the principal axis, causes a stronger magnetisation than the same force acting in a direction perpendicular to this axis. If we construe (the directions of the axes being the same as before) two other ellipsoids of revolution, one with axes in the ratio of the square roots of the values for the magnetisation in the two cases mentioned, the other with axes in the ratio of those values themselves, then, for a given direction of magnetic force the direction of magnetisation is indicated by the radius vector towards the point where the tangent plane $\perp$ the magnetic force touches the first ellipsoid; the relative value of the magnetisation is measured by the length of this radius vector to where it meets the second ellipsoid [1]).

The strength of the HALL-effect in a plane plate, made of a crystal of bismuth, depends on the component of magnetisation $\perp$ the plane of the plate. When the

[1]) In the original communication was erroneously stated, that both direction and magnitude of magnetisation might be found with the aid of *one* ellipsoid.

direction of magnetisation coincides with the principal axis the HALL-coefficient for the plane perpendicular to it will have a smaller value, than when the magnetisation is in a direction perpendicular to that axis. Very likely we shall, for an arbitrary position of the plane for which we wish to know the HALL-coefficient, find that coefficient with the aid of the ellipsoid of revolution construed with the extreme values.

The magnetic increase of resistance we know to be much smaller in the direction of the magnetic force than in the directions perpendicular to it. Hence we put forward the following *hypothesis*: in the magnetic field the resistance is increased only in all directions of a plane perpendicular to the *magnetisation*; in this plane all resistances are increased in equal proportion. The magnitude of the increase of resistance depends on the direction of magnetisation. We find the smallest increase when this direction coincides with the principal axis, and the greatest when it is perpendicular to the principal axis. Very likely for an arbitrary position the coefficient of increase of resistance will be found with the aid of an ellipsoid of revolution, construed with the extreme values.

After this increase of resistance the ellipsoid of resistances in general will possess three unequal axes. The resistances in directions at right angles in a plane section will generally be increased in different proportions; hence the possibility exists of dissymmetry of the HALL-effect.