Topological Properties of Quantum States of Condensed Matter: some recent surprises.

F. D. M. Haldane
Princeton University
and
Instituut Lorentz

1. Berry phases, zero-field Hall effect, and "one-way light"
2. Anomalous and Spin Hall effect, Topological insulators
3. Non-abelian FQHE states

Bands with both time-reversal and spatial-inversion symmetry:

- Bands are doubly-degenerate at generic points in the Brillouin zone
- Bands at special k-points where $2\boldsymbol{k}_j = \boldsymbol{G}$ are classified by **inversion symmetry** $I_i(\boldsymbol{k}_j) = +1$ or -1 about inversion center i in the real-space unit cell.
- In 2D (3D) there are 4 (8) special k-points and 4 (8) distinct inversion centers; the product over all bands below the Fermi energy is

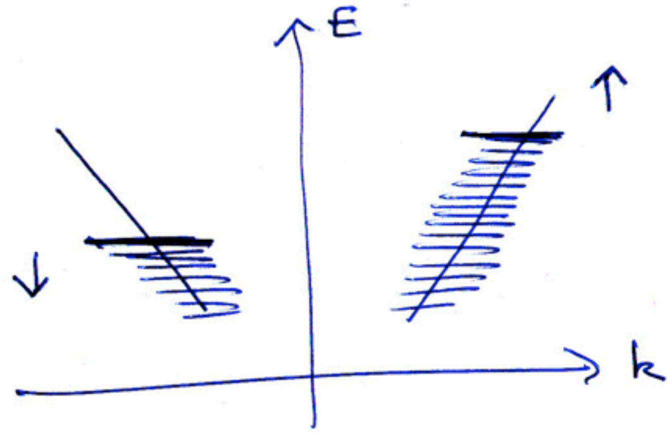
$$\eta_0 = \prod_j I_i(\boldsymbol{k}_j) = \pm 1$$

Fu and Kane, 2006

- In 2D and above this is independent of which inversion center i is chosen

This is clearly a "topological invariant", but Kane and Fu's recent (2006) result shows the not-so-obvious fact that it must have the value $+1$ in the absence of spin-orbit coupling.

"Quantum Spin-Hall effect"

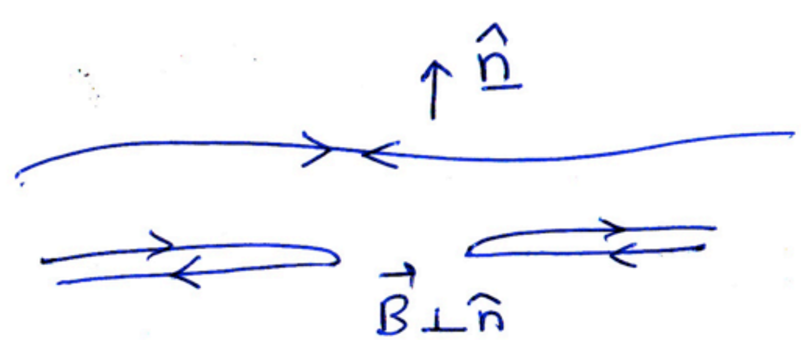


Current-carrying State has a finite magnetization:

$\vec{M} \propto I$

- (But not quantized)

