Quantum Optics
& Quantum Information:
the “quantum repeater”

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University of Innsbruck

Austrian Academy of Sciences

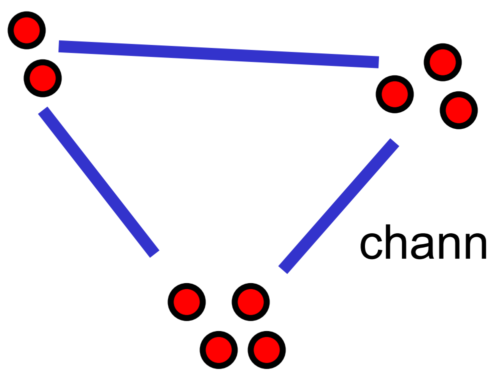
SFB
Coherent Control of Quantum Systems

€U TMR & IP

Institute for Quantum Information

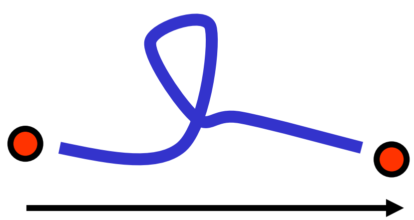
motivation slide in lecture 1

Quantum Network



- Nodes: local quantum computing
  - store quantum information
  - local quantum processing

- Channels: quantum communication
  - transmit quantum information

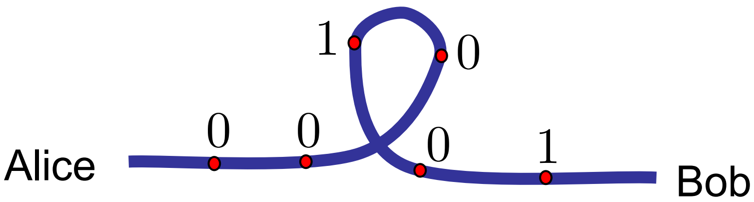


1. Introduction

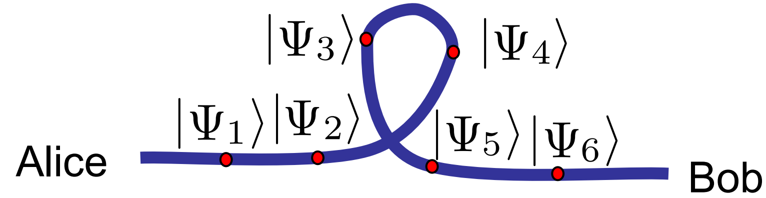
- basic ideas
  - quantum communication & quantum cryptography
  - need for a quantum repeater

Quantum Communication

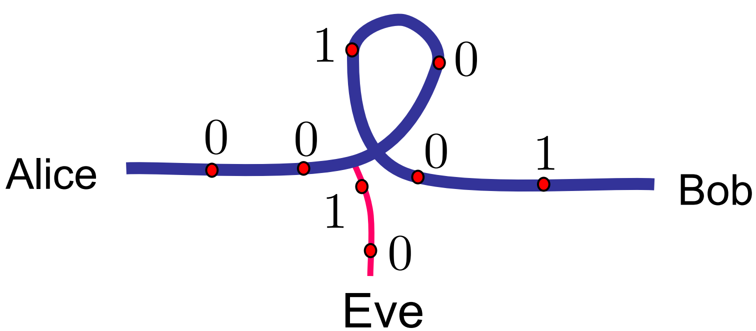
- classical communication

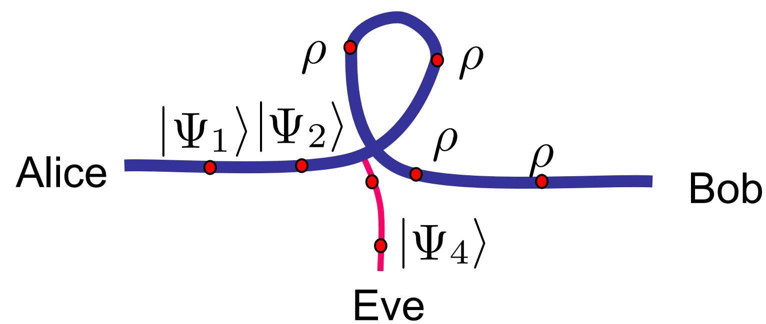


- quantum communication

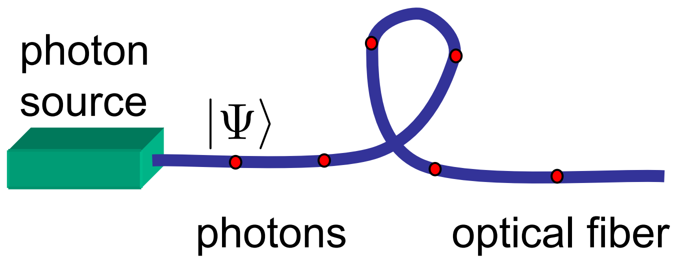


- ✓ quantum networks
- ✓ cryptopgraphy





- implementation: photons



$|0\rangle = |\updownarrow\rangle$ vertical polarization

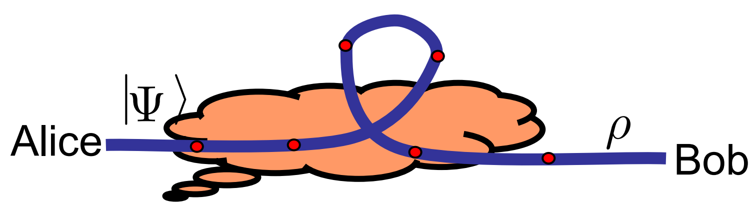
$|1\rangle = |\leftrightarrow\rangle$ horizontal polarization

- problem: decoherence

1. photons are absorbed:

- probability a photon arrives: $P = e^{-L/L_0}$
- quantum communication is limited to short distances (< 100 Km).

2. states are distorted:



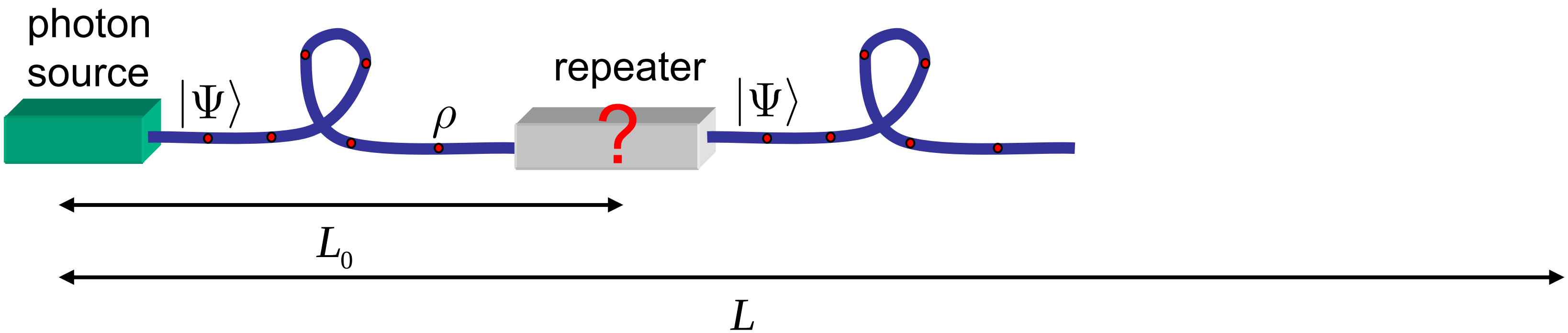
fidelity $F = \langle\Psi|\rho|\Psi\rangle < 1$

We cannot know whether this is due to decoherence or an eavesdropper.

... to regain fidelity we want:

Quantum Repeater

- goal



- properties:
  - overall fidelity $F = \langle\Psi|\rho|\Psi\rangle \simeq 1$
  - scaling of resources, e.g. communication time $\sim L^\eta < e^{L/L_0}$ with L length of communication channel

- Q.: concept of a repeater? implementation?

H. Briegel et al., PRL 98, PRA 99

Entanglement based quantum communication schemes

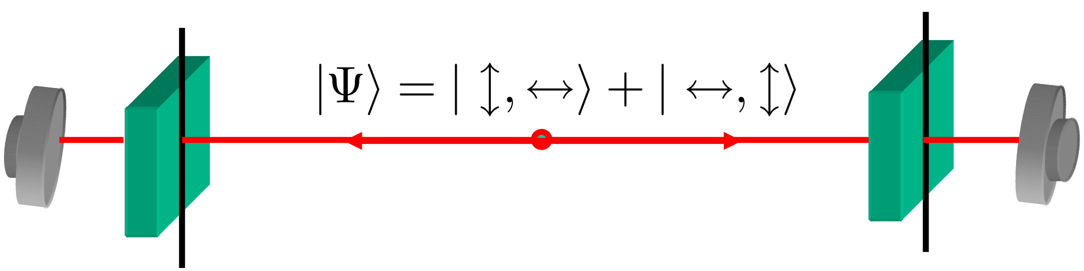
- entangled state

$$|\Psi\rangle = |0,1\rangle + |1,0\rangle$$

Alice ———————— Bob

EPR correlations

- example: photon pair



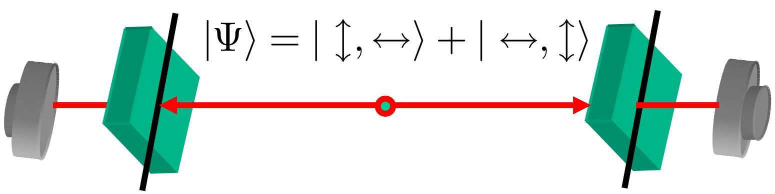
Application:

- secret communciation using entangled states: Ekert protocol

1. Check that particles are indeed entangled.

Correlations in all directions.

$$|\Psi\rangle = |\updownarrow, \leftrightarrow\rangle + |\leftrightarrow, \updownarrow\rangle$$



No eavesdropper present

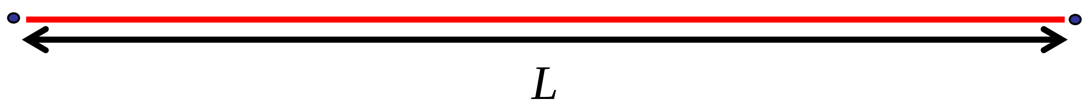
2. Measure in A and B (z direction):

Alice	Bob
0	0
1	1
1	1
1	1
0	0

Send secret messages

Quantum Repeater

- Q.: Long distance quantum cryptography?
- We can do (long) distance quantum cryptography if we have a (long) distance high fidelity EPR pair.



- Q.: How to generate a (long) distance high fidelity EPR pair over a noisy channel?
- Quantum repeater: generate a high fidelity EPR pair $F \sim 1$ over a channel of length L with a small number of trials $\sim L^{\eta}$

2. Quantum communication over a noisy channel

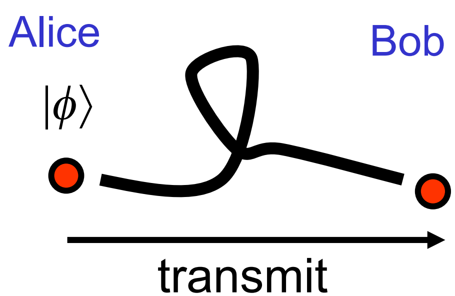
is based on …

- Teleportation
- Nested purification protocol

= quantum repeater protocol

2.1 Quantum Communication over a Noisy Channel

- **goal:** transmit an unknown quantum state reliably



Example:

✓ qubit

✓ entangled with rest

- **noise / decoherence:** as the basic problem

✓ measure of reliability: fidelity

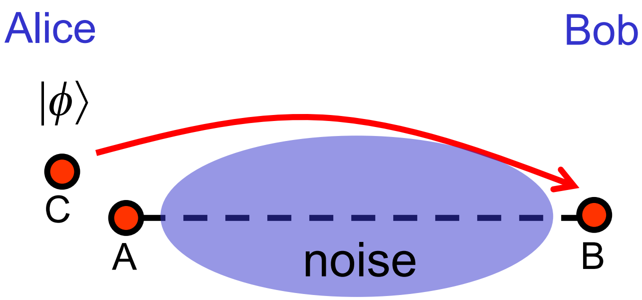
$$F = \langle\phi|\rho|\phi\rangle = \begin{cases} 1 & \text{perfect} \\ \sim 1 & \text{good} \\ \ll 1 & \text{bad} \end{cases}$$

✓ F scales with the distance

$$F \sim e^{-L/L_0}$$

- **teleportation**

  If Alice and Bob share a singlet (EPR) pair as a resource, we can teleport the unknown quantum state



$$|\text{EPR}\rangle \sim |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B$$

Protocol:

✓ CNOT between A&C

✓ measure A&C

✓ classical communication Alice to Bob

✓ rotate B

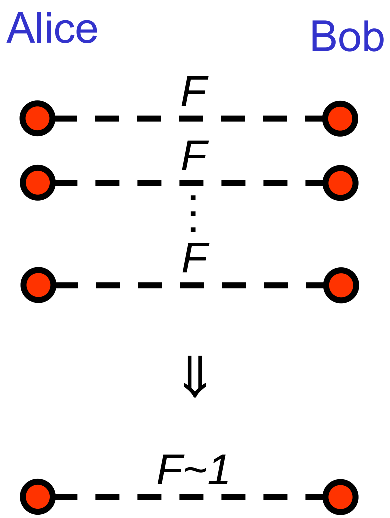
- **Summary:**

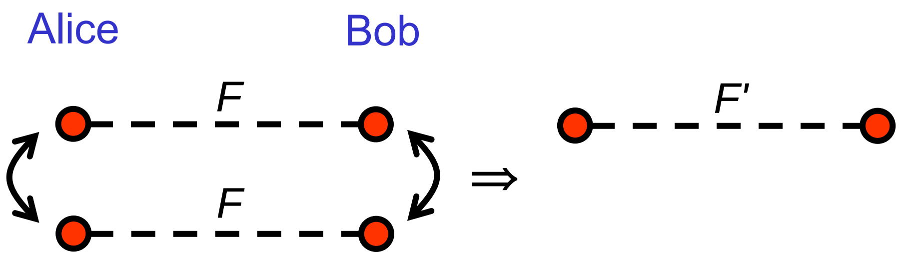
  If we can create a distant EPR pair *F~1*, then the problem of quantum communication is solved.

