Quantum optics & quantum information

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Introduction / Motivation / Overview

- Quantum information
  - quantum computing, quantum communication etc.
- Zoo of quantum optical systems
  - ions, neutral atoms, CQED, atomic ensembles
- Theoretical Tools of Quantum Optics
  - quantum optical systems as open quantum systems
1.1 Quantum information processing

- quantum computing

\[ |\psi_{\text{out}}\rangle = U|\psi_{\text{in}}\rangle \]

- quantum communication

transmission of a quantum state
Quantum computing

- **quantum memory**
  - \(|0\rangle\)
  - \(|1\rangle\)
  - N spin-1/2 systems
  - quantum register
  - qubit

- **quantum gates**
  - single qubit gate:
    - \(\hat{U}_1\) = rotation of a single qubit
    - \(U_1\)
  - two-qubit gate:
    - \(\hat{U} = |0\rangle_1\langle 0| \otimes \hat{1}_1 + |1\rangle_1\langle 1| \otimes \hat{U}_1\)
    - control
    - target

- **read out**
- **[no decoherence]**
Our goal ... implement quantum networks

- quantum network

[Diagram of a network with nodes and channels]

- Nodes: local quantum computing
  - store quantum information
  - local quantum processing
  - measurement

- Channels: quantum communication
  - transmit quantum information
  - local / distant

Goals:
- map to physical (quantum optical) system
- map quantum information protocols to physical processes
1.2 Zoo of quantum optical systems

- trapped ions
  - Few particle system with complete quantum control:
    - spin-1/2s coupled to harmonic oscillator(s)

- CQED
  - quantum state engineering:
  - quantum computing
  - state preparation & measurement
  - collective modes
  - cavity decay
  - spontaneous emission
  - laser
• **from BEC to Hubbard models**
  – strongly correlated systems
  – time dependent, e.g. quantum phase transitions
  – ...
  – exotic quantum phases (?)

• **quantum information processing**
  – new quantum computing scenarios, e.g. "one way quantum computer"
  
  "quantum simulator"
... measurements beyond standard quantum limit

- N independent atoms

\[ \Delta \omega_{\text{SQL}} = \frac{1}{T \sqrt{n_{\text{rep}}}} \frac{1}{\sqrt{N}} \]

standard quantum noise limit

- N entangled atoms

\[ \Delta \omega_{\text{ent}} = \frac{1}{T \sqrt{n_{\text{rep}}}} \frac{1}{f(N)} \geq \frac{1}{T \sqrt{n_{\text{rep}}}} \frac{1}{N} \]

Heisenberg limit: maximally entangled state \(|0000\rangle + |1111\rangle\)

- Entanglement via collisions: spin squeezing

BEC

laser

\[ |0\rangle^\otimes N \]

product state

[\( \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]^\otimes N

product state

[\( \sum c_n |0^n\rangle |1^{N-n}\rangle \]

\[ = |0000\rangle + |1111\rangle \]

measurements beyond standard quantum limit
• cascaded quantum system: transmission in a quantum network
• **atomic ensembles**

atomic / spin squeezing; quantum memory for light; continuous variable quantum states

quantum repeater: establishing long distance EPR pairs for quantum cryptography and teleportation
1.3 Quantum optical systems as *open* quantum systems

- example: trapped ion

\[ |n = 0 \rangle \quad \otimes \quad |n = 1 \rangle \quad \otimes \quad |n = 2 \rangle \quad \ldots \]

\[ \Delta \quad \Omega \quad \Gamma \]

\[ \text{atom} \rightarrow \text{harmonic trap} \]

\[ \text{empty radiation modes} \rightarrow \text{phonon dissipation} \]
1.3 ... *Open Quantum System*

**role of coupling to environment:**
- noise / dissipation (decoherence)
- quantum optics … state preparation (e.g. laser cooling)

**Quantum Markov processes:**
- quantum stochastic Heisenberg and Schrödinger equations
- master equations etc.

*This is valid in quantum optics*
1.3 ... *Open Quantum System*

role of coupling to environment:
• continuous observation:
  
  clicks ↔ quantum jumps
  & preparation
Outline: Quantum Computing & Communication with …

1. trapped ions

2. neutral atoms
   - optical lattices, cold collisions, Rydberg gates etc.