- on edge, There is a local magnetisation direction



$\vec{B} \propto \hat{n}$ produces current

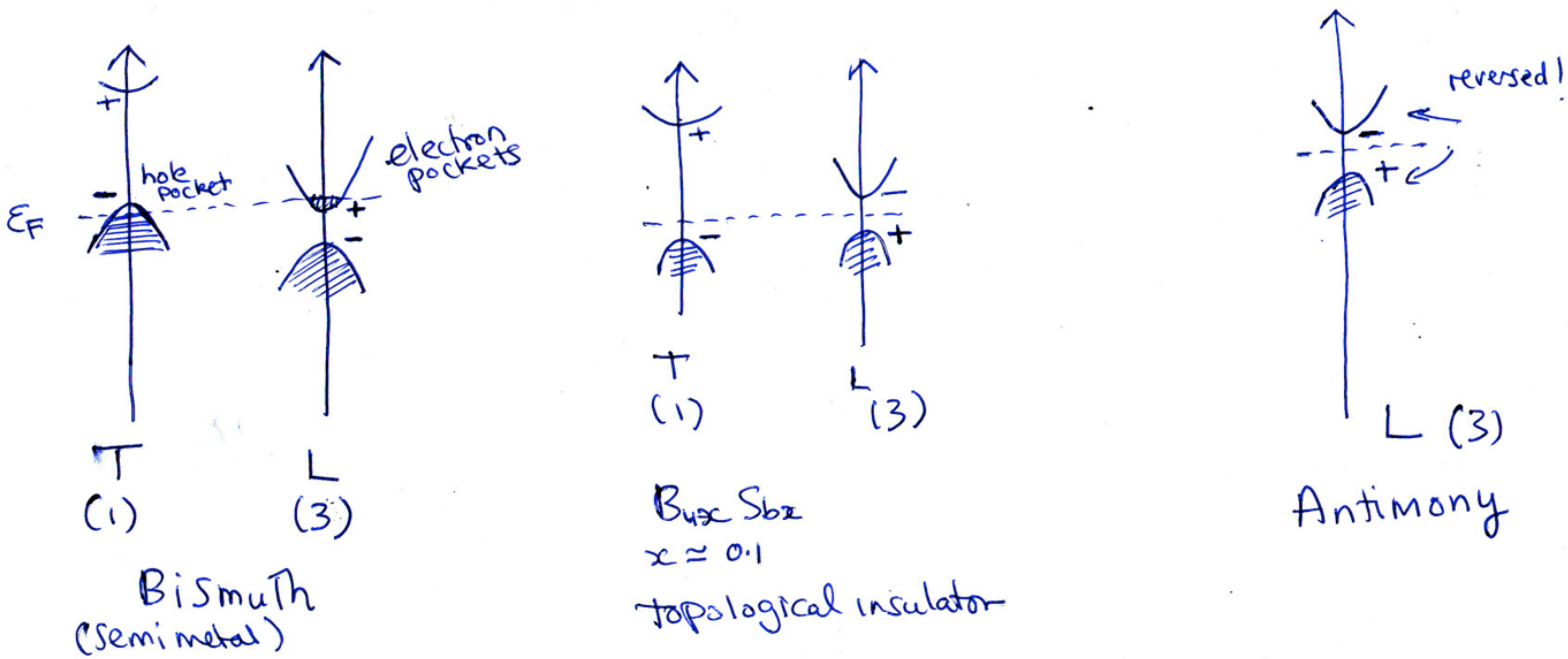
$\vec{B} \perp \hat{n}$ backscatters

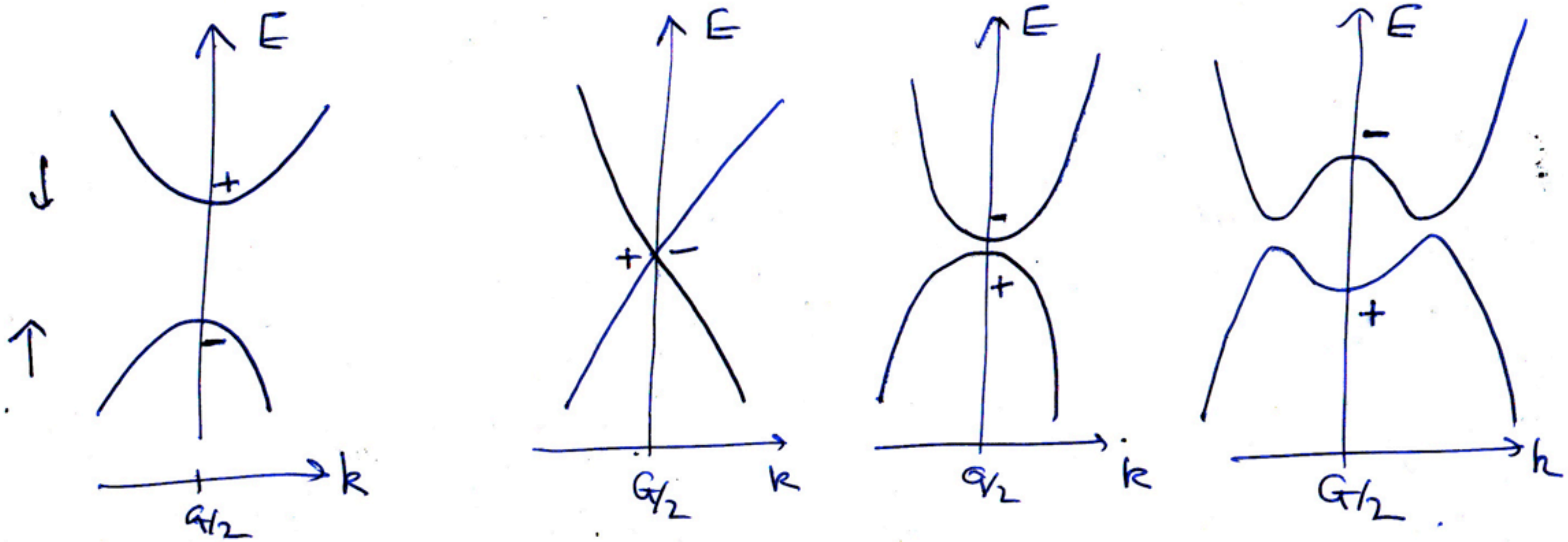
$\left(B \, \psi_R^{+} \psi_L \, e^{i\phi} + hc \right)$

Kane + Fu
Moore + Balents

3D topological insulators.

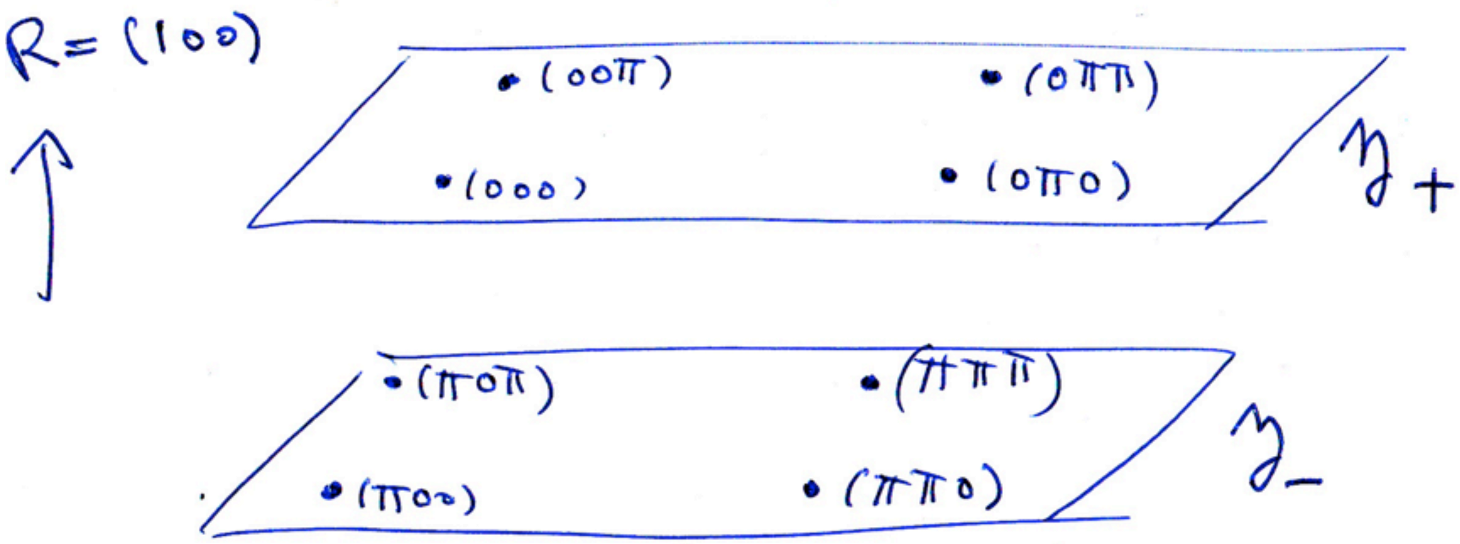
1) if inversion symmetry is present, (as well as time reversal symmetry), just count number of odd parity bands below Fermi level at points $\vec{k} = \vec{G}/2$ ($\vec{G}$ a reciprocal lattice vector)





reversal of order of bands with opposite
Inversion symmetry

3D Z_2 invariant (general case)



k-space plane includes (000)

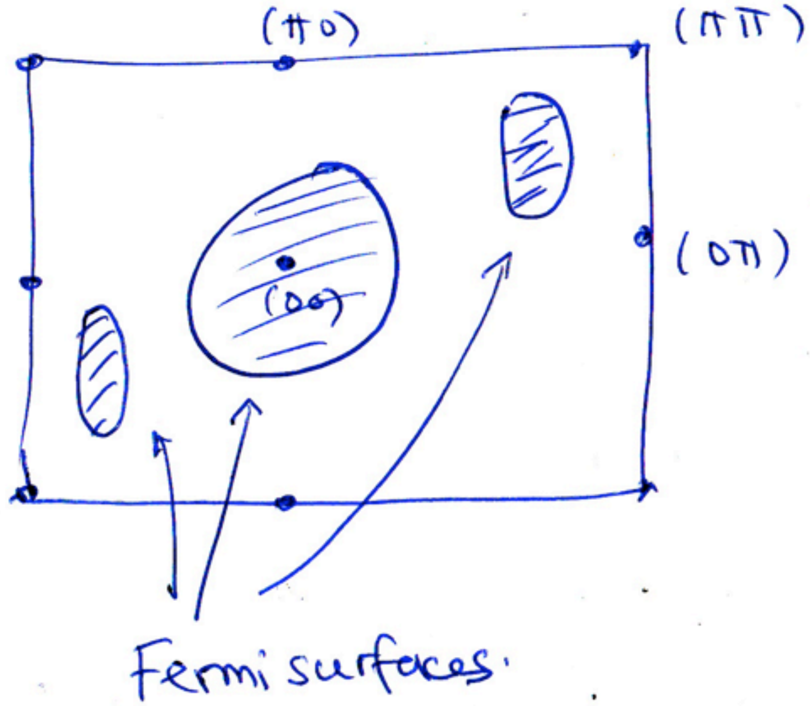
plane without (000)

$$\eta_+ \eta_- = -1$$ (however $\vec{R}$ is chosen)

lattice translation ⊥ to k space planes

$$\boxed{\text{Can calculate } \eta_\pm \text{ from Berry phase integral over 2D k-space plane}}$$

- on every crystal face, find a 2D metal



$$\prod_{\nu} e^{i\phi_{\nu}^{B}} = e^{i\phi^{B}_{TOTAL}}$$

Berry phase for moving around Fermi surface

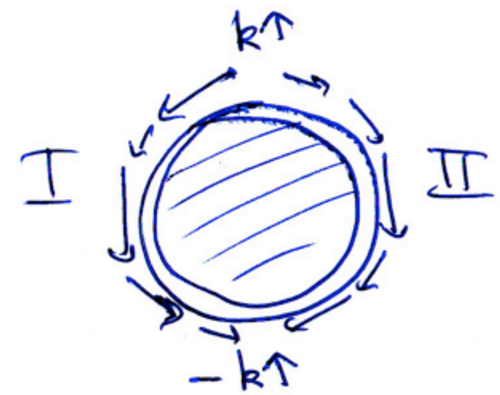
Karplus Luttinger: $$\sigma_{H}^{xy} = \frac{e^2}{h}\left(\underbrace{\underbrace{\frac{\phi^{B}_{Total}}{2\pi}}_{\text{Bulk}} + \underbrace{\text{integer}}_{\text{surface}}}_{= \frac{1}{2} + \text{integer}} \right)$$

Time reversal: $$e^{i\phi^{B}_{TOT}} = e^{-i\phi^{B}_{TOT}}$$

$$\boxed{e^{i\phi^{B}_{TOT}} = \eta = \underset{\text{topological insulator}}{-1}}$$

Localization is absent!

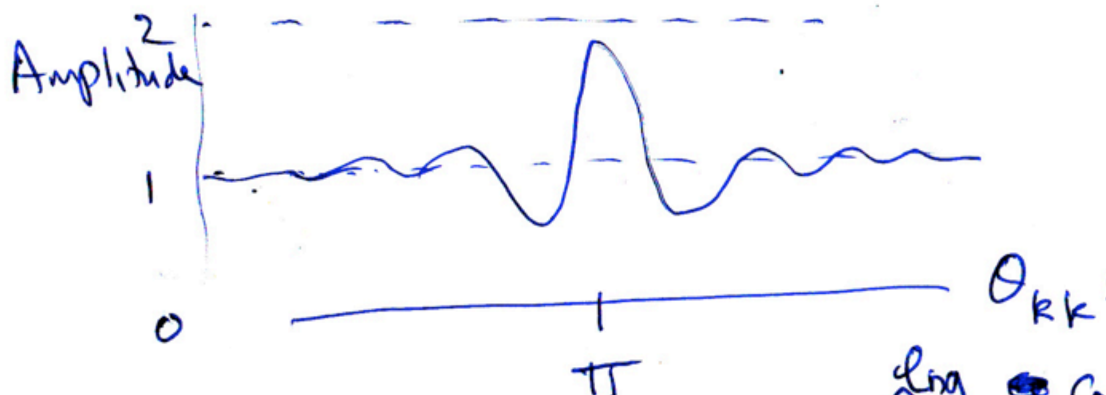
(a)



No SOC

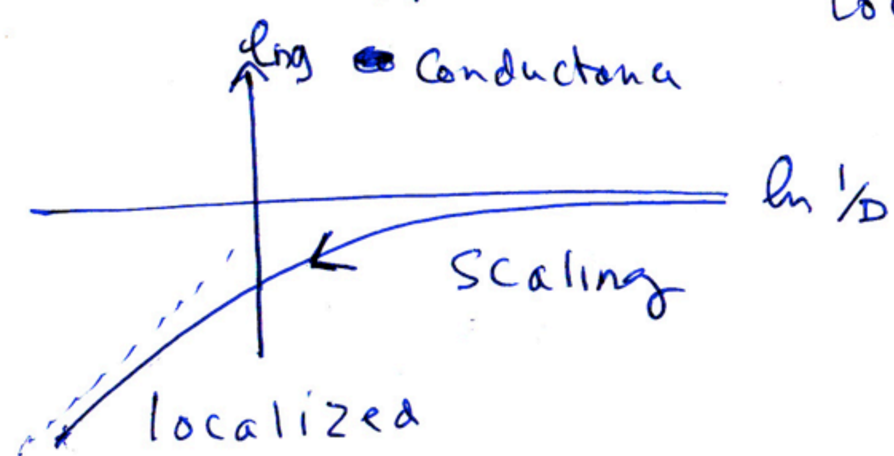
Berry phase + 1 around fermi surface which is $k \leftrightarrow -k$ symmetric.

Time-reversed paths I and II from $k\uparrow$ to $-k\uparrow$ constructively interfere