  However, a noisy channel produces only EPR pairs with *F<1*.

IBM, Oxford '96

- **purification**



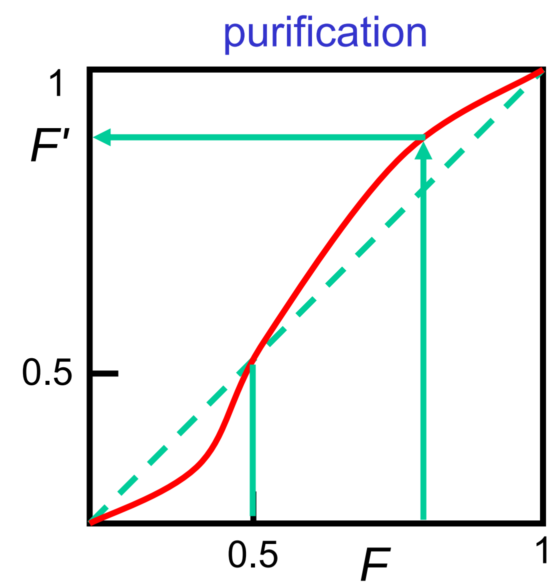


Protocol:

✓ CNOT & measurement A_2& B_2

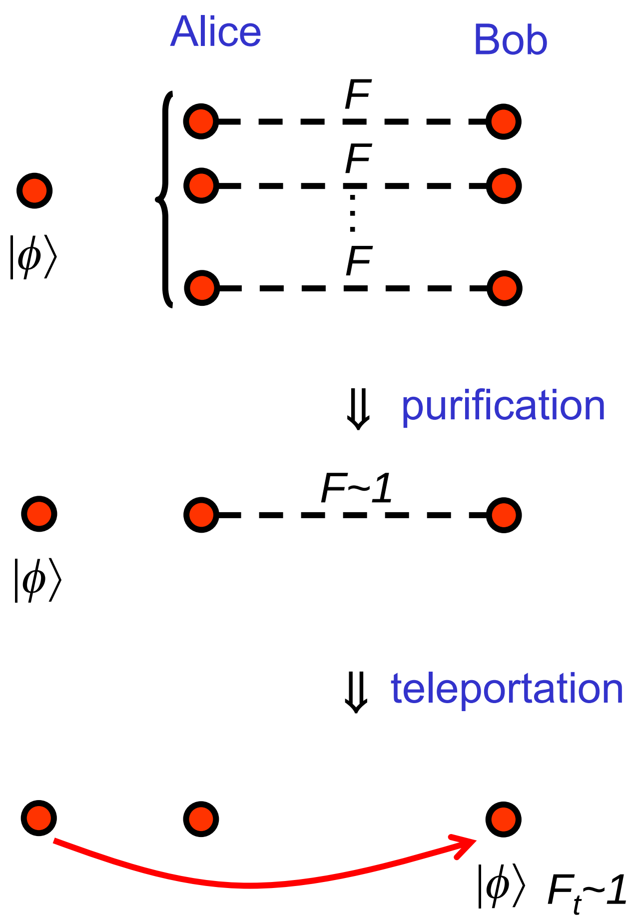
✓ keep if 00 or 11, and throw away if 01 or 10

✓ Alice performs rotation



If F> ½ then F'>F

- **Summary: quantum communication via noisy channel**



Remarks:

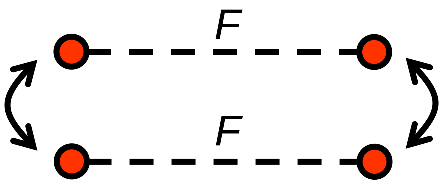
- purification condition F> ½ imposes a limitation on the length of the channel

$$F \sim e^{-L/L_0} > \frac{1}{2} \Rightarrow L < L_0$$

- interactions and measurements will lead to errors

Remark 1: Purification Schemes

- **Scheme A: Bennett et al. (IBM)**



$$\sigma = \rho_W(F)^{(A_1B_1)} \otimes \rho_W(F)^{(A_2B_2)}$$

$$\rho_W(F) = x|S\rangle\langle S| + \tfrac{1-x}{4}\,\hat{1} \quad \text{with} \quad x = \tfrac{4F-1}{3}$$

Werner state

Bell states

$$|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \equiv |\psi^-\rangle \quad \text{singlet}$$

$$\left.\begin{aligned} |T_0\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \equiv |\psi^+\rangle \\ |T_1\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \equiv |\phi^+\rangle \\ |T_2\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) \equiv |\phi^-\rangle \end{aligned}\right\} \text{triplet}$$

Note: depolarization step to Werner state = random homogeneous unitary rotation

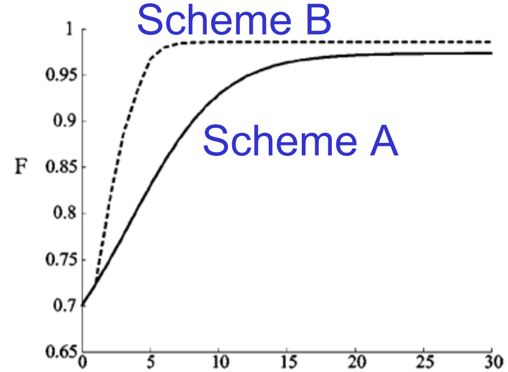
$$\rho \longrightarrow \mathcal{D}^{(AB)}(\rho) = \int d\mu_U (U_A \otimes U_B)\rho(U_A \otimes U_B)^\dagger$$

- **Scheme B: Deutsch et al. (Oxford)**

$$\rho(x) = x_0|\phi^+\rangle\langle\phi^+| + x_1|\phi^-\rangle\langle\phi^-| + x_2|\psi^+\rangle\langle\psi^+| + x_3|\psi^-\rangle\langle\psi^-|$$

comparing efficiency:

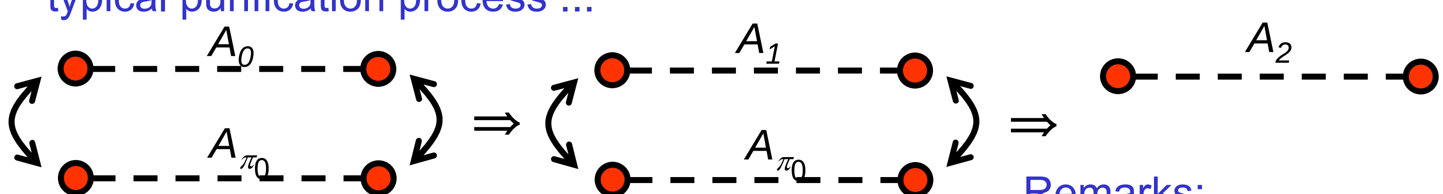
fidelity vs. number of successful purification steps



number of purification steps

- **Scheme C:** scheme B, but always purify from auxiliary pair π_0 with *constant* fidelity A_{π_0}

typical purification process ...



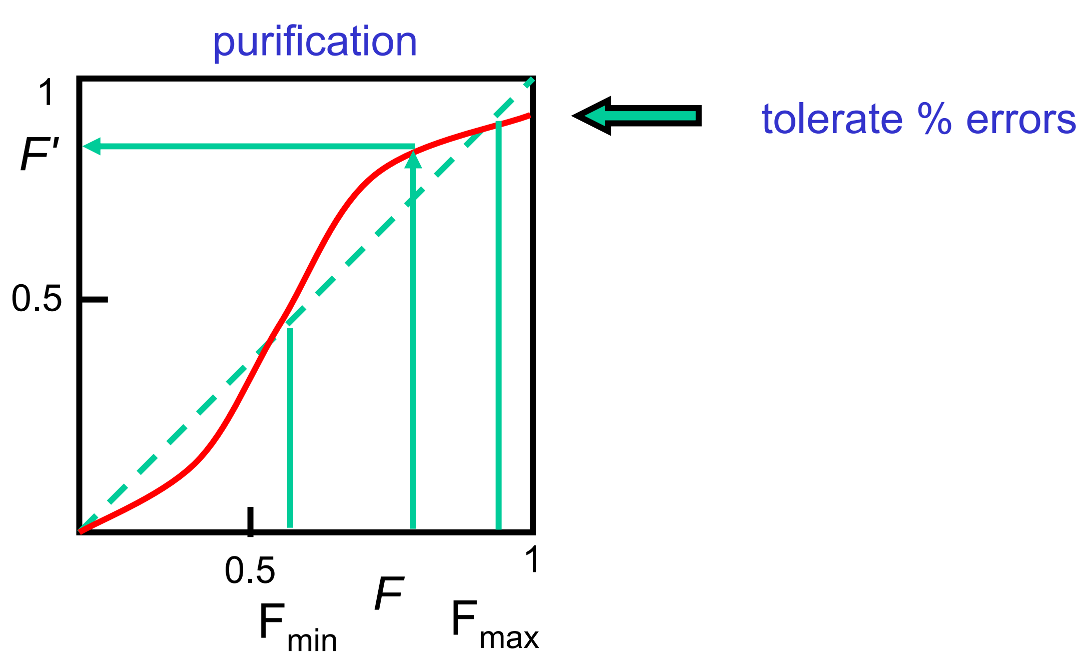
Remarks:

✓ very low qubit resources

✓ purifies to a *finite* final fidelity

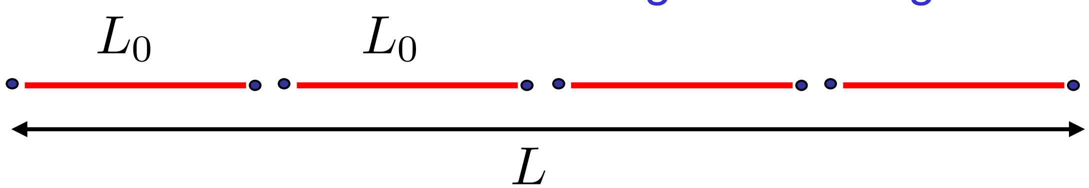
Remark 2: Errors in local operations and measurements

- errors in local operations and measurements are chararcterized by a fidelity parameter
  - F_{CNOT} ~ probability that in a CNOT we obtain the ideal state if we start with a pure state
  - F_{rot} ~ single bit rotation
  - F_{meas}~measurement
- nested purification protocol in the presence of errors

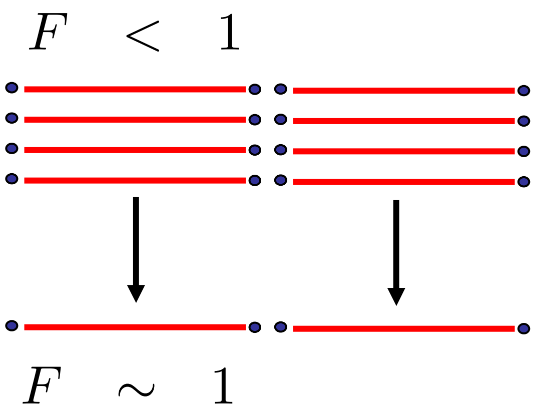


2.2 Quantum repeater: the concept

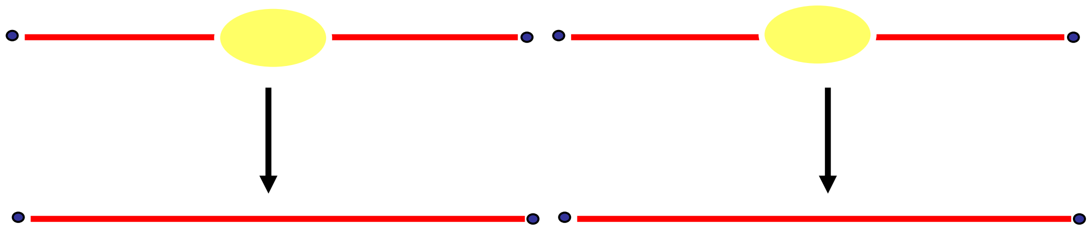
- **goal:** generate *long distance entangled pairs* with fidelity *F ~ 1* in a small number of trials $\sim L^{\eta}$
- **key ideas:**
  - divide transmission channel into segments and generate pairs



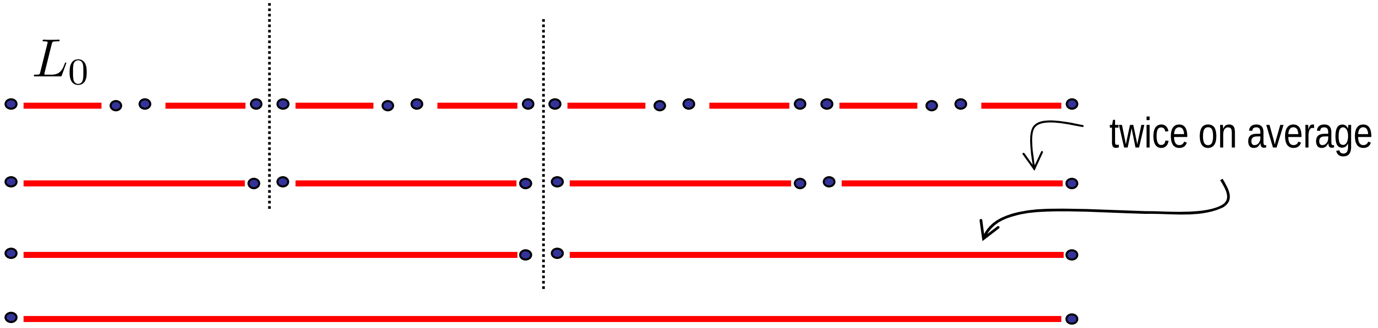
  - purification



– connect pairs to extend length by entanglement swapping



- putting all of this together



$$L = 2^n L_0$$

- efficiency:
  – number of elementary operations ~ L
  – with purification $\sim L^{\log_2 L}$

... a few numbers

- Resources and time needed for creating a distant EPR pair via optical fibers

TABLE I. Resources and time needed for creating a distant EPR pair via optical fibers. See text for more details.

	Continental scale		Intercontinental scale	
local resources at one node →	Resources	Time	Resources	Time
A	1.58×10^{9}	3.88×10^{-2}	9.01×10^{12}	0.298
B	329	1.34×10^{-2}	4118	0.103
C	7	0.77	10	15.69

$N=2^7=128 \approx 1000$ km $\quad$ $N=2^{10}=1024 \approx 10000$ km

parameters:

$F \sim 0.96$
errors = 0.5%
$l_{segment} = 10$ km
$\tau_{pair} \approx 10^{-4}$ sec

$\tau_{operation} \approx 10^{-5}$ sec

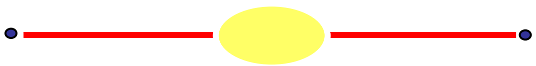
3. Quantum Repeater: Implementations with Photons and Atoms

- deterministic quantum state transfer (with high Q cavities)
- probabilistic EPR pair generation (with atomic ensembles)

Quantum repeater: implementation

- **Requirements:**
  - generate entanglement

  - store entangled states and
    perform collective local operations



- **Probabilistic protocols**

  Since the quantum repeater protocol to build the EPR resource is probabilistic,

  - *probabilistic* generation of entanglement,
  - *probabilistic* entanglement swapping
  - …

  is sufficient.

Generation of distant EPR pairs

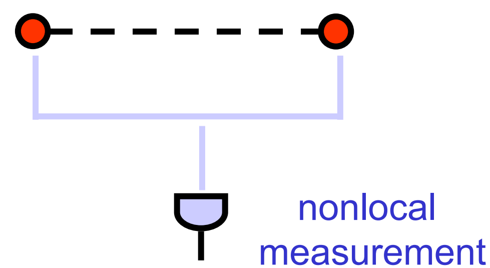
- Deterministic vs. probabilistic generation of EPR pairs

1. "quantum engineering"= deterministic

$$|0\rangle_A|0\rangle_B \longrightarrow |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B$$

difficult

2. "quantum gambling"=probabilistic

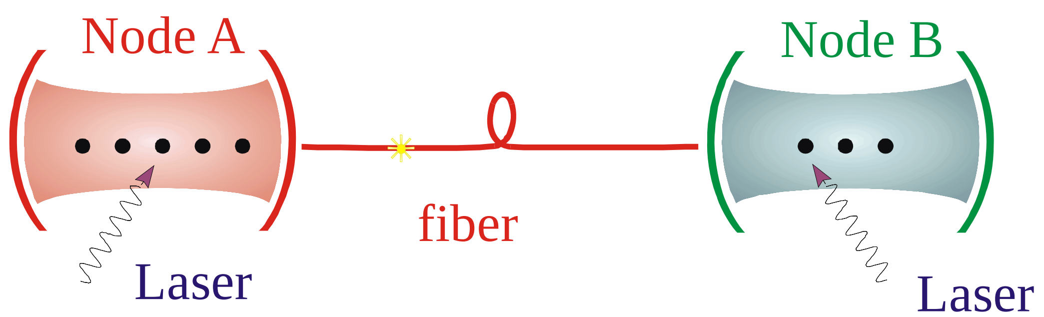


comparatively easy

- Note: purification is probabilistic, and thus a probabilistic generation of EPR pairs is sufficient

Deterministic transmission (& EPR generation)

- In the early days (~1997): **deterministic transmission** of qubits

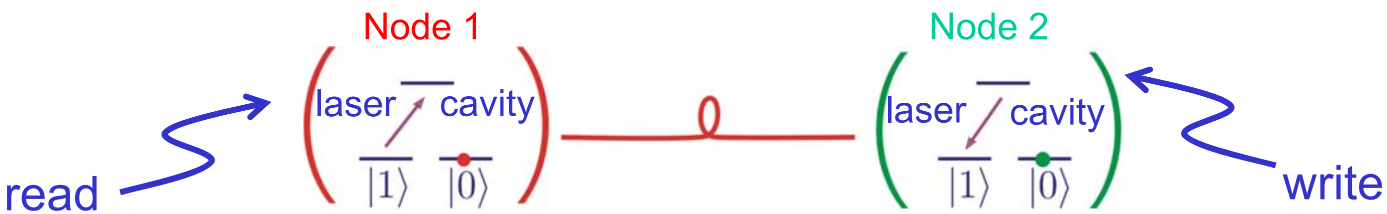


$|\phi\rangle \xrightarrow[\text{transmit}]{\hspace{6em}}$

- memory: atoms → • databus: photons → • memory: atoms

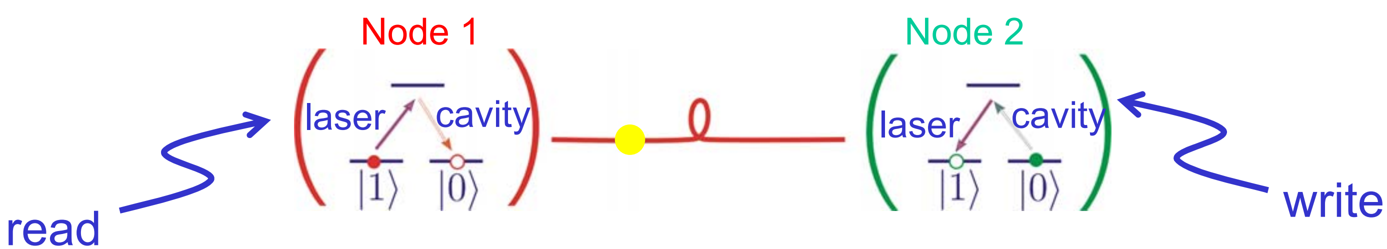
- requirements: ☹
  - ✓ single atoms
  - ✓ high-Q cavities

- **sending the qubit in state 0**



$$|0\rangle|0\rangle \quad \rightarrow \quad |0\rangle\ |0\rangle$$

- **sending the qubit in state 1**



$$|1\rangle|0\rangle \quad \rightarrow \quad |0\rangle\ |1\rangle$$

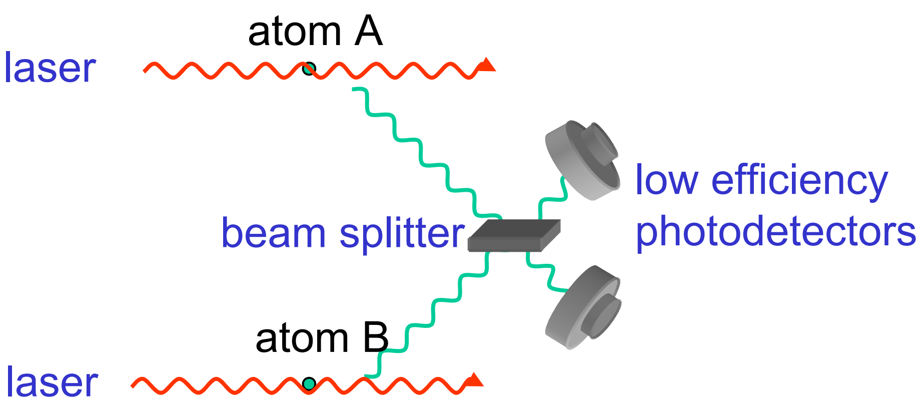
- **superpositions**

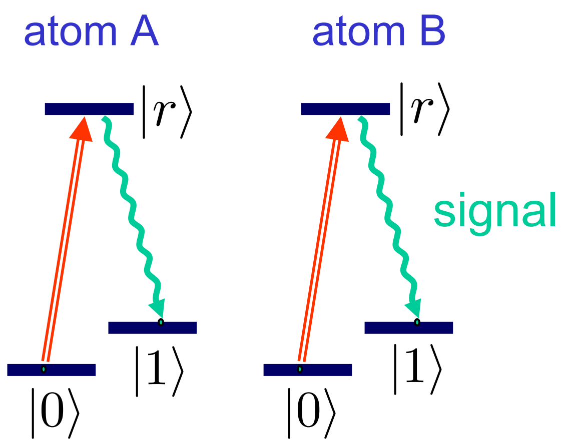
$$[\alpha\ |0\rangle + \beta|1\rangle]\ \ |0\rangle \quad \longrightarrow \quad |0\rangle\,[\alpha\ |0\rangle + \beta\ |1\rangle]$$

Cabrillo, IC &PZ, PRA 99

Probabilistic entanglement of single atoms

- entanglement generation





- Weak (short) laser pulse, so that the excitation probability is small.

- If no detection, pump back and start again.

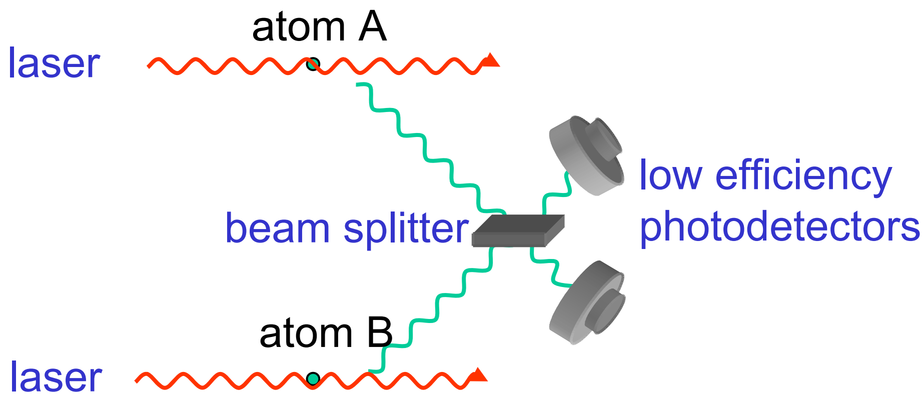
- If detection, an entangled state is created.

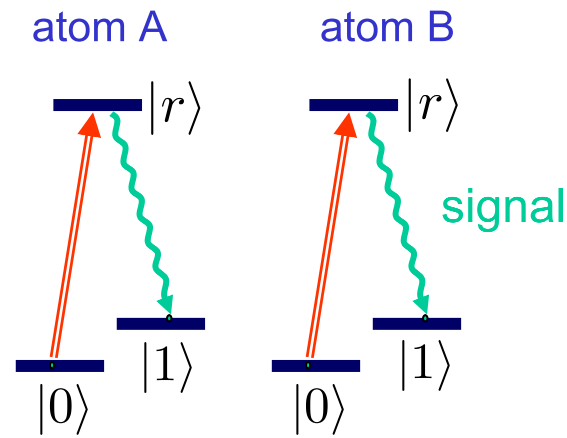
$$\sim |0,1\rangle + |1,0\rangle$$

[Note: with photon loss an exponentially large number of repetitions in L]

Probabilistic entanglement of single atoms

- entanglement generation



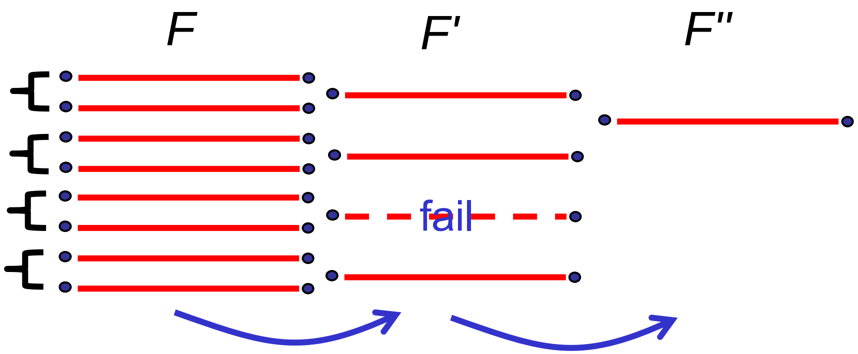


- Initial state: $|0,0\rangle\,|\text{vac}\rangle$
- After laser pulse: $(|0,0\rangle + \epsilon\underline{|0,r\rangle} + \epsilon\underline{|r,0\rangle} + O(\epsilon^2))|\text{vac}\rangle$
- Evolution: $|0,0\rangle|\text{vac}\rangle + \epsilon\sum_k (b_k|\underline{0,1}\rangle + a_k|\underline{1,0}\rangle)\,|1_k\rangle + O(\epsilon^2)$
- Detection: $b_k|0,1\rangle \pm a_k|1,0\rangle \simeq \underline{|0,1\rangle \pm |1,0\rangle}$

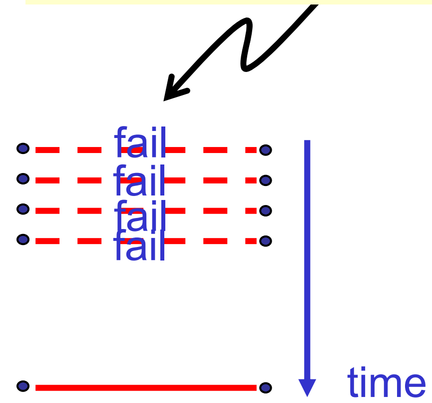
Remark: … built in purification in time

our entanglement generation scheme by detecting photon emission

- we can (also) purify by repeated trials *in time*

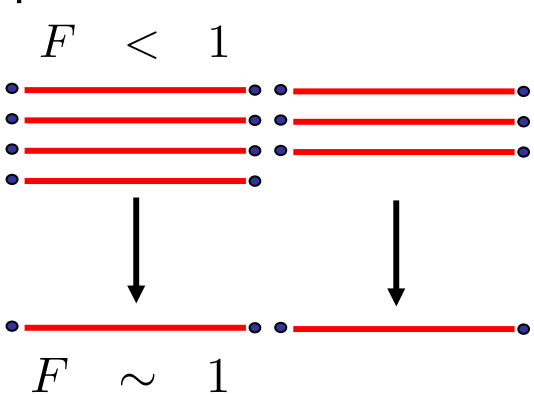


or



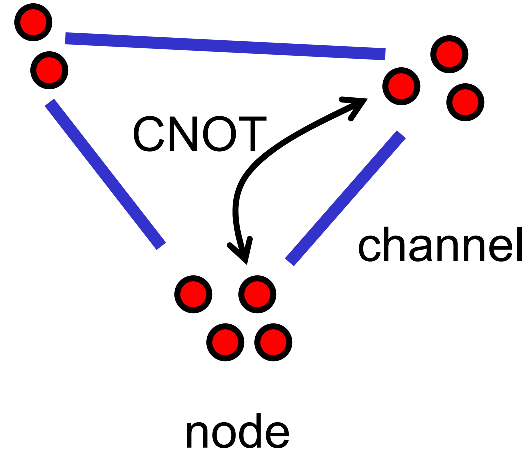
repeated trials in time

- *quantum memory* is essential because purification protocols are probabilistic



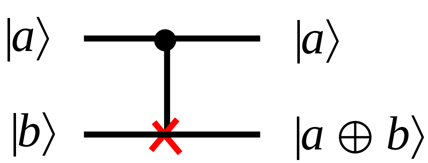
different segments will be finished at different times!

atoms in our entanglement generation scheme provide quantum memory



Probabilistic EPR creation as a resource for quantum computing

- *remote* CNOT quantum gate



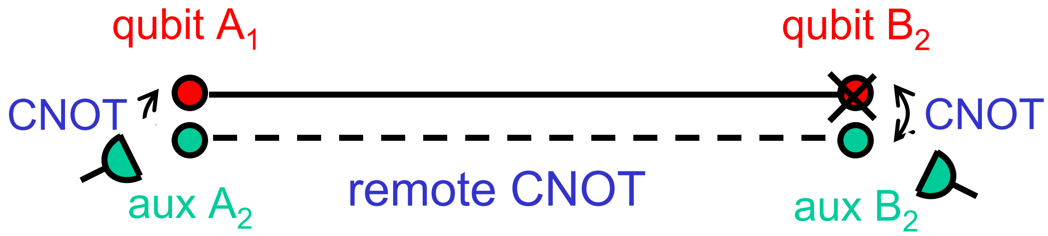
CNOT

truth table

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

Remote CNOT

- remote CNOT gate between qubits A_1 and A_2



protocol for remote CNOT:

- generate distant EPR pair probabilistically A_2&B_2
- apply local CNOT(A_1, A_2) and CNOT(B_1, B_2)
- measure A_2 in basis $|0\rangle, |1\rangle$, and B_2 in basis $|\pm\rangle \sim |0\rangle \pm |1\rangle$
- rotate A_1, B_1 depending on measurement outcome

Quantum repeaters with atomic ensembles

- atomic ensembles

- issues:
  - entanglement generation
  - connection
  - decoherence and imperfections
  - applications

requirements: ☺
✓ensembles
✓low-Q cavities

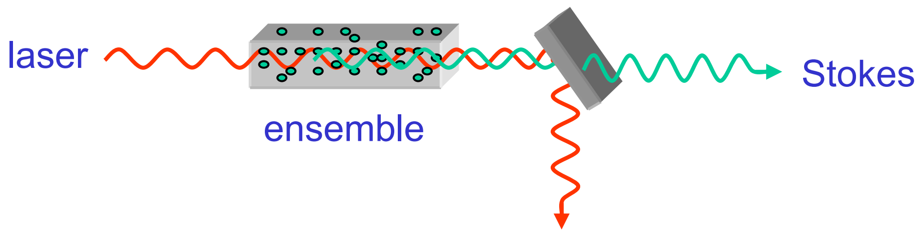
L. M. Duan et al., Nature Nov 2001

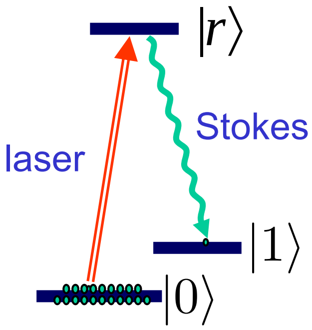
- first experimental results at Harvard, Caltech, Georgia Tech

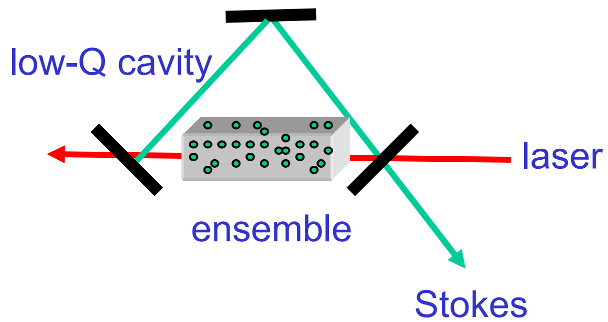
L. M. Duan et al., Nature Nov 2001

Atomic Ensembles

- system: cloud of cold atoms

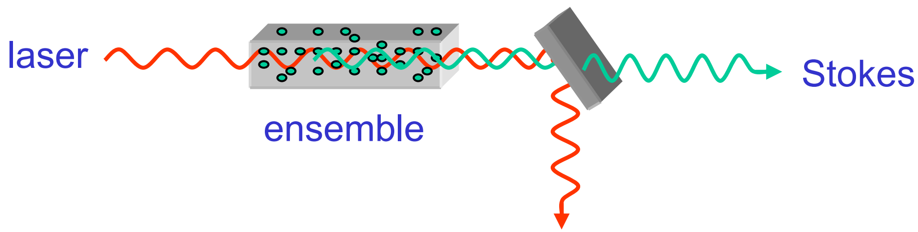


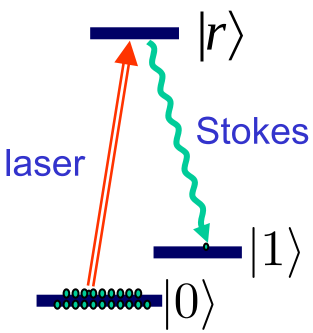




Atomic Ensembles

- system: cloud of cold atoms





- Raman process:

$|0\rangle^{\otimes N} \equiv |\text{vac}\rangle$ (atomic ground state)

$$\rightarrow \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} |0_1 \ldots 1_i \ldots 0_{N_a}\rangle \equiv a^\dagger |\text{vac}\rangle \quad \text{(single atomic excitation)}$$

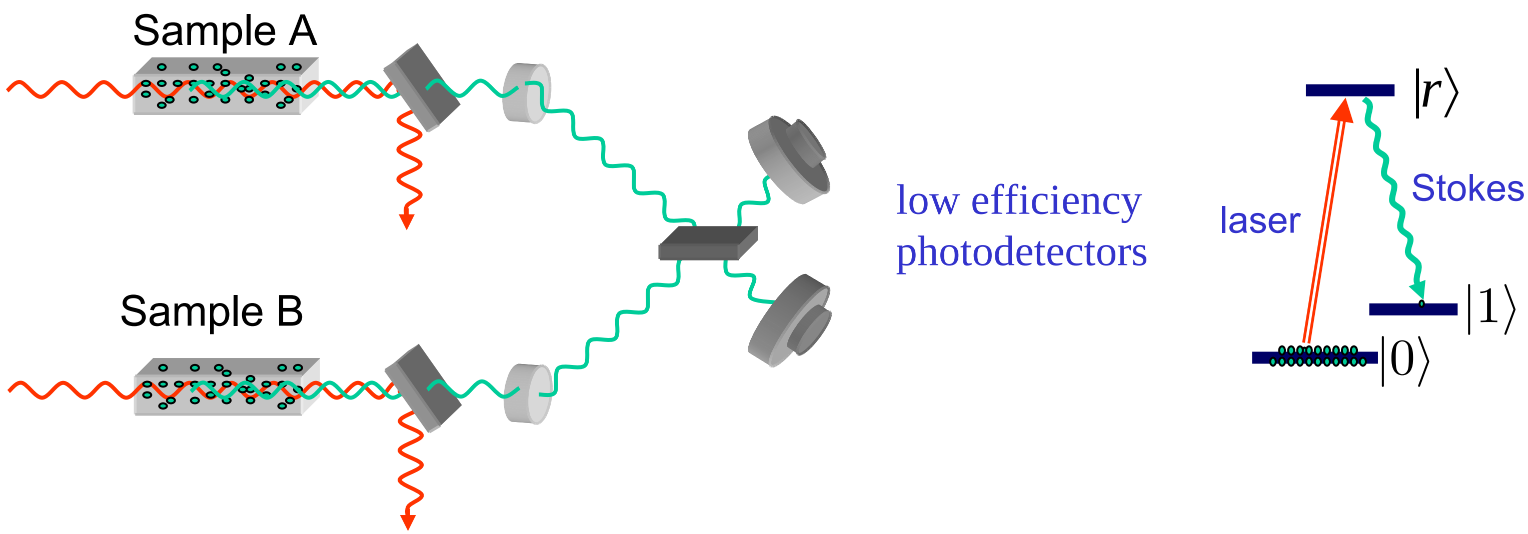
$$[a, a^\dagger] \approx 1$$

- state of atomic collective mode + Stokes photon

$$|\phi\rangle = |\text{vac}\rangle + \sqrt{p_c}\, a^\dagger c^\dagger_{\text{Stokes}} |\text{vac}\rangle + O(\sqrt{p_c}^2) \quad (p_c \ll 1)$$

... analogous to parametric downconversion

A. Generation of entanglement



$$|\phi\rangle_A \otimes |\phi\rangle_B = \left(|\text{vac}\rangle_A + \sqrt{p_c}\, a^\dagger c_s^\dagger |\text{vac}\rangle_A\right) \otimes \left(|\text{vac}\rangle_B + \sqrt{p_c}\, b^\dagger c_s^\dagger |\text{vac}\rangle_B\right)$$

measurement gives

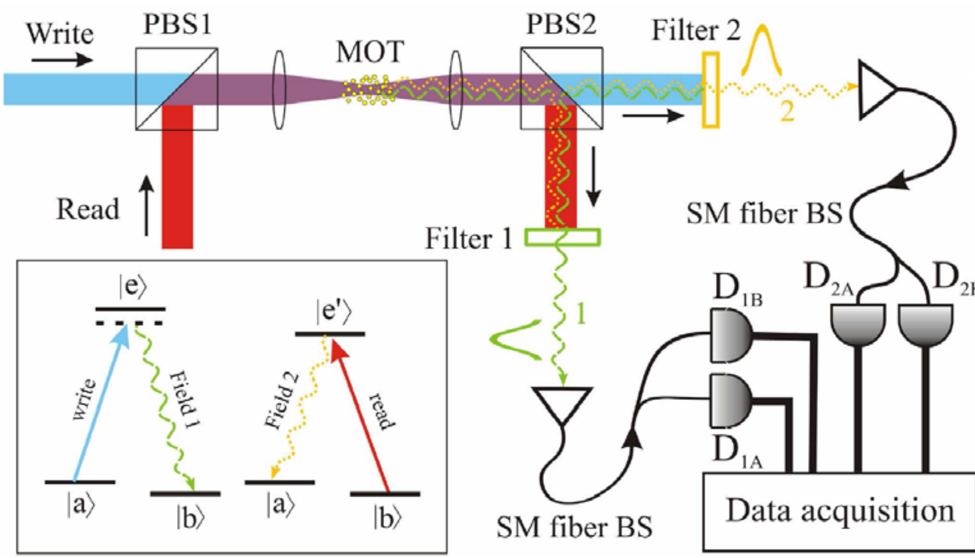
$$\begin{aligned} |\psi_{AB}^{\pm}\rangle &= (a^\dagger \pm b^\dagger)|\text{vac}\rangle \\ &\equiv |1_a, 0_b\rangle \pm |0_a, 1_b\rangle \end{aligned}$$

We have generated entanglement between collective atomic states

Experiments on write & read

M. Lukin & R. Walsworth (Harvard),
J. Kimble (Caltech),
Guo et al. (Hefei)

- **Harvard:** light storage, write & read N-photon states
- **Caltech:** single-photon generation from stored excitation in an atomic ensemble (preprint 2004)



photon state:

$$|\Phi_{12}\rangle = |00\rangle + \sqrt{\chi}\,|11\rangle + \chi|22\rangle + \ldots \quad (\chi \ll 1)$$

$$\tilde{g}_{1,1} = \tilde{g}_{2,2} \simeq 2$$

$$\tilde{g}_{1,2} = 1 + 1/\chi$$

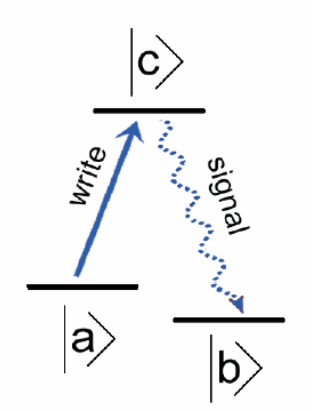
- $\tilde{g}_{1,1}$ and $\tilde{g}_{2,2}$ autocorrelation 1 and 2
- $\tilde{g}_{1,2}$ crosscorrelation between (1,2)

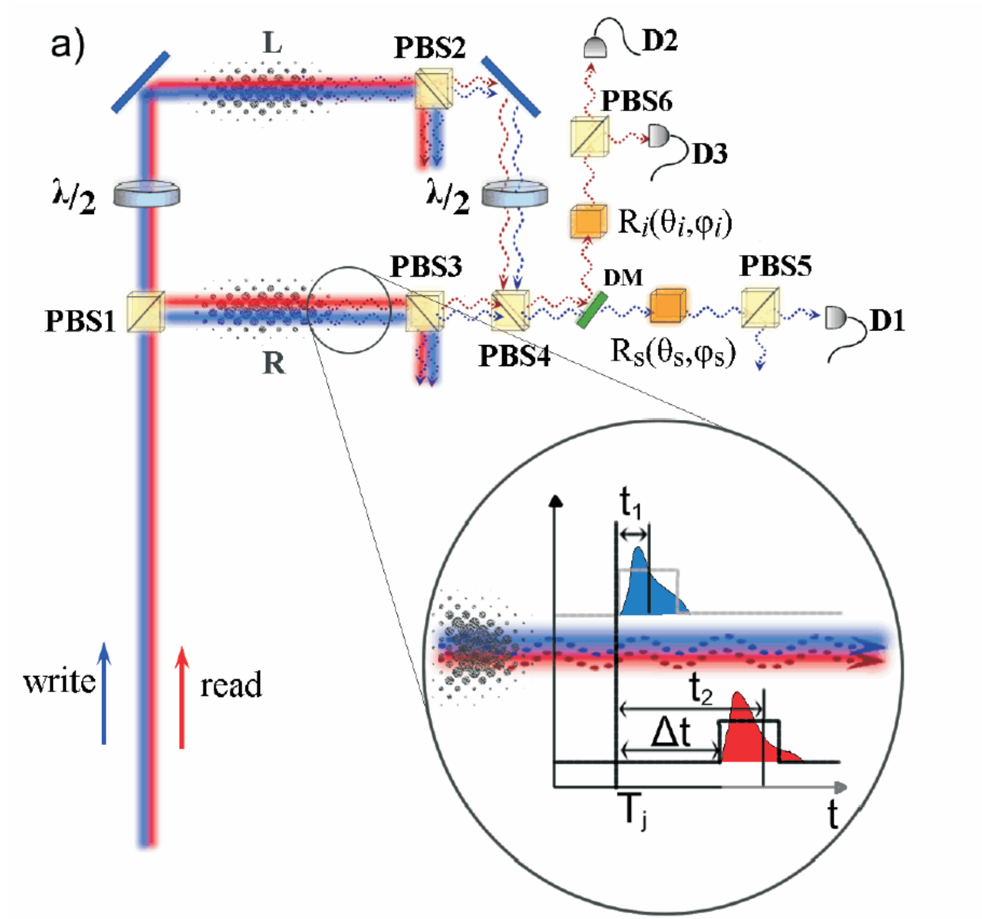
violation of Cauchy-Schwarz inequality:

$$R \equiv \frac{\tilde{g}_{1,2}^2}{\tilde{g}_{1,1}\tilde{g}_{2,2}} \leq 1 \qquad R_{\mathrm{exp}} \approx 53$$

- … and the related Hefei experiment

Experiment: Matsukevich & Kuzmich, Science 2004





- **write pulse**

$$|\Psi\rangle \sim |a_1\rangle \ldots |a_{N_L+N_R}\rangle |0_p\rangle_L |0_p\rangle_R + \chi(|L_a\rangle|1_p\rangle|0_p\rangle + |R_a\rangle|0_p\rangle|1_p\rangle)$$

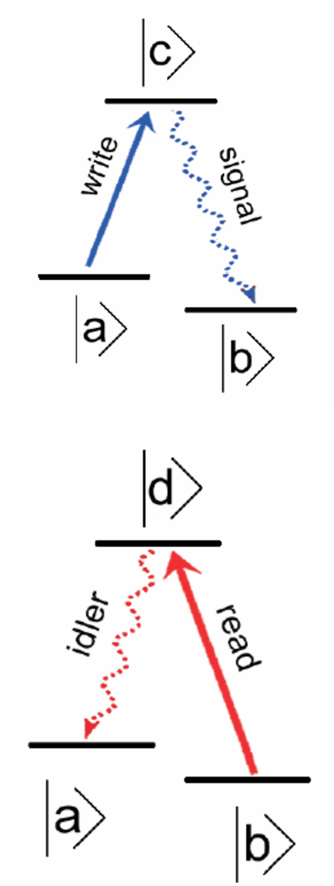
- scattered light: spatial → polarization

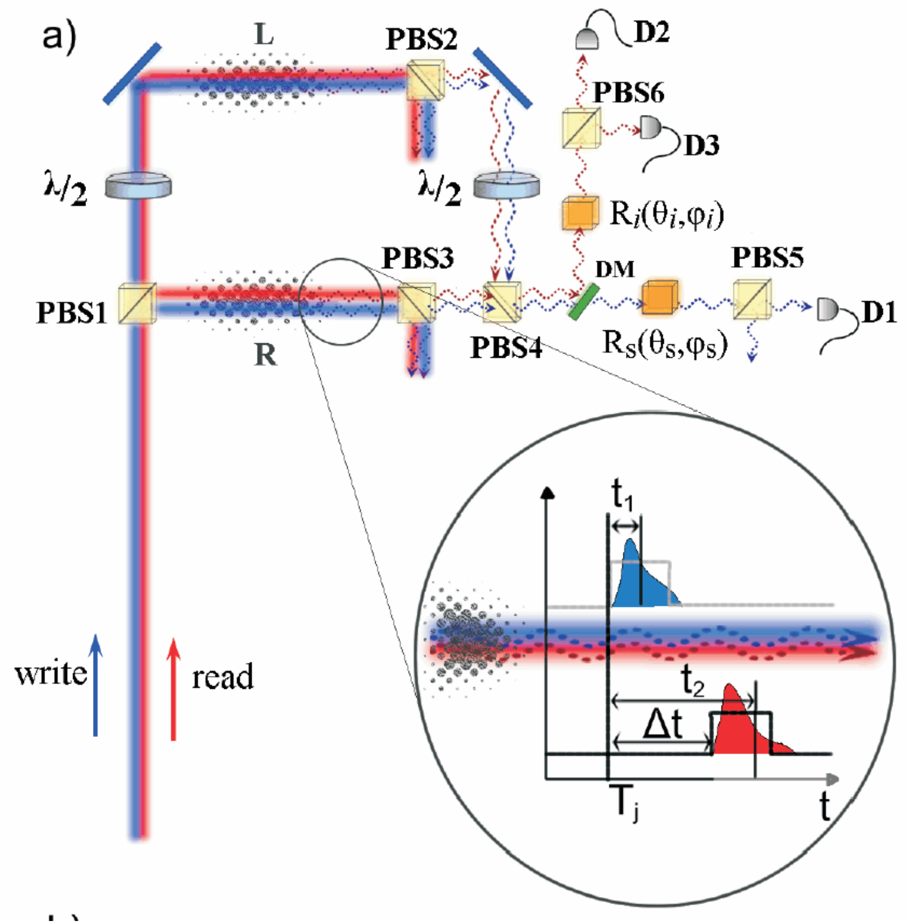
$$|1_p\rangle_L \rightarrow |H\rangle_s; \quad |1_p\rangle_R \rightarrow |V\rangle_s$$

- detect single photon + polarization

$$|\Psi\rangle_A = \cos(\theta_s)e^{-i\phi_s}|L_a\rangle + \sin(\theta_s)e^{i\eta_s}|R_a\rangle$$

Experiment: Matsukevich & Kuzmich, Science 2004

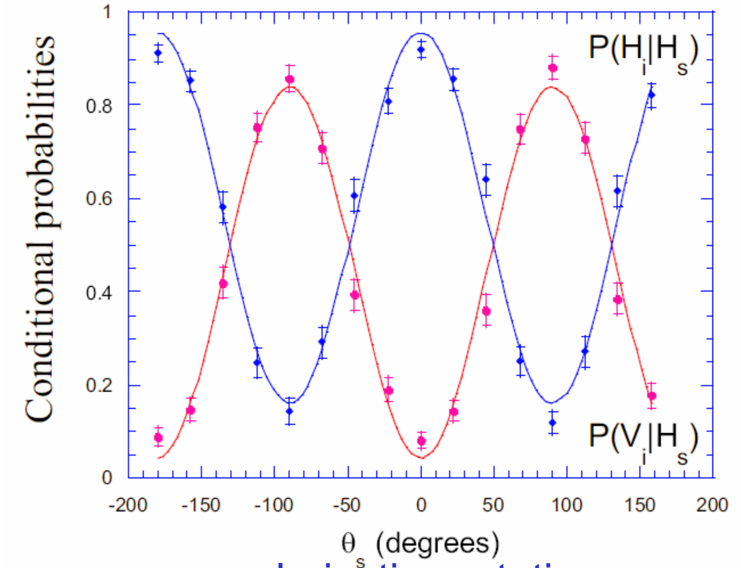




- **read pulse:** convert atomic excitation into a single photon

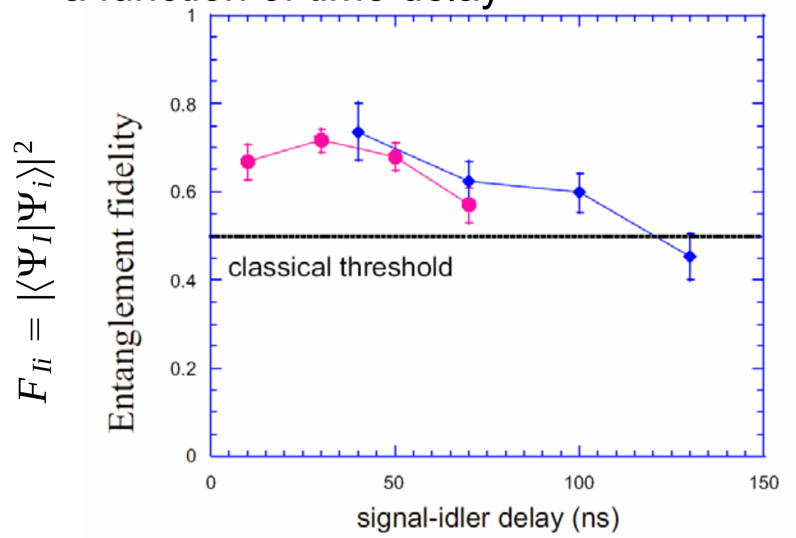
$$|\Psi\rangle_A = \cos(\theta_s)e^{-i\phi_s}|L_a\rangle + \sin(\theta_s)e^{i\eta_s}|R_a\rangle \rightarrow |\Psi\rangle_i = \cos(\theta_s)e^{-i\phi_s}|H\rangle_i + \sin(\theta_s)e^{i(\eta_i+\eta_s)}|V\rangle_i.$$

- conditional probabilities vs polarization rotation

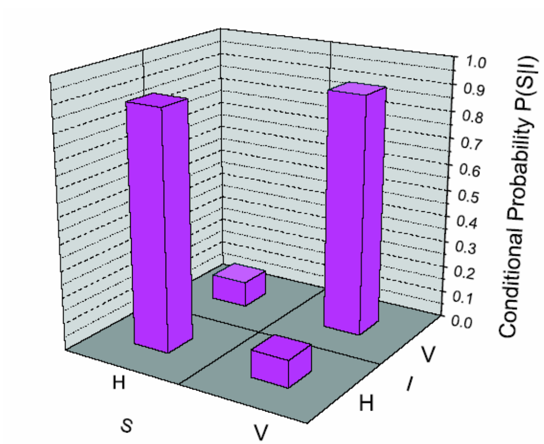


polarization rotation

- fidelity of reconstruction of the intended quantum memory state as a function of time delay

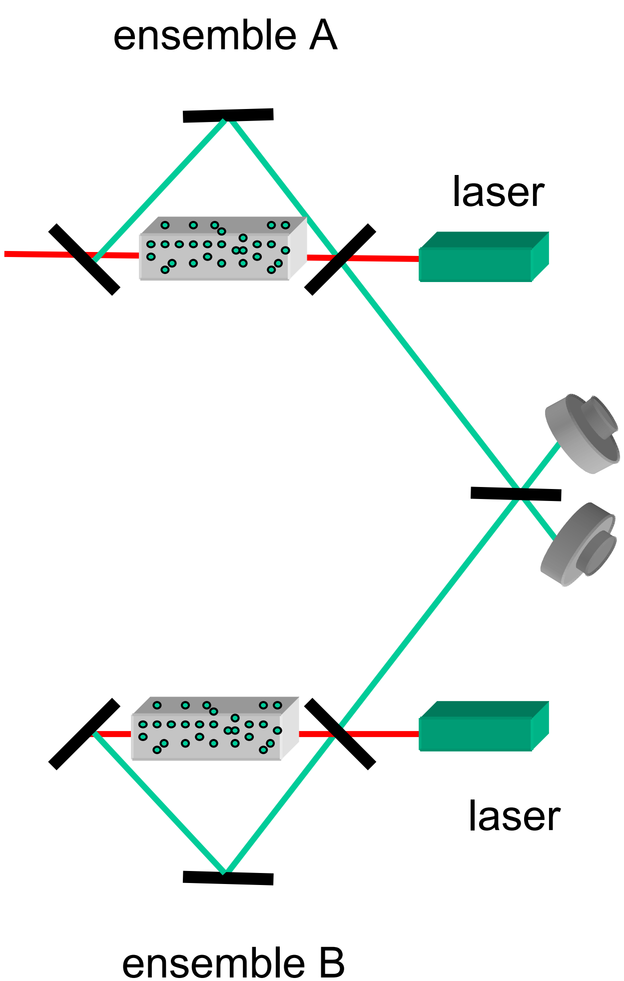


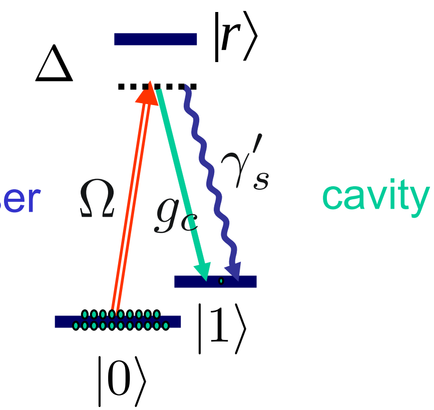
- measured conditional probabilities at the point of highest correlation



Appendix: technical part

Alternative model: atomic ensemble in a cavity



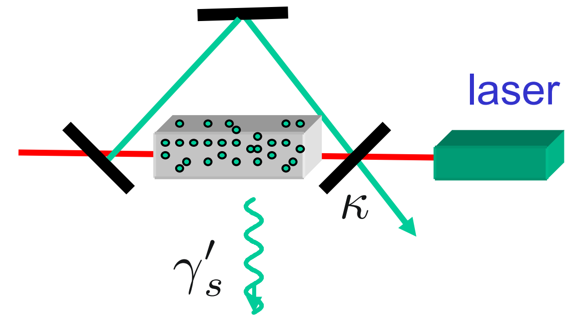


- Master equation: $\dot{\rho} = \ldots$

  - Interaction with the laser
  - Interaction with cavity mode
  - Spontaneous emission.
  - Cavity damping.

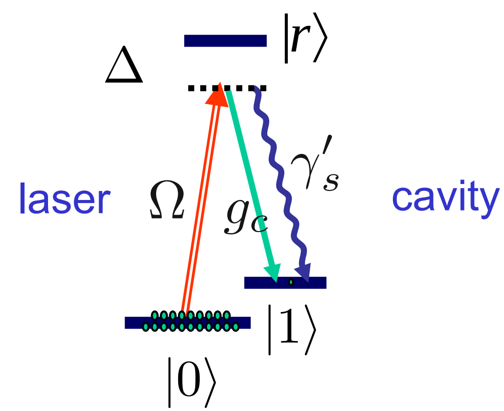
... an atomic ensemble improves the signal to noise ☺

Master equation: atoms in cavity



- master equation

$$\dot{\rho} = -i\,[H,\rho] + \Lambda\rho$$



- Hamiltonian

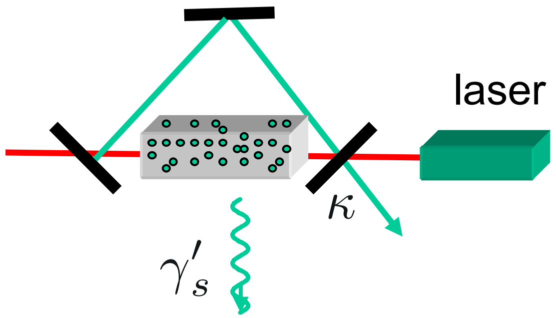
$$H = \hbar\frac{\sqrt{N_a}\Omega g_c}{\Delta} a^\dagger c^\dagger + \text{h.c.}$$

atoms   cavity

$$a \equiv \frac{1}{\sqrt{N_a}}\sum_{i=1}^{N_a} a_i \quad (a_i = |0\rangle_i \langle 1|)$$

$$|\phi\rangle \sim \sum_n \tanh r_c^n \frac{\left(c^\dagger\right)^n}{\sqrt{n!}} \frac{\left(a^\dagger\right)^n}{\sqrt{n!}} |0_a\rangle\,|0_p\rangle$$

two-mode squeezed atom + cavity state

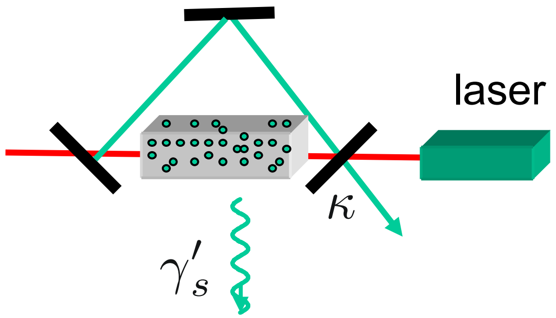


- cavity damping and spontaneous emission

$$\Lambda\rho = \frac{1}{2}\kappa(2c\rho c^\dagger - c^\dagger c\rho + \rho c^\dagger c) \qquad \text{cavity damping}$$

$$+\frac{1}{2}\gamma_s'\sum_i(2a_i^\dagger\rho a_i - a_i a_i^\dagger\rho - \rho a_i a_i^\dagger) \qquad \text{spontaneous emission}$$

Master equation: bad cavity limit



- bad cavity limit $\kappa \gg \dfrac{\sqrt{N_a}\,|\Omega g_c|}{\Delta}$
- adiabatic elimination

$$\dot{\rho}_a = \frac{1}{2}\kappa'(2a^\dagger \rho_a a - a a^\dagger \rho_a - \rho_a a a^\dagger)$$

$$\kappa' = \frac{4N_a\,|\Omega g_c|^2}{\Delta^2 \kappa}$$

"good" Stokes emission

$$+\frac{1}{2}\gamma_s' \sum_i (2a_i^\dagger \rho_a a_i - a_i a_i^\dagger \rho_a - \rho_a a_i a_i^\dagger)$$

$$N_a \gamma_s'$$

bad spontaneous emission

- Q.: condition for good to bad ??

Master equation: collective atomic operators

- collective atomic operators

$$a_\mu \equiv \frac{1}{\sqrt{N_a}} \sum_{j=0}^{N_a} a_j e^{ij\mu\,/\,N_a} \qquad (\mu = 0, 1, \cdots, N_a - 1)$$

- master equation

$$\dot{\rho}_a \quad = \quad \frac{1}{2} (\underbrace{\kappa'}_{\text{good}} + \underbrace{\gamma'_s}_{\text{bad}}) \left(2a^\dagger \rho_a a - a a^\dagger \rho_a - \rho_a a a^\dagger\right)$$

Stokes emission

good

bad

$$\cancel{+\gamma'_s \sum_{\mu \neq 0} (2a^\dagger_\mu \rho_a a_\mu - a_\mu a^\dagger_\mu \rho_a - \rho_a a_\mu a^\dagger_\mu)}$$

other modes

trace out!

- signal to noise

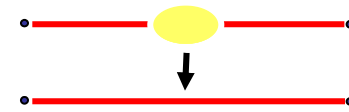
$$R_{\rm sn} = \frac{\kappa'}{\gamma'_s} \sim \frac{4 N_a \left|g_c\right|^2}{\kappa \gamma_s}$$

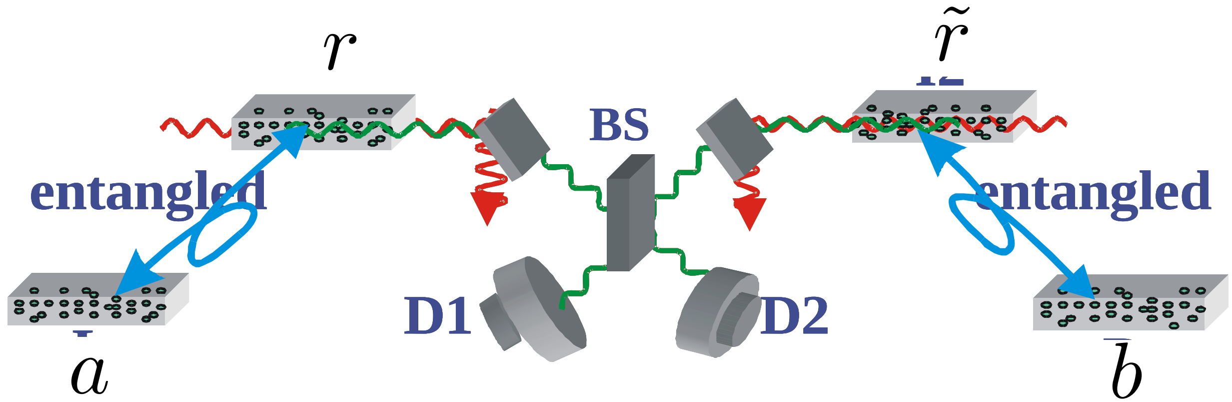
☺

ensembles help!

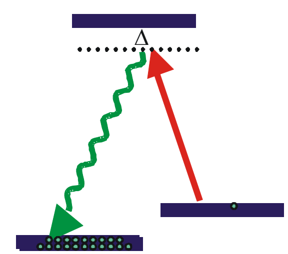
End technical part

B. Connection





- steps
  - apply a red laser pulse to transfer atomic excitation to optical excitation



  - succeeds if D1 *or* D2 registers *one* photon: distance doubled!

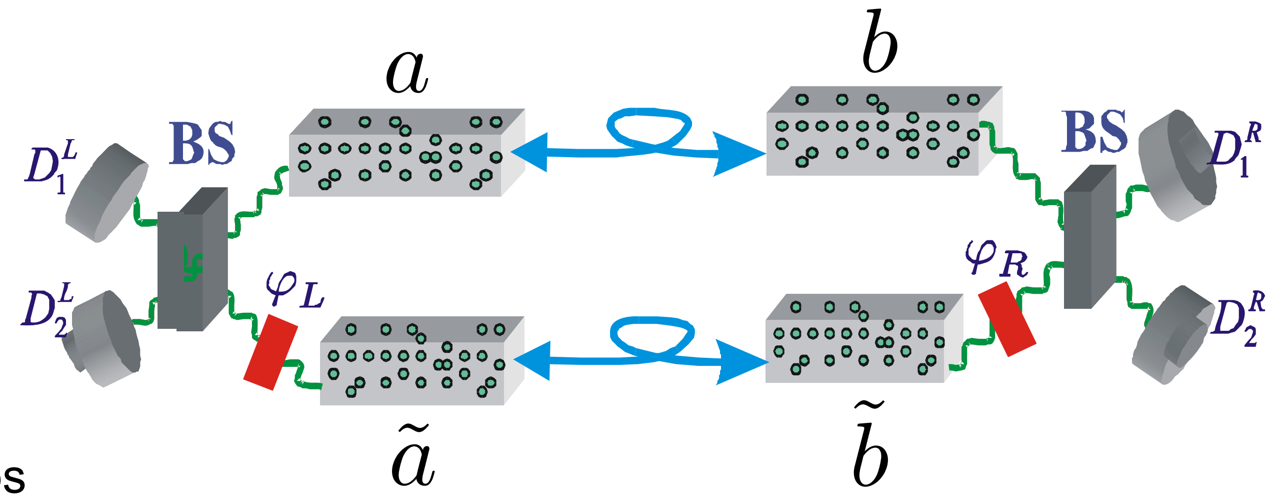
$$(a^\dagger + r^\dagger)(b^\dagger + \tilde{r}^\dagger)|\text{vac}\rangle \longrightarrow (a^\dagger + b^\dagger)|\text{vac}\rangle$$

(ideal)

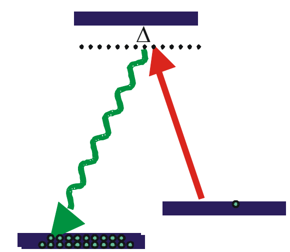
click=apply the operator:$(r + \tilde{r})$

  - fails otherwise: repeat everything starting from entanglement generation

C. Application: quantum cryptography



- steps
  - we generate two pairs
  - transfer atomic excitation to optical excitation, detect after phase shifter and beamsplitter



  - succeeds if D1 or D2 registers one photon on the left side and one photon on the right side.

$$(a^\dagger + b^\dagger)(\tilde{a}^\dagger + \tilde{b}^\dagger)|\mathrm{vac}\rangle \xrightarrow{\text{post selection}} (a^\dagger \tilde{b}^\dagger + \tilde{a}^\dagger b^\dagger)|\mathrm{vac}\rangle$$

equivalent to a photon polarization entangled state $\sim \; |\updownarrow \longleftrightarrow\rangle + |\longleftrightarrow \updownarrow\rangle$

  - role of phase shifter: single-bit rotation

apply Ekert protocol!

Imperfections:

- Spontaneous emission into other modes:

No effect, since they are not measured.

- Detector efficiency, photon absorption in the fiber, etc:

More repetitions.

- Dark counts:

More repetitions

- Technial note: anaylsis based on *effective maximally entangled state*

$$\rho = \frac{1}{c_0 + 1} \left(c_0 \left| \mathrm{vac} \right\rangle_{LR} \left\langle \mathrm{vac} \right| + \left| \Psi \right\rangle_{LR}^{+} \left\langle \Psi \right| \right)$$

✓ entanglement part decreases only linearly with L (instead of exponential)

✓ vacuum part drops out in quantum cryptography protocol

Scaling

- Fix the final fidelity: F

- Number of repetitions: $\sim L^{\log_2 L}$

- Example:

  Detector efficiency: 50%

  Length L=100 L_0

  Time $T=10^6 T_0$

  (to be compared with $T=10^{43} T_0$ for direct communication)

Quantum Repeater with Atomic Ensembles: Conclusions

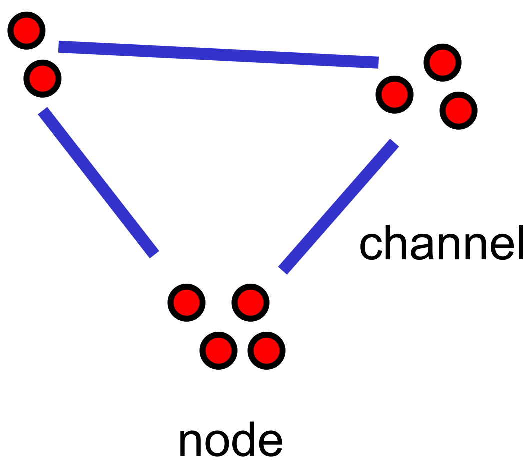
- Quantum repeaters allow to extend quantum communication to long distances.
- They can be implemented with trapped single atoms or atomic ensembles.
- Atomic ensembles (Duan et al. scheme)
  - ✓ No trapping/cooling is required.
  - ✓ No (high-Q) cavity is required.
  - ✓ Atomic collective effects make it more efficient.
  - ✓ No high efficiency detectors are required.
- Extension and application of these ideas to
  - ✓ quantum computing: remote gates
  - ✓ other implementations: solid state, …

JM Taylor
W Dür
PZ
A Yacoby
CM Marcus
MD Lukin
preprint 2005

3. Solid state circuits entanglement generation and purification

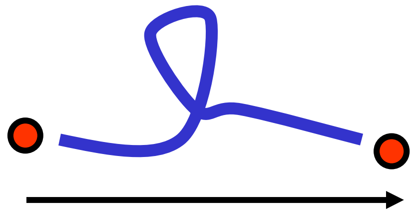
motivation slide in lecture 1

Quantum Network

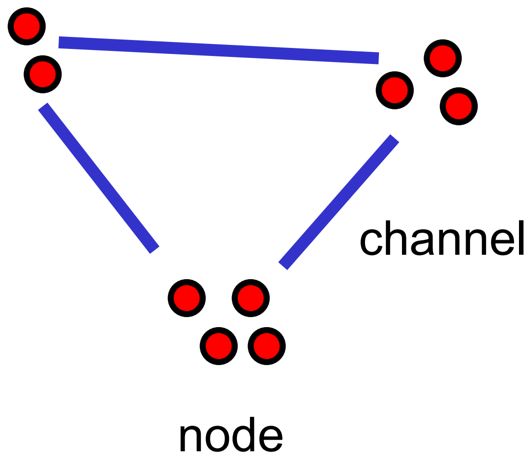


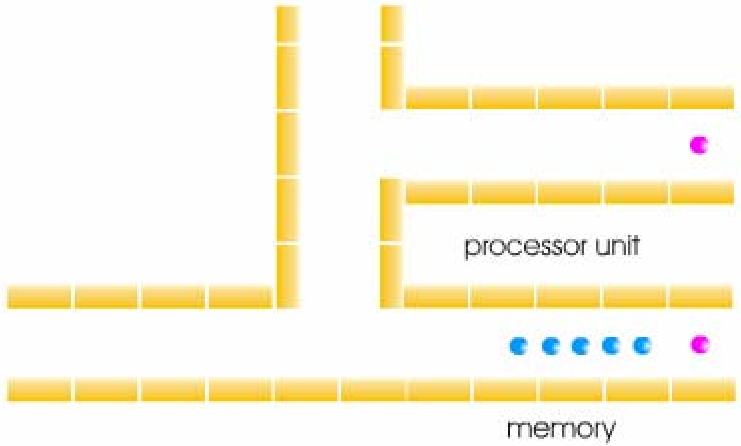
- Nodes: local quantum computing
  - store quantum information
  - local quantum processing

