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ANALYTIC RESULTS IN INTERMEDIATE VOLUMES FOR PURE SU(2) GAUGE THEORY

Jeffrey KOLLER

CALTECH, Pasadena, CA 91125, U.S.A.

and

Pierre van BAAL

Theoretical Physics Division, CERN, 1211 Geneva 23, Switzerland

We briefly discuss the general principles of analytic calculations in finite volume gauge theory up to volumes of at least five Compton wavelengths of the scalar glueball. We discuss recent Monte Carlo results in the finite volume context and present new analytic results on the E^- and T_2^- masses. Some speculations on going to larger volumes are presented.

We have performed calculations[1,2] based on the observation that the zero-momentum effective Hamiltonian derived by Lüscher[3] is still applicable at "large" values of the renormalized coupling constant. The non-trivial topology of configuration space, probed increasingly at growing coupling as the wave functional spreads, can be taken into account approximately by imposing boundary conditions in the space of zero momentum field configurations. Gribov ambiguities, associated with the non-trivial topology of configuration space, are avoided here by using more than one co-ordinate patch[4].

We start with pure SU(2) gauge theory in the Hamiltonian formulation on a finite cubic volume[3]. The vector potentials are taken periodic and the classical vacuum (after dividing out the gauge freedom) forms a three-dimensional vacuum-valley (also known as toron-valley[5]). At eight gauge inequivalent points within a connected component of the vacuum-valley, the quadratic approximation for the transverse fluctuations vanishes, thus preventing a simple one-loop calculation of an effective three-dimensional Hamiltonian for the "slow" vacuum-valley co-ordinates. At these eight points the classical potential is quartic, and since the vacuum-valley is widest here, this is where the wave functional will concentrate in perturbation theory. Lüscher[3] therefore derived an effective Hamiltonian for the nine zero-momentum modes $A_i(\vec{x}) = c_i/L$, with c_i a spatially constant SU(2) Lie algebra element. These are the modes in which the classical potential is quartic when expanding around the perturbative vacuum $A_i = 0$.

The other seven gauge inequivalent perturbative vacua are related by homotopically non-trivial gauge transformations with homotopy Z_2^3, characterized by 't Hooft's twist[6]. Wave functionals thus fall into representations of this homotopy group, labelled by 't Hooft's electric flux[6] $\vec{e} \in Z_2^3$. A perturbative energy level then splits into four levels: (i) a singlet with $\vec{e} = 0$, (ii) a triplet with one unit of electric flux, i.e., $\vec{e} = (1,0,0)$, $(0,1,0)$ or $(0,0,1)$, (iii) a triplet with two units of electric flux, i.e. $\vec{e} = (1,1,0)$, $(1,0,1)$ or $(0,1,1)$ and (iv) a singlet with three units of electric flux $\vec{e} = (1,1,1)$. The degeneracies are due to the cubic symmetry and

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the split in energy can be understood as tunnelling through a quantum induced potential barrier[7].

The symmetries induced by the above-mentioned homotopically non-trivial gauge transformations can be reformulated as boundary conditions on the wave functional. This is in analogy to the double-well $V(x) = (x^2-1)^2$, where levels split into even and odd parity states, characterized by the boundary conditions $\nabla_x\Psi(0) = 0$ and $\Psi(0) = 0$ respectively. In an adiabatic approximation[2], the boundary conditions can be written in terms of just the zero-momentum vector fields c_i. This leads to only a minor modification of Lüscher's effective Hamiltonian, which we extended to higher orders mainly to study stability of the results. In the course of this extension we discovered appropriate co-ordinates for the vacuum-valley within the set of zero-momentum vector potentials, thereby technically allowing the progress we made.

The Hamiltonian cannot be solved exactly, but a simple Rayleigh-Ritz basis allowed us to compute the matrix for the Hamiltonian analytically. Truncating the basis will yield an upper bound for the energies, and (with slightly more work) a rigorous lower bound[2]. In this way we can claim better than three-digit accuracy for most of our results. The energy levels are classified by the electric flux $\vec{e}$ and by the irreducible representations of the cubic group (or the subgroup which leaves $\vec{e}$ invariant). Following the notation of Ref. 8, there are for each parity two singlets ($A_1^\pm$ and $A_2^\pm$), one doublet ($E^\pm$) and two triplets ($T_1^\pm$ and $T_2^\pm$). For non-zero electric flux we denote the lowest energy state with i units of electric flux by e_i^+. The results of our calculations can be found in Refs. 1 and 2.