3. atomic ensembles
   - quantum repeater with atoms / qdots
   - teleportation with ensembles

Theoretical Tools: Quantum noise
- decoherence, state preparation (by “quantum jumps, read out
- from quantum operations to stochastic Schrödinger equations, continuous measurement and all that
Quantum Computing with Trapped Ions

• basics: quantum optics of single ions & many ions
  – develop toolbox for quantum state engineering
• 2-qubit gates
  – from first 1995 gate proposals and realizations
  – ... geometric and „best“ coherent control gates
• spin models
1. A single trapped ion

- a single laser driven trapped ion

\[ H = H_0 + H_0A + H_1 \]

\( H_0T = \frac{\hat{p}^2}{2M} + \frac{1}{2} Mv^2 \hat{X}^2 \equiv \hbar v(a^\dagger a + \frac{1}{2}) \)

\( H_{0A} = -\hbar \Delta \langle e|e \rangle \)

\( H_1 = -\frac{1}{2} \hbar \Omega e^{-ikL\hat{X}} \langle e|g \rangle + h.c. \)

\[ H = \frac{\hat{p}^2}{2M} + \frac{1}{2} Mv^2 \hat{X}^2 + \hbar \omega_{eg} \langle e|e \rangle - \hbar (-\frac{1}{2} \Omega e^{ik\hat{X} - i\omega t} \langle e|g \rangle + h.c.) \]
- Laser absorption & recoil

\[ |g\rangle \rightarrow |e\rangle e^{i k L \hat{X}} |\text{motion}\rangle \]

\[ H_1 = -\frac{1}{2} \hbar \Omega e^{i k L \hat{X}} a^\dagger a |e\rangle |g\rangle + \text{h.c.} \]

Laser photon recoil: couples internal dynamics and center-of-mass

- Lamb-Dicke limit

\[ a_0 = \sqrt{\frac{\hbar}{2 M v}} \]

Lamb-Dicke expansion

\[ e^{i k L \hat{X}} = e^{i \eta (a + a^\dagger)} = 1 + i \eta (a + a^\dagger) + \cdots \]

\[ \eta = 2 \pi a_0 \lambda_L \equiv \sqrt{\frac{\epsilon_R}{hv}} \sim 0.1 \]
- **spectroscopy**: atom + trap

\[ \frac{1}{2} \Omega e^{ikLX} |e \rangle \langle g| = \frac{1}{2} \Omega |e \rangle \langle g| + i \frac{1}{2} \Omega a^\dagger |e \rangle \langle g| + i \frac{1}{2} \Omega a |e \rangle \langle g| + \ldots \]

- **processes**: "Hamiltonian toolbox for phonon-state engineering"

- laser interaction

\[ \frac{1}{2} \Omega e^{ikLX} |e \rangle \langle g| = \frac{1}{2} \Omega |e \rangle \langle g| + i \frac{1}{2} \Omega a^\dagger |e \rangle \langle g| + i \frac{1}{2} \Omega a |e \rangle \langle g| + \ldots \]
• example: "laser tuned to red sideband"

\[ H_{JC} = \hbar \nu a^\dagger a - \hbar \Delta |e\rangle\langle e| - \frac{1}{2} i\hbar (\Omega \eta) |e\rangle\langle g| a + \text{h.c.} \]

• Remark: CQED

\[ H_{JC} = \nu a^\dagger a + \omega_{eg} |e\rangle\langle e| - i g |e\rangle\langle g| a + \text{h.c.} \]

Jaynes-Cummings model

vacuum Rabi frequency

~ laser (switchable)
[Dissipation: spontaneous emission]

- sideband cooling... as optical pumping to the ground state

\[ |g, 0 \rangle \xrightarrow{\nu} |e, 0 \rangle \xrightarrow{\Gamma} |e, 1 \rangle \xrightarrow{\nu} |e, 2 \rangle \xrightarrow{\cdots} \]

preparation of pure states

\[ \rho_{\text{atom}} \otimes \rho_{\text{motion}} \rightarrow |g\rangle\langle g| \otimes |0\rangle\langle 0| \]

- measurement of internal states: quantum jumps ...

qubit read out
Excercises in quantum state engineering

- **Example 1**: single qubit rotation
  \[(\alpha |g\rangle + \beta |e\rangle) \otimes |0\rangle \rightarrow (\alpha' |g\rangle + \beta' |e\rangle) \otimes |0\rangle\]
  
  (1) we can rotate the qubit without touching the phonon state

- **Example 2**: swapping the qubit to the phonon mode
  \[(\alpha |g\rangle + \beta |e\rangle) \otimes |0\rangle \rightarrow |g\rangle \otimes (\alpha |0\rangle + \beta |1\rangle)\]

  (2) Using a laser pulse we can swap qubits stored in ions to the phonon modes (and vice versa)
• Example 3: engineering arbitrary phonon superposition states

$$|g\rangle \otimes |0\rangle \xrightarrow{U} |\Psi\rangle = |g\rangle \otimes \sum_{n=0}^{N} c_n|n\rangle$$

given coefficients $c_n$

✓ Fock states
✓ squeezed & coherent states
✓ Schrödinger cat states
✓ ...

|g,0⟩

|g,1⟩

|g,2⟩

|e,0⟩

|e,1⟩

|e,2⟩

|g\rangle \otimes \sum_{n=0}^{n_{\text{max}}} c_n|n\rangle

• Idea: we will look for the inverse $U$ which transforms $|\Psi\rangle$ to $|g\rangle \otimes \sum_{n=0}^{n_{\text{max}}} c_n|n\rangle$

Law & Eberly, Gardiner et al., Wineland et al.
2. Many Ions

- 2 ions & collective phonon modes

\[ v_r = \sqrt{3} v_c \]  

- example: classical ion motion

(3) We can swap a qubit to a collective mode via laser pulse
• Example: 2 ions in a 1D trap kicked by laser light

\[ H = \nu_c a^\dagger a + \nu_r b^\dagger b + \frac{1}{2} \Omega(t) \sigma_1^+ e^{i\eta_c (a^\dagger + a)} + \frac{1}{2} \eta_r (b^\dagger + b) + \frac{1}{2} \Omega(t) \sigma_2^+ e^{i\eta_c (a^\dagger + a)} - \frac{1}{2} \eta_r (b^\dagger + b) + \text{h.c} \]
Ion Trap Quantum Computer '95

- Cold ions in a linear trap

Qubits: internal atomic states
1-qubit gates: addressing ions with a laser
2-qubit gates: entanglement via exchange of phonons of quantized collective mode

State vector

\[ |\Psi\rangle = \sum c_x |x_{N-1}, \ldots, x_0\rangle_{\text{atom}} |0\rangle_{\text{phonon}} \]

- QC as a time sequence of laser pulses
- Read out by quantum jumps
Level scheme

| $r_1$ \rangle | r_0 \rangle 
\hline
\text{auxiliary level}
\text{addressing with different light polarizations}

\text{qubit}

\text{state measurement via quantum jumps}
Two-qubit phase gate

- step 1: swap first qubit to phonon

\[ |g,0\rangle \xrightarrow{\pi\text{ pulse}} |g,1\rangle \]

\[ |r_0,0\rangle \xrightarrow{\hat{U}_{\pi,0}^m} |r_0,1\rangle \]

\[ |g\rangle_m|0\rangle \rightarrow |g\rangle_m|0\rangle \]

\[ |r\rangle_m|0\rangle \rightarrow -i|g\rangle_m|1\rangle \]
• step 2: conditional sign change

second atom: n

\[
\begin{align*}
|g,0\rangle & \rightarrow |g,1\rangle \\
|g,m,0\rangle & \rightarrow |g,m,1\rangle \\
-i|g,m,0\rangle & \rightarrow -i|g,m,1\rangle \\
-i|g,m,0\rangle & \rightarrow -i|g,m,1\rangle \\
\end{align*}
\]
• step 3: swap phonon back to first qubit
• summary: we have a phase gate between atom m and n

\[ |g\rangle|g\rangle |0\rangle \rightarrow |g\rangle|g\rangle |0\rangle, \]
\[ |g\rangle|r_0\rangle |0\rangle \rightarrow |g\rangle|r_0\rangle |0\rangle, \]
\[ |r_0\rangle|g\rangle |0\rangle \rightarrow |r_0\rangle|g\rangle |0\rangle, \]
\[ |r_0\rangle|r_0\rangle|0\rangle \rightarrow - |r_0\rangle|r_0\rangle|0\rangle. \]

\[ |\epsilon_1\rangle|\epsilon_2\rangle \rightarrow (-1)^{\epsilon_1 \epsilon_2} |\epsilon_1\rangle|\epsilon_2\rangle \quad (\epsilon_{1,2} = 0, 1) \]

Rem.: this idea translates immediately to CQED
• (addressable) 2 ion controlled-NOT + tomography

Realization of the Cirac–Zoller
controlled-NOT quantum gate

Ferdinand Schmidt-Kaler, Hartmut Häffner, Mark Riebe, Stephan Gudde,
Gavin P. T. Lancaster, Thomas Deuschle, Christoph Becher,
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Experimental demonstration of a
robust, high-fidelity geometric
two ion-qubit phase gate

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J. Britton*, W. M. Itano*, B. Jelenković*,§, C. Langer*, T. Rosenband*
& D. J. Wineland*

• teleportation Innsbruck / Boulder

• decoherence: quantum memory DFS 20 sec

truth table CNOT
Innsbruck

|S, S⟩ |S, D⟩ |D, S⟩ |D, D⟩ |
input

|S, S⟩ |S, D⟩ |D, S⟩ |D, D⟩ |
output

|φ⟩

EPR pair
Scalability

- key idea: moving ions … without destroying the qubit
Two-qubit gate … the “wish list”

• fast: \textit{max} \# operations / decoherence \textit{[what are the limits?]}

• NO temperature requirement: “hot” gate, i.e. NO ground state cooling

\[
|\psi\rangle\langle\psi| \otimes \rho_{\text{motion}} \rightarrow \text{entangle qubits via motion} \rightarrow |\psi\rangle\langle\psi| \otimes \rho'_{\text{motion}}
\]

qubits \hspace{1cm} \text{motional state:}
\hspace{1cm} \text{e.g. thermal}

• NO individual addressing

addressing: \hspace{1cm} \text{large distance}
\hspace{1cm} \text{vs.}
\hspace{1cm} \text{strong coupling: small distance}
Speed limits

• In all present proposals the speed limit for the gate is given by the trap frequency

\[ T_{\text{gate}} \sim \frac{1}{\eta \nu} \]

- trap frequency
- Lamb Dicke parameter \( \eta = \sqrt{\frac{e R}{\nu}} \)

\[ T_{\text{gate}} \sim \frac{1}{\sqrt{\nu}} \quad \nu \sim 10 \text{ MHz}, \text{i.e.} \ T_{\text{gate}} \sim \mu \text{ s} \]

limits given by trap design
The rest of the lecture …

- Push gate

- Geometric phase gates

- Optimal Control Gates
  - what is the best gate for given resources?

- [Examples]
  - fast gate with short laser pulses
  - fast gate with continuous laser pulses
  - engineering spin Hamiltonians …

J.I. Cirac & PZ

D. Leibfried et al.
NIST

J. Garcia-Ripoll
J.I. Cirac,
PZ
Another example for a 2-qubit gate …

Push gate

- converting "spin to charge"

\[
\beta^2 x^2(t) \quad d \quad \bar{x}_1(t) \quad \bar{x}_2(t)
\]

- spin dependent optical potential

\[
\begin{align*}
&\text{different AC Stark shifts} \\
&\text{fine structure}
\end{align*}
\]

- state dependent interaction

- accumulate different energy shifts along different trajectories: 2-qubit gate

- \text{robust: temperature insensitive} 😊

V(R)

\[
\begin{array}{c}
\text{1} \\
\text{2}
\end{array}
\]

state dependent interaction
Push gate

- converting "spin to charge"
- Hamiltonian

\[
H = \sum_{i=1}^{N} \left[ \frac{1}{2m} p_i^2 + V_{e,i}(x_i) - F_i(t) \sigma_z^i x_i \right] + \sum_{i<j} \frac{e^2}{4\pi\varepsilon_0} \frac{1}{|x_i - x_j|}
\]
- spin dependent optical potential
- qubit dependent displacement of the ion
- different AC Stark shifts
Geometric Phase [Gate]: One Ion

• Goal: geometric phase by driving a harmonic oscillator
• Hamiltonian

\[ H = \frac{1}{2} \hbar \omega (\hat{p}^2 + \hat{x}^2) - f(t) \hat{x} \]

• Time evolution

\[ |\psi_0\rangle = |z_0 = x_0 + ip_0\rangle \longrightarrow |\psi_t\rangle = e^{i\phi_t} |z_t = x_t + ip_t\rangle \]

• Solution

\[ \frac{d}{dt} z = -i\omega z + i \frac{1}{\sqrt{2}} f(t) \quad \rightarrow \quad z_t = e^{-i\omega t} \left( z_0 + \frac{i}{\sqrt{2}} \int_0^t d\tau e^{i\omega \tau} f(\tau) \right) \]

\[ \frac{d}{dt} \phi = \frac{1}{2\sqrt{2}} f(t)(z^* + z) \quad \text{classical evolution} \]

phase space

phase space

\( (x_0, p_0) \) \( (x_t, p_t) \)
• Condition:

After a given time $T$ the coherent wavepacket is restored to the freely evolved state

$$\int_0^T d\tau \ e^{i\omega \tau} f(\tau) \neq 0$$
• Rotating frame: \( \tilde{z}_t = \tilde{x}_t + i\tilde{p}_t = e^{i\omega t} z_t \)

\[
\frac{d\tilde{z}}{dt} = ie^{i\omega t} \frac{1}{\sqrt{2}} f(t) \\
\frac{d\phi}{dt} = \frac{d\tilde{p}}{dt} \tilde{x} - \frac{d\tilde{x}}{dt} \tilde{p} = 2 \frac{dA}{dt}
\]

• Phase

\[
\phi(T) = \text{Im} \frac{i}{\sqrt{2}} \int_0^T d\tau e^{i\omega \tau} f(\tau) \tilde{z}_\tau^* \\
= \text{Im} \frac{i}{\sqrt{2}} \left[ \int_0^T d\tau e^{i\omega \tau} f(\tau_1) \right] \tilde{z}_0^* + \frac{1}{2} \text{Im} \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 e^{i\omega (\tau_1 - \tau_2)} f(\tau_1) f(\tau_2)
\]

The phase does not depend on the initial state, \((x_0, p_0)\)
• Example

unperturbed    forced

\[ F(t) \propto \sin(2\omega t) \]

• The phase does not depend on the initial state, \((x_0, p_0)\) (temperature independent)
Geometric Phase Gate: Single Ion

- Hamiltonian

\[
H = \frac{1}{2} \hbar \omega (\hat{p}^2 + \hat{x}^2) - |1\rangle \langle 1| f(t) \hat{x}
\]

- Time evolution operator

\[
U(T) = e^{i\phi |1\rangle \langle 1|}
\]

\[
(\alpha |0\rangle + \beta |1\rangle) \otimes |z_0\rangle 
\]

\[
U(T) \rightarrow (\alpha |0\rangle + \beta e^{i\phi} |1\rangle) \otimes |z_T\rangle
\]

single ion phase gate

motion factors out
NIST Gate: Leibfried et al Nature 2003

• 2 ions in a running standing wave tuned to $\omega_r$

\[ H = \omega_r a^\dagger a - F(t)(\sigma^1_z + \sigma^2_z)(a_r + a_r^\dagger) \]

• If $F(t)$ is periodic with a period multiple of $\omega_r$, after some time the motional state is restored, but now the total phase is

\[ \phi = A\sigma^1_z\sigma^2_z \quad U(T) = \exp(i\phi\sigma^z_1\sigma^z_2) \]

• To address one mode, the gate must be slow $\ddot{\circ}$

\[ T \gg 2\pi/\omega_r \]
NIST Gate: Leibfried et al. Nature 2003

\[
2^{-1/2} \left[ \left| \downarrow\downarrow \right> - i \left| \uparrow\uparrow \right> \right]
\]

\[
230 \times T_{\text{trap}}
\]
Best gate?

• What is the best possible gate?

  requirements: ...

  constraints: ...

• ... an optimal control problem
N Ions

- We will consider $N$ trapped ions (linear traps, microtraps...), subject to state-dependent forces:

$$H = \sum_{i=1}^{N} \left[ \frac{1}{2m} p_i^2 + V_{e,i}(x_i) - F_i(t) \sigma_i^z x_i \right] + \sum_{i<j} \frac{e^2}{4\pi\varepsilon_0 |x_i - x_j|} \frac{1}{P}$$

- normal modes

$$H = \sum_i \left[ \frac{1}{2m} P_i^2 + \frac{1}{2} m v_k^2 Q_k^2 \right] - \sum_k F_i(t) \sigma_i^z M_{ik} Q_k$$

- unitary evolution operator

$$U(T) = \exp \left( i \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z \right)$$

- constraints on forces

$$\int_0^T d\tau e^{i\omega_k \tau} F_i(\tau) = 0, \quad \forall i, \forall k$$

general Ising interaction
Quantum Control Problem

- **Target:** the Ising interaction, is a function of the forces

\[ J_{ij} = \frac{1}{2m\hbar} \int_0^T \int_0^T d\tau_1 d\tau_2 \, F_i(\tau_1) F_j(\tau_2) G_{ij}(\tau_1 - \tau_2). \]

The kernel \( G \) depends only on the trapping potential.

- **Constraints:** displacements, \( z_k \), depend both on the forces and on the internal states. To cancel them, we must impose

\[ \int_0^T d\tau \, e^{i\omega_k \tau} F_i(\tau) = 0, \quad \forall i, k \]

- **Additional constraints:** the total time, \( T \); smoothness & intensity of the forces, no local addressing of ions …

**fastest gate?**
More results

• **Theorem:** For N ions and a given Ising interaction $J_{ij}$, it is always possible to find a set of forces that realize the gate

$$\exp\left(-iT \sum_{ij} J_{ij} \sigma^i_z \sigma^j_z \right),$$

although now the solution has to be found numerically.

• **Applications:** Generation of cluster states, of GHZ states, stroboscopic simulation of Hamiltonians, adiabatic quantum computing,…

The time, $T$, is arbitrary!

|cluster state| $|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_z + |1\rangle_z)$ |
|---|---|
|GHZ state| $|\phi\rangle_{GHZ} \sim e^{-ij^2t} |+\rangle \equiv e^{-i\sum_i \frac{1}{2} \sigma^i_z} \sim |00 \ldots\rangle + |11 \ldots\rangle$ |
Engineering cluster and GHZ states

Cluster state N=10

GHZ state N=20

These examples use a common force: $F_i(t) = x_i g(t)$

Juanjo Garcia-Ripoll has calculated this up to N=30 ions
A final remark:
Analogies with Condensed Matter Hamiltonians

• Cavity QED: optical / microwave CQED / ion trap vs. JJ + transmission line
  see Yale & Delft

• Trapped Ion vs. Nanomechanical Systems + Quantum Dot / Cooper pair box
Trapped ion
• trapped ion driven by laser

Nano-mechanical system
• quantum dot in a phonon cavity

I. Wilson-Rae, PZ, A. Imamoglu, PRL 2004
Spectroscopy of Quantum Dots

$|M = -\frac{1}{2}\rangle$ $|M = +\frac{1}{2}\rangle$

25 meV

$\sigma^+$ $\sigma^-$

heavy holes

$|M = -\frac{3}{2}\rangle$ $|M = +\frac{3}{2}\rangle$

light holes

see A. Imamoglu's lecture
Quantum dot in a phonon cavity

• system

• Thin rod elasticity: $\lambda_p \sim L >> b,d$

four branches with no infrared cutoff:

✓ flexural & in-plane bending $\omega \sim q^2$

$$\frac{\partial^2 u}{\partial t^2} + \frac{EI_2}{\rho} \frac{\partial^4 u}{\partial y^4} = 0$$

✓ torsional & compression modes $\omega \sim q$

Q = 25,000 has been measured for modes with $\omega = 2 \pi \times 200$ MHz.
Hamiltonian: single mode coupled to a QD via deformation coupling

\[ H = \hbar \omega_0 b_0^\dagger b_0 + \hbar [\Delta - \omega_0 \eta (b_0 + b_0^\dagger)] |e\rangle \langle e| + \hbar \frac{1}{2} \Omega (|e\rangle \langle g| + \text{h.c.}) \]
• unitary transformation to polaron representation: \( B = e^{\eta (b_0 - b_0^\dagger)} \)

\[
H = \hbar \omega_0 b_0^\dagger b_0 - \hbar \Delta |e\rangle \langle e| + \hbar \frac{1}{2} \Omega \left( e^{\eta (b_0 - b_0^\dagger)} |e\rangle \langle g| + \text{h.c.} \right)
\]

\[
\uparrow
\]

looks like ion trap Hamiltonian with effective Lamb-Dicke parameter (replacing the recoil): \( \eta \sim 0.1 \)

\[
H = \frac{p^2}{2M} + \frac{1}{2} M v^2 X^2 - \Delta |e\rangle \langle e| - \frac{1}{2} \Omega (e^{i k L X}|e\rangle \langle g| + \text{h.c.})
\]

\[
\equiv e^{i \eta (a + a^\dagger)}
\]
• another example: Cooper pair box

cooling: I. Martin, S. Shnirman; L. Tian, …

"cavity QED": K. Schwab et al.