- Channels: quantum communication
  - transmit quantum information



Quantum Network

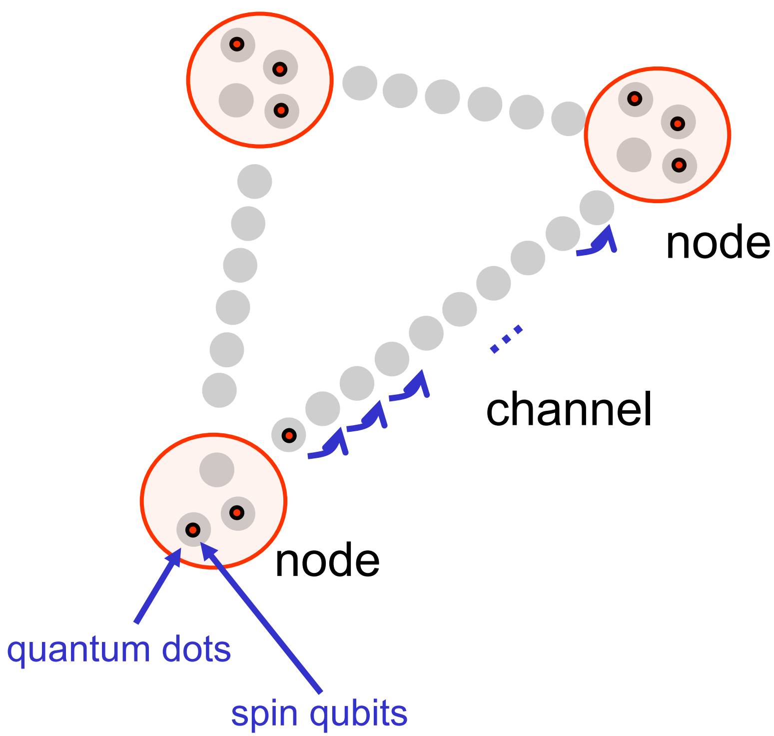




Movie by D. Leibfried NIST

Ion trap quantum computer:
scalability via segemented traps = transporting the ions around

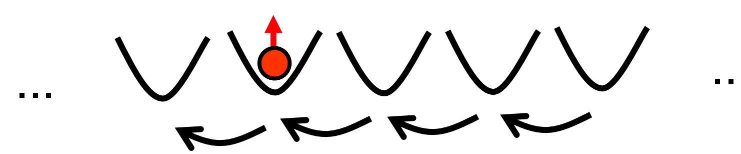
Mesoscopic quantum networks



- Nodes: local quantum computing
  - spin qubits
  - Loss & DiVincenzo: exchange gates etc.

- Channels: quantum communication
  - mesoscopic distance

arrays of quantum dots:



here: *deterministic* transport spin qubits by switching gate electrodes:

„bucket brigade“

Remark on scalability:

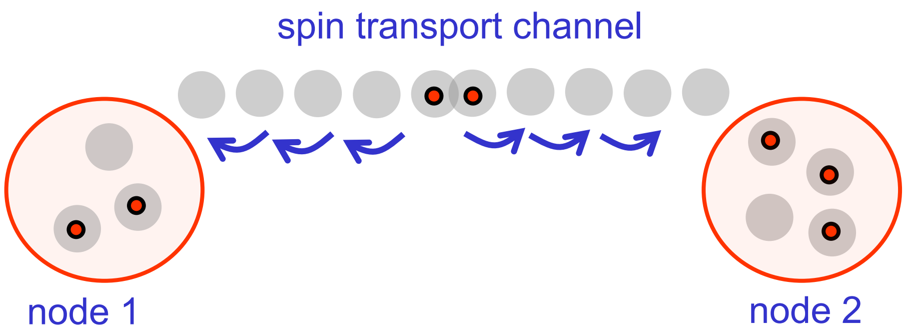
✓ fault tolerant threshold for nearest neighbor interactions extremely high

✓ transport ...

Basic idea

Quantum repeater protocol ... with available resources

- **Version 1** (simple):
  - logical qubit = single electron spin



Protocol:

✓ singlet generation & transport

✓ purification

✓ quantum repeater

Basic idea

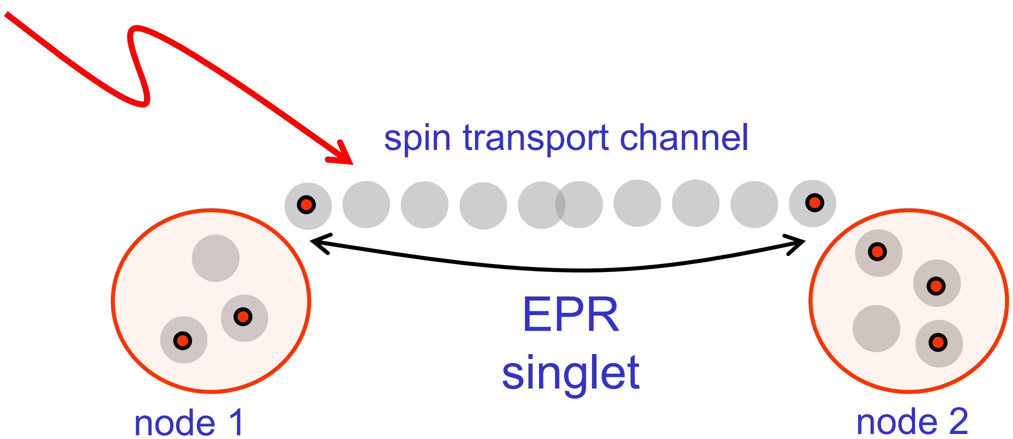
Quantum repeater protocol ... with available resources

- **Version 1** (too simple?):
  - logical qubit = single electron spin
  - problem quantum memory (@ nodes) ?

dominant transport error:

✓ nuclear spin (hyperfine)
fidelity F= 0.9 for 20 μm

✓ [charge, phonons, ...]



Protocol:

✓ singlet generation & transport

✓ purification

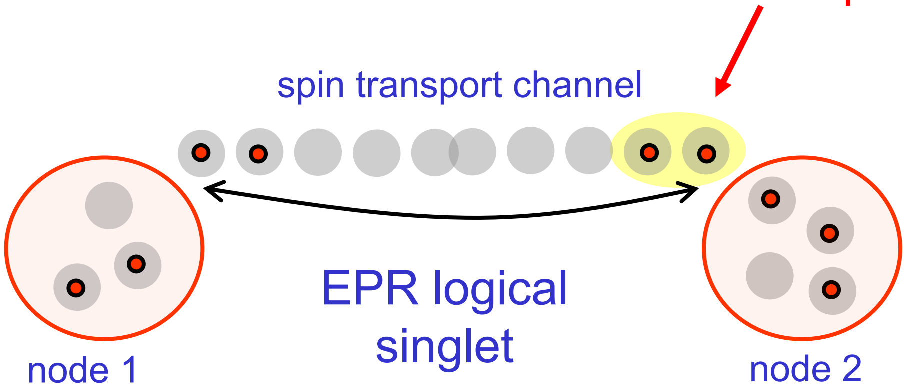
✓ quantum repeater

Basic idea

Quantum repeater protocol ... with available resources

- **Version 2** (advanced):
  - logical qubit = two electron spin
  - decoherence free subspace

logical qubit in decoherence free subspace

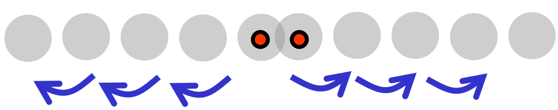


Protocol:

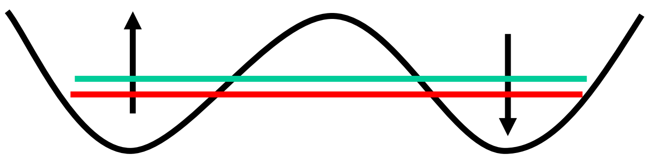
✓ singlet generation & transport
✓ purification
✓ quantum repeater

Version 1: A single electron quantum repeater

- physical system:
  - electrically gated quantum dots
  - electron spin = qubit
- spin transport channel



- local quantum computing a la Loss & DiVincenzo
  - Exchange gate



$$H_{\text{ex}} = J\,\hat{\vec{S}}^{A} \cdot \hat{\vec{S}}^{B}$$

  - *local* qubit rotations difficult ☹
- Measurement

...= resources $\Longleftrightarrow$ protocol:

✓ singlet generation

✓ purification

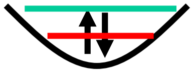
✓ connection of pairs

= quantum repeater

A. Generation of singlet states

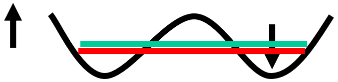
- scheme

  loading two electrons in a
  (double-)QDot & cooling

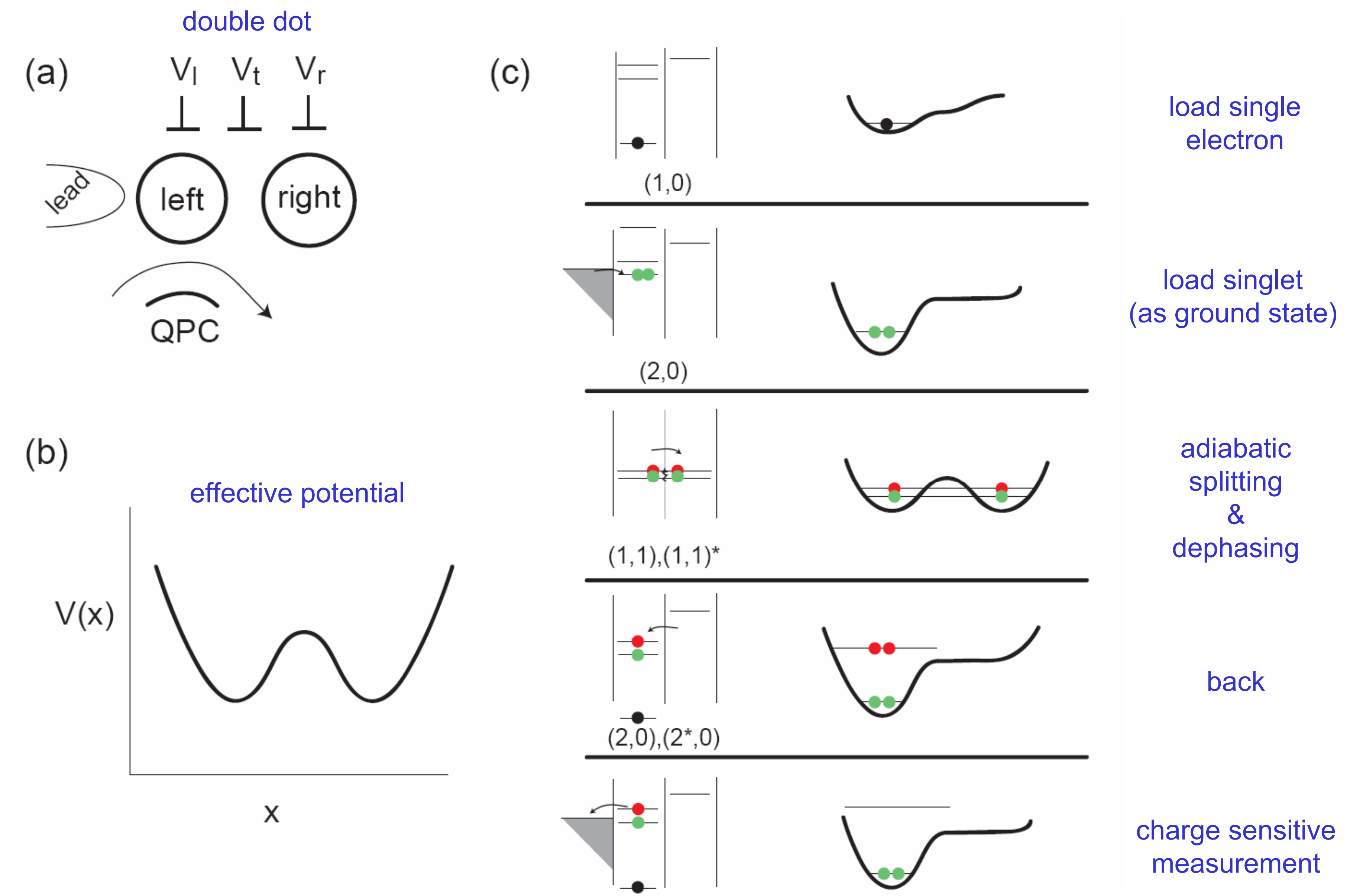


$|S{=}0,M{=}0\rangle \sim |\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle$

  split two electrons



- ... also provides a singlet measurement

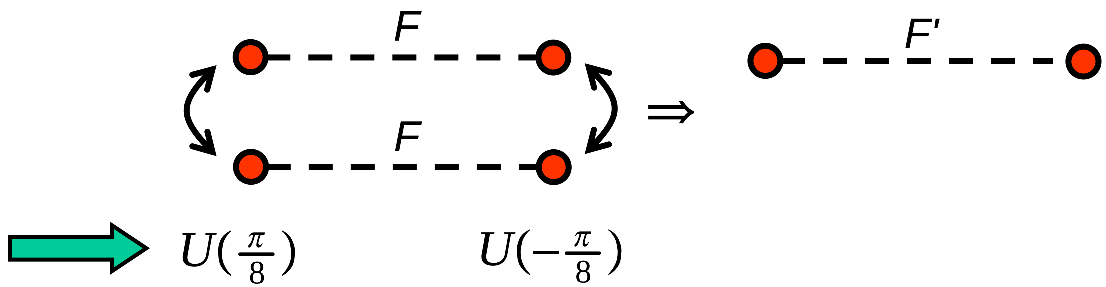


$$|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \equiv |\psi^-\rangle$$

$$|T_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \equiv |\psi^+\rangle$$

B. Entanglement purification

- Bennett et al. protocol: with exchange and local single qubit rotations

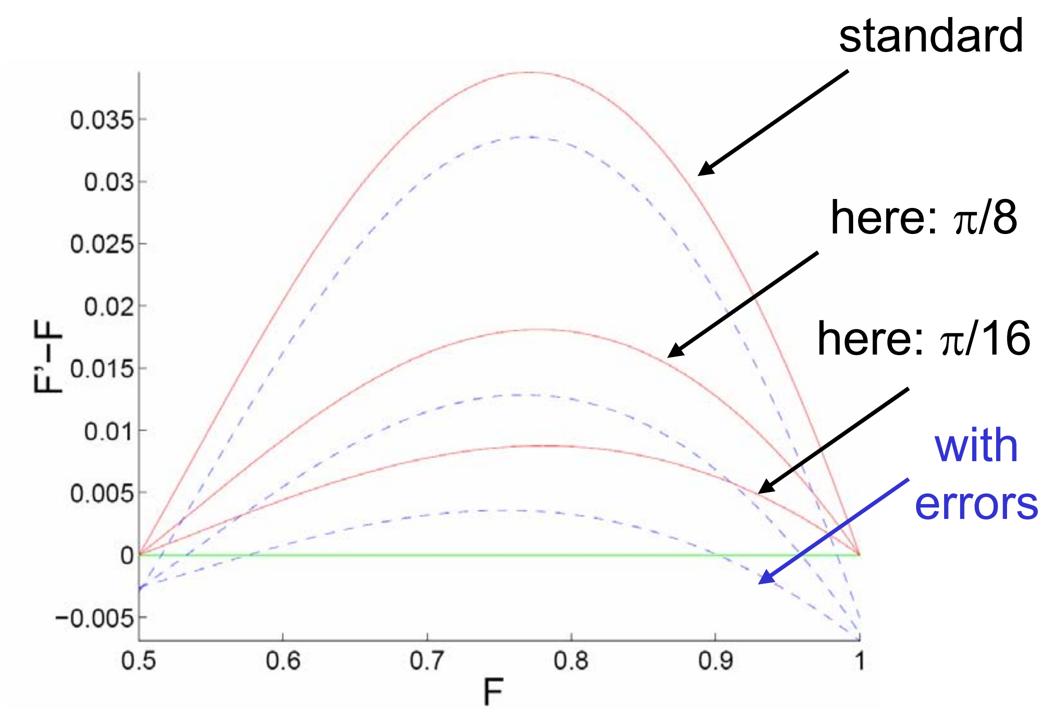


Exchange as a resource:

$$U(\phi) = \exp(-i\phi H_{\mathrm{ex}}/J)$$

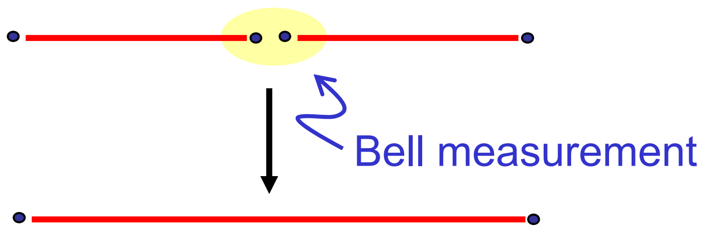
$$U(\frac{\pi}{4}) \sim \mathrm{SWAP}$$

$$U(\pm\frac{\pi}{8}) \sim \mathrm{SWAP}^{\pm 1/2}$$



C. Connection process

- connection

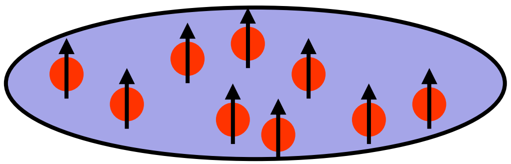


- QDots: *probabilistic* singlet measurement

  prob = 1/4

Limits / errors: hyperfine

- dephasing due to the nuclear spins



$$V_{hf} = A\sum \alpha_k \hat{\vec{I}}^k \cdot \hat{\vec{S}}$$

$$\rightarrow A\sum_k \alpha_k \hat{I}_z^k S_z$$

$$B_{\text{eff}} = A\sum_k \alpha_k \hat{I}_z^k / g^* \mu_B$$

Overhauser field:
mean field

dipole-dipole

$$\langle B_{\text{eff}}(t+\tau) B_{\text{eff}}(t) \rangle \simeq \left(\frac{A}{g^* \mu_B \sqrt{N}} \right)^2 e^{-f(\gamma_{dd}\tau)}$$

Physical picture:

✓ quasistatic (on spin manipulation time)

✓ rms strength ~10-50 mT

✓ $\gamma_{dd} \simeq 6\ \text{ms}^{-1}$

✓ short electron spin memory time $A/\hbar\sqrt{N} \sim 100\mu\text{s}^{-1}$

Version 2: Two electron encoding

- encoding in a decoherence free subspace $\alpha|S\rangle + \beta|T_0\rangle$

$$|0_L\rangle \equiv |S\rangle = 1/\sqrt{2}\,(|\uparrow\ \downarrow\rangle - |\downarrow\ \uparrow\rangle)$$
$$|1_L\rangle \equiv |T_0\rangle = 1/\sqrt{2}\,(|\uparrow\ \downarrow\rangle + |\downarrow\ \uparrow\rangle)$$

$$|\Psi_+\rangle_{ab} = (|0_L\rangle_a|0_L\rangle_b + |1_L\rangle_a|1_L\rangle_b)/\sqrt{2}$$
$$= (|\uparrow\ \downarrow\rangle_a|\uparrow\ \downarrow\rangle_b + |\downarrow\ \uparrow\rangle_a|\downarrow\ \uparrow\rangle_b)/\sqrt{2}$$

- make $|S\rangle, |T_0\rangle$ immune against quasistatic hyperfine field

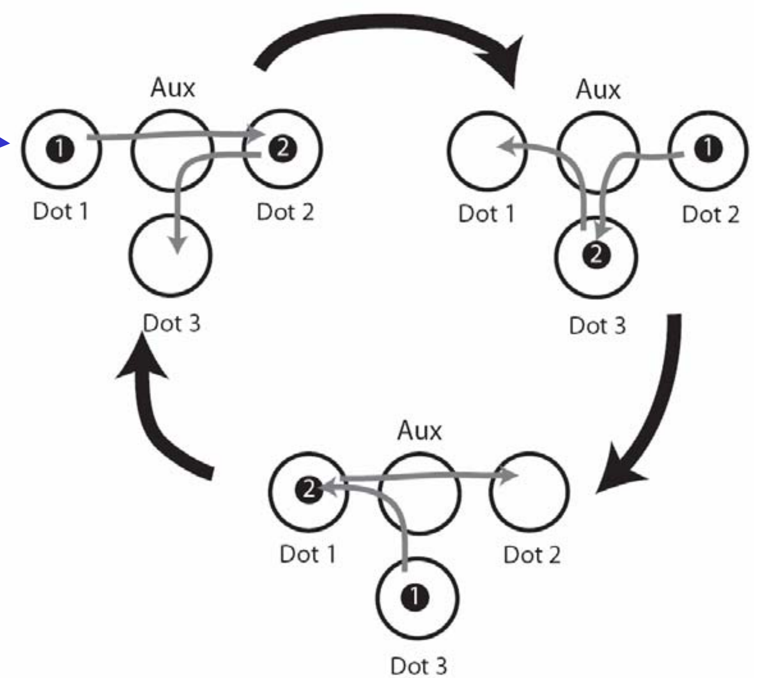
Protocol:

✓ electron encounters

$$\frac{g^*\mu_B}{\hbar}\int dt\, B_{\rm eff}^{a,b}(t)$$

✓ shuttle on scale

$$t \ll 1/\gamma_{dd}$$



- ... whole quantum repeater protocol in decoherence free subspace

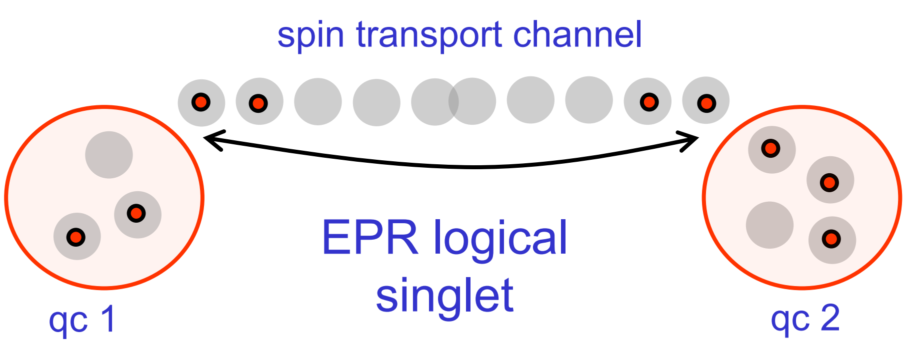
Quantum repeater protocol ... ingredients

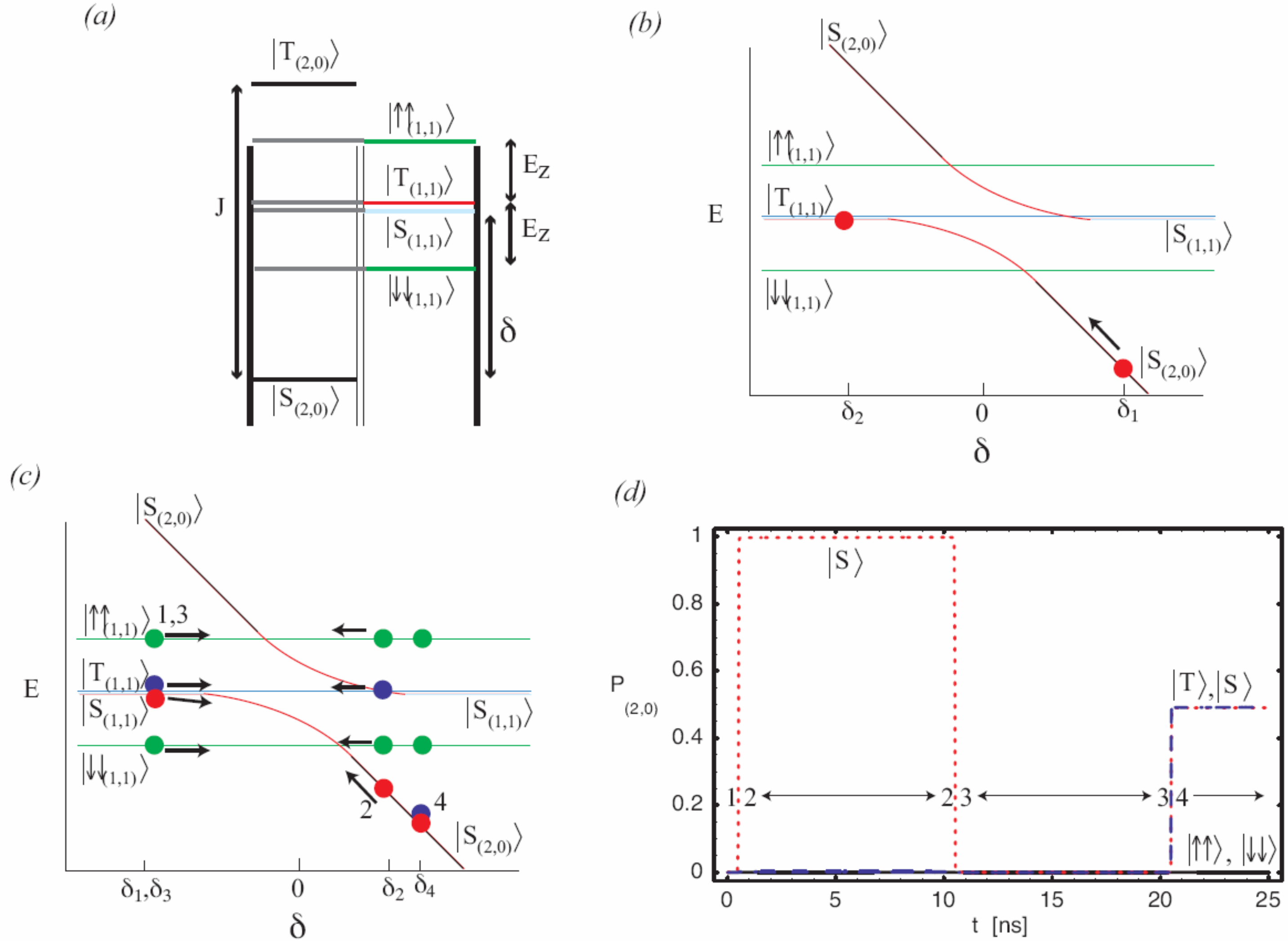
- EPR pair in decoherence free subspace $|\Psi_+\rangle_{ab} = (|0_L\rangle_a|0_L\rangle_b + |1_L\rangle_a|1_L\rangle_b)/\sqrt{2}$
- SWAP operation: physical exchange of two spin-qubits
- measurement on pair on spin-qubits
  - Singlet $|S\rangle$
  - Triplet $|T_0\rangle$ $(m_S = 0)$
  - Triplet $|m_S| = 1$
- results:
  - probabilistic generation of logical EPR singlet
  - probabilistic CNOT on pair of logical qubits
  - probabilistic connection

= Oxford type protocol

Summary

- Quantum repeater protocol for quantum communication
  - nested purification
- Physical implementation
  - deterministic: quantum state transfer via atom / high-Q cavity
  - probabilistic: atomic ensembles / photons
- Solid state circuits for entanglement generation, purification + quantum repeater protocol




(a)
|T(2,0)⟩
|↑↑(1,1)⟩
|T(1,1)⟩
|S(1,1)⟩
|↓↓(1,1)⟩
|S(2,0)⟩
J
EZ
EZ
δ
(b)
|S(2,0)⟩
|↑↑(1,1)⟩
|T(1,1)⟩
E
|S(1,1)⟩
|↓↓(1,1)⟩
|S(2,0)⟩
δ2
0
δ1
δ
(c)
|S(2,0)⟩
|↑↑(1,1)⟩ 1,3
|T(1,1)⟩
E
|S(1,1)⟩
|S(1,1)⟩
|↓↓(1,1)⟩
2
4
|S(2,0)⟩
δ1,δ3
0
δ2 δ4
δ
(d)
1
0.8
0.6
0.4
0.2
0
P(2,0)
|S⟩
|T⟩,|S⟩
1 2
2 3
3 4
|↑↑⟩, |↓↓⟩
0
5
10
15
20
25
t [ns]