Dr. L. H. SIERTSEMA. *On the effect of pressure on the natural rotation of the plane of polarisation in solutions of cane-sugar (Continued).*

1. To the measurements, described in the former communication [1]) new ones, with a greater concentration have been added. In making them the two quartz-plates, used on a former occasion have been employed together for the compensation.

The following results have been obtained:

III. Thickness of the quartz-plates 6.88 + 13.835 mM. = 20.715 mM.

c	$\Delta\lambda$	α	$\Delta\alpha_k$	$\Delta\alpha_s$	α_s	$\Delta\alpha_s/\alpha_s$	n
27.84	601	+ 1°.007	+ 0°.086	1°.093	439°	+ 0.00249	10
»	539	1°.455	0°.108	1°.563	555°	282	15
»	»	1°.399	»	1°.507	»	272	15
»	»	1°.440	»	1°.557	»	281	15

Conc. 27.84, mean value of $\frac{\Delta\alpha_s}{\alpha_s} = + 0.00273 \pm 0.00007$

In the same manner as before we find for the relative variation per unit of length:

c	$\Delta\beta/\beta$
27.84	0.00270

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[1]) Communication No. 35.

From this result, compared with the preceding ones, it appears that a relation of this quantity to the concentration cannot be deduced from these measurements.

2. It is interesting to test the results here obtained by a theory proposed by TAMMANN [1]). According to this theory the coefficient of variation of the specific rotatory power by external pressure is equal to the coefficient of variation by internal pressure, which latter can be varied by adding sugar, or any inactive salt.

We therefore will proceed to the calculation of the specific rotatory power γ. This quantity is related to the rotation per unit of length by the equation $\beta = c\,\gamma$. If now the external pressure varies, all three quantities, appearing in this equation, will vary, and we find

$$\frac{\triangle\beta}{\beta} = \frac{\triangle c}{c} + \frac{\triangle\gamma}{\gamma}$$

No measurements are known of the quantity $\frac{\triangle c}{c}$, which is equal to the coefficient of compressibility $-\frac{\triangle v}{v}$. Accepting the hypothesis of TAMMANN, we can, in the manner pointed out by him, deduce this coefficient from the researches of AMAGAT on the compressibility of water at high pressures [2]).

For doing this let us take a volume v_0 of the solution at an external pressure $= 0$, in which the internal pressure is greater by $\triangle K$ atm. than in water and further an equal volume $w_{\triangle K}$ of water at an external pressure

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[1]) See e. g. TAMMANN, Zeitschr. f. phys. Ch. XIV, p. 433 (1894).

[2]) TAMMANN, Zeitschr. f. phys. Ch. XVII, p. 620 (1895).

$\triangle K$, so that $v_0 = w_{\triangle K}$. Supposing we increase for both fluids the external pressure with p, according to the hypothesis of TAMMANN both volumina will still be equal, hence $v_p = w_{\triangle K + p}$. Defining the coefficient of compressibility μ for water by the relation

$$\mu = -\frac{1}{w_0} \cdot \frac{dw_p}{dp} \quad ^{1)}$$

in which μ is a function of p, we find

$$w_p = w_0 \left(1 - \int_0^p \mu dp\right)$$

and further

$$v_0 = w_{\triangle K} = w_0 \left(1 - \int_0^{\triangle K} \mu dp\right)$$

$$v_p = w_{\triangle K + p} = w_0 \left(1 - \int_0^{\triangle K + p} \mu dp\right)$$

and for the wanted quantity $\frac{\triangle c}{c}$ we find

$$\frac{\triangle c}{c} = -\frac{\triangle v_0}{v_0} = \frac{v_0 - v_p}{v_0} = \frac{\int_{\triangle K}^{\triangle K + p} \mu\, dp}{1 - \int_0^{\triangle K} \mu dp}$$

in which we must take $p = 100$.

Assuming moreover with TAMMANN the empiric relation

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[1]) Our μ is identical with $\frac{\partial v}{\partial p}$ of TAMMANN, see l. c. p. 622.

$\mu = \dfrac{A}{p + B}$, in which A and B are constants, we find

$$\frac{\Delta c}{c} = \frac{A\, l\, \dfrac{B + \Delta K + p}{B + K}}{1 - A\, l\, \dfrac{B + \Delta K}{B}}.$$

The values of ΔK have been deduced by Tammann from observations of Marignac on thermal expansion [1]); the constants A and B have likewise been calculated by Tammann from the researches of Amagat.

With these values (mean temperature 10°) we find:

c	ΔK	$\dfrac{\Delta c}{c}$	$\dfrac{\Delta \beta}{\beta}$	$\dfrac{\Delta \gamma}{\gamma} = \dfrac{\Delta \beta}{\beta} - \dfrac{\Delta c}{c}$
9.48	238	+ 0.00449	+ 0.00268	— 0.00181
18.70	465	418	252	166
27.84	669	398	270	128

3. We now have to test the theory of Tammann by comparing the quantity $\dfrac{\Delta \gamma}{\gamma}$ with the coefficients of variation by increasing concentration. This comparison will be made in the following manner. We begin with deducing from experimental data the coefficient of variation of the specific rotatory power. Assuming with Tammann that the coefficient of variation found by us agrees with a variation of the internal pressure $\Delta K = 100$, we shall find from this how much the internal pressure changes for $\Delta c = 1$, and this value we shall compare with the one which follows from the values deduced by Tammann in different other ways.

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[1]) Tammann, Zeitschr. f. phys. Ch. XIII, p. 179 (1894); XXI, p. 532 (1896).

From the formulae of Tollens on the relation between specific rotatory power and concentration we deduce

c	$\dfrac{\Delta \gamma}{\gamma}$ (for $\Delta c = 1$)	$\dfrac{\Delta \gamma}{\gamma}$ (observed)	Δk
9.48	— 0.000238	— 0.00181	13.1
18.70	243	166	14.7
27.84	068	128	5.6

This last quantity represents the value of ΔK for $\Delta c = 1$, which according to Tammann follows from the formulae of Tollens. The values of ΔK, deduced by Tammann from other phenomena, give for this the much greater value 23.4.

4. Further we can test the theory of Tammann by researches of Farnsteiner [1]) on the variation of the salt. Knowing the variation of $\dfrac{\Delta \gamma}{\gamma}$ which agrees with $\Delta K = 100$ from our measurements, connected with the hypothesis of Tammann, we can calculate ΔK from some of the measurements of Farnsteiner, and then compare these values with those deduced by Tammann from other phenomena.

The results of this comparison are found in the following table. In this we represent by

A the values of $\dfrac{\Delta \gamma}{\gamma}$ for $\Delta K = 100$, as found above,

B the values of $\Delta \gamma$ from measurements of Farn-

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[1]) Farnsteiner, Ueber die Einwirkung einiger anorganischen Salze auf das optische Drehungsvermögen des Rohrzuckers. Diss. Jena, 1890.

STEINER, for an addition of a weight 1 of salt to n of water,

C the values of $\frac{\triangle\gamma}{\gamma}$, following from B,

$\triangle K$ the increase of the internal pressure, calculated from our measurements for the addition mentioned above,

$\triangle K_T$ the values of this quantity according to those found by TAMMANN in other ways.

	c	A	B	n	C	$\triangle K$	$\triangle K_T$
NaCl	9.48	—0.00181	1.42	9.93	—0.0213	1120	1090
	18.70	166	3.01	4.73	452	2720	2030
KCl	9.48	—0.00181	1.04	9.93	—0.0156	862	636
	18.70	166	2.06	4.73	309	1860	1150
$BaCl_2$	6.48	—0.00181	0.39	9.93	—0.00584	323	526
	18.70	166	0.57	4.73	855	515	990
$CaCl_2$	9.48	—0.00181	1.13	9.93	—0.0169	934	1140
	18.70	166	2.38	4.73	357	2150	2400

The agreement is satisfactory for some salts, in other cases on the contrary we find important differences.

5. Finally we come to the conclusion that the variation of the specific rotatory power by pressure, by variation of concentration and by the addition of an inactive salt is a more complicated phenomenon than we are led to suppose by the hypothesis of TAMMANN.

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COMMUNICATIONS

FROM THE

PHYSICAL LABORATORY

AT THE

UNIVERSITY OF LEIDEN

BY

PROF. DR. H. KAMERLINGH ONNES,

Director of the Laboratory.

No. 39.

(REPRINT).

A. VAN ELDIK. Measurements of the capillary ascension of the liquid phase of a mixture of two substances in equilibrium with the gaseous phase (*With two plates*).

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