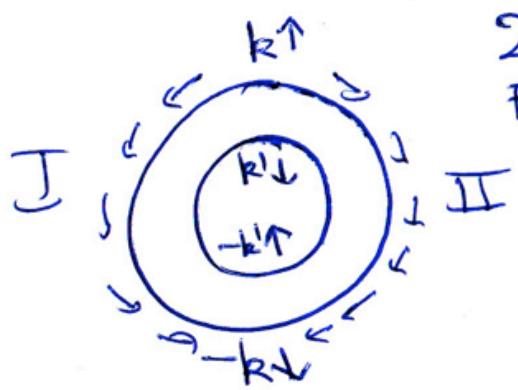


extra 180° backscattering

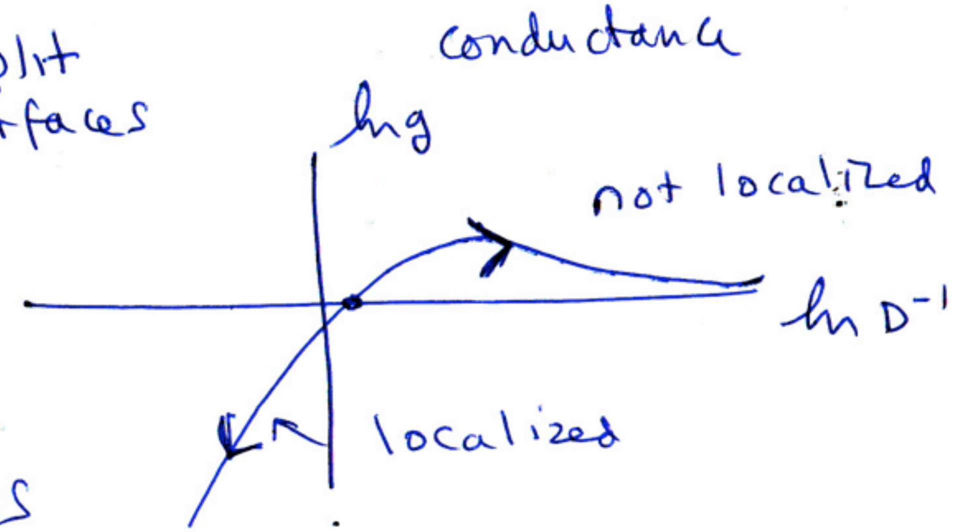
→ Weak (Anderson) localization



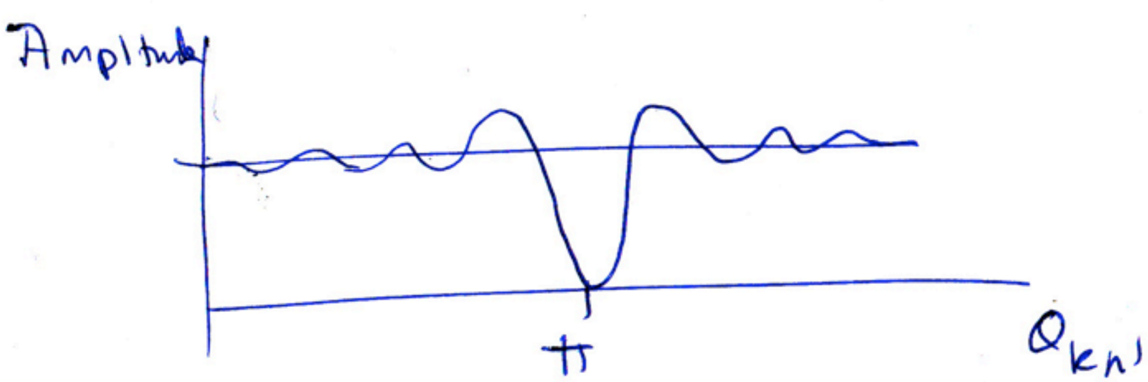
(b) usual case with SOC (symplectic)



2 spin-split Fermi surfaces



- weak scattering allows only paths from k↑ to −k↓.
- Berry phase −1
- destructive interference

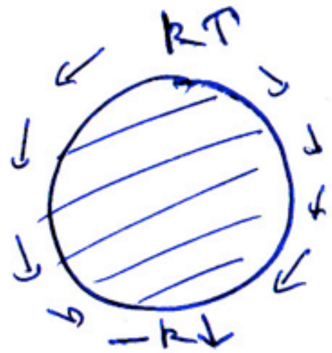


- Strong disorder allows scattering from

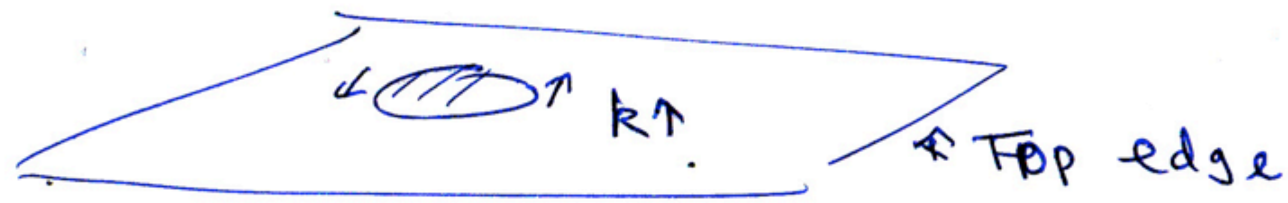
  k↑ to −k'↑

  (no interference)
- Also interactions affect weak antilocalization

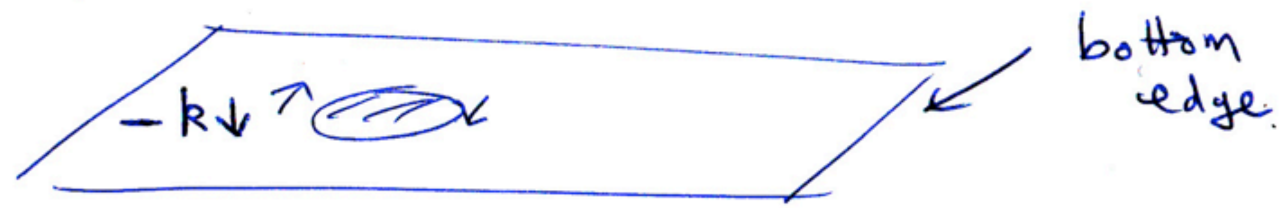
But, if The "other" spin-split fermi Surface is absent



NO destruction of weak antilocalization by strong disorder



Topological insulator.



k↑ and −k↑ are on opposite faces of sample "Sandwich"

- <u>open questions</u>

  nature of conduction of <u>interacting</u> surface of topological insulator

- Magnetic / Superconducting order.

- Majorana fermions, proximity effect.