For larger values of g the energies become more sensitive to the higher order corrections in the effective Hamiltonian. However, the energy difference with the vacuum (i.e., the A_1^+ ground state), are remarkably stable. We consider the renormalization group invariant quantities $z_R = L\cdot(E_R-E_{A_1^+})$, where $E_{A_1^+}$ is the vacuum energy and E_R is the ground state energy of the representation R (for $R = A_1^+$, E_R is the first excited A_1^+ energy). These quantities can be measured in Monte Carlo calculations on $L^3\times T$ lattices, by use of time-time correlations of the appropriate operators (with T large enough to ensure zero temperature). See Fig. 2 of Ref. 1 for a comparison with the Monte Carlo data of Ref. 9. Elsewhere in this volume, Berg presents new data for $\sqrt{z_{e_1^+}/z_{E^+}}(=\sqrt{K}/m(E^+))$ and $z_{A_1^+}/z_{E^+}(=m(A_1^+)/m(E^+))$ as a function of z_{E^+}, which agree still better with our results, even up to $z_{E^+} \sim 7.5$. Remaining deviations, especially at lower z_{E^+}, could be due, among other things, to lattice artefacts in the Monte Carlo data (lattices as small as 4^3 in the spatial directions are used) or to the non-adiabatic corrections in our analytic calculations. We investigated the latter issue carefully in Ref. 2 and found that this non-adiabatic deviation is biggest at intermediate distances and surprisingly improves beyond about $z_{E^+} \sim 3$.

Nevertheless, we expect our zero momentum approximation to break down somewhere, to restore rotational invariance and have the energy of electric flux behave as predicted by a string picture. Let us quickly review the status of these issues. Our results yield $z_{T_2^+}/z_{E^+} \sim 0.5$ for $z_{E^+} \geq 2.5$. If rotational invariance is restored, the irreducible representations of the cubic group have to combine into angular momentum multiplets in a well prescribed way. As an important example, a $J^P = 2^+$ quintet is composed of an E^+ doublet and a T_2^+ triplet. A necessary condition for rotational invariance is therefore that $z_{T_2^+}/z_{E^+} = 1$. Unfortunately, no reliable Monte Carlo results for T_2^+ are available yet to confirm our prediction, but we can confidently say one has to at least go beyond $z_{E^+} = 5$ to restore rotational invariance. This seems very counter-intuitive, since naively one expects the boundary conditions to be irrelevant for volumes larger than the Compton wavelength of the smallest mass. However, one must remember that gauge theories have a reasonably large length scale, set by the deconfining phase transition (yielding $z_{E^+} \sim 5$). The best way to understand the relevance of this distance scale in the context of zero temperature and finite volume, is to use 't Hooft's duality transformation[6]. However, it will not do more than indicate one should expect a "cross-over" in the mass-ratios at this distance scale, and certainly does not predict the precise value of z, or the size of the effect. Still, recent Monte Carlo data for the ratio $z_{E^+}/z_{A_1^+}$ [10] at large volumes do seem to confirm a strong cross-over at $z_{A_1^+} \sim 6$. Figure 1 collects the Monte Carlo results known to us <u>before</u> the conference. For SU(3), however, the same ratio was computed by two groups[11,12]. They agree for operator-operator correlations, but the cold source method[12] gives a much lower E^+ mass (with large statistical error bars). It was suggested[11] that for $Z_{A_1^+} > 6$ the operators used in Ref. 10 might couple to an excited E^+ state rather than to the E^+ ground state. In the light of this we

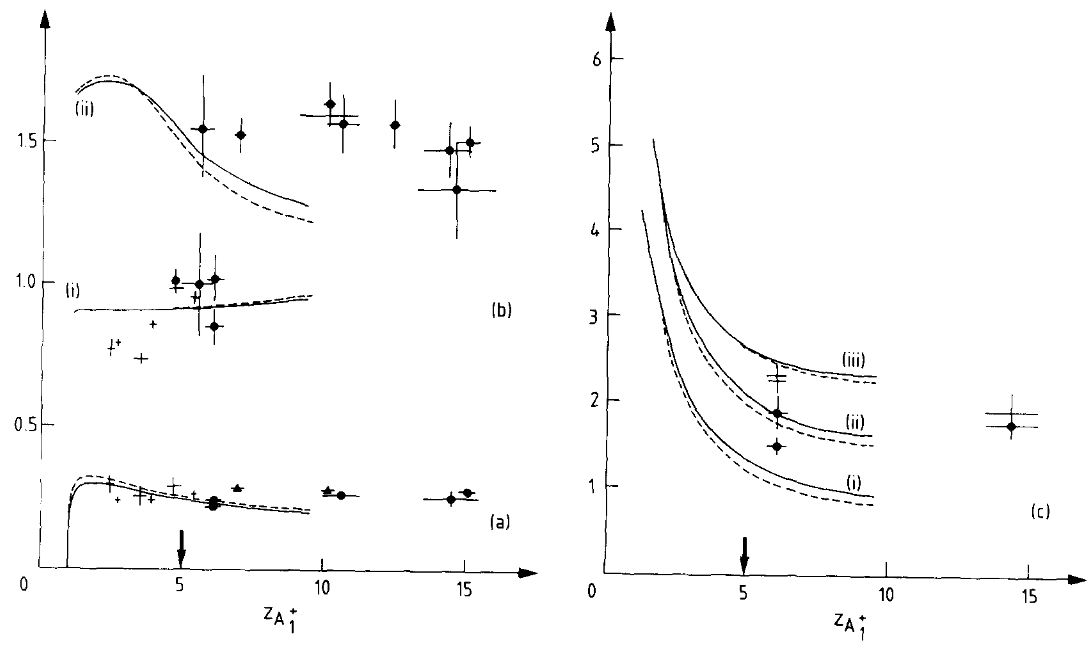

FIGURE 1

Comparing results for mass ratios. The solid curves are our analytic results; the dashed lines include the higher order corrections[1,2]. In (a) we give the results for $\sqrt{z_{e_1^+}/z_{A_1^+}}$ and in (b) for $z_{E^+}/z_{A_1^+}$ where curve (i) is for the E^+ ground state and curve (ii) is for the first excited E^+ state. The data (✚) are from Ref. 9, (●) from Ref. 10 and (▲) combines results of Ref. 9 and Ref. 10. In (c) we compare negative parity results. Curve (i) is for $z_{A_1^-}/z_{A_1^+}$, curve (ii) for $z_{T_2^-}/z_{A_1^+}$ and curve (iii) for $z_{E^-}/z_{A_1^+}$. The data [(●) for $z_{A_1^-}$ $z_{A_1^+}$ and (✚) for $z_{E^-}/z_{A_1^+}$] are from Ref. 10. The arrow at $z_{A_1^+} = 5$ is the distance beyond which we naively expect our approximations to become inaccurate.

examined our results for the first excited E^+ state, and find at $z_{A_1^+} \sim 6$ a ratio for the excited E_+ mass to the A_1^+ mass of approximately 1.4. Hence our results are consistent with the above conjecture, but we are hesitant to consider our results reliable beyond $z_{A_1^+} \sim 5$. In Fig. 1 we also give analytic results obtained "on request" after the conference for $z_{E^-}/z_{A_1^+}$ and $z_{T_2^-}/z_{A_1^+}$. (For T_1^- we found $z_{T_1^-}/z_{E^-} \sim 1.4$-$1.5$.)

Concerning the behaviour of the energy of electric flux, our calculations predict roughly $z_{e_i^+}/z_{e_1^+} \sim i$ [1], whereas a string picture would predict $z_{e_i^+}/z_{e_1^+} \sim \sqrt{i}$ [6]. This is such a marked contrast that these ratios of different units of electric flux can be used as a very good indicator for the transition to the confining domain. Very recent Monte Carlo data by Berg[13] confirm our predictions for these electric flux energy ratios in the intermediate volume range, and no string-like behaviour is observed up to $z_{E^+} \sim 6$-7.

Let us finally speculate on the mechanism whereby the zero-momentum approximation breaks down. As mentioned above, our detailed analysis indicates that the adiabatic approximation seems to improve for increasing coupling. We therefore consider it more likely that it will again be the topological non-triviality of configuration space that causes a sudden change. This time it should be the good old (non-zero action) instantons which are responsible. It is important to stress that it will be the instantons with a size of the total volume which will dominate first (remember that the coupling constant is set at the scale $\mu = 1/L$). In the Hamiltonian picture it simply means that a wave functional in one connected component of the vacuum-valley starts to see the other components. This is necessarily outside of the zero momentum sector. Therefore, it might be possible that suitable boundary conditions in the first few non-zero momentum fluctuations will approximately take these effects into account. We believe that Gribov horizons will again prevent these fluctuations from becoming arbitrarily large. One expects this effect to be drastic, since increase of the coupling will make the volume bigger so that larger instantons fit in, and will also enhance the likelihood that smaller instantons contribute. One might envision this causing the vacuum energy to develop a local minimum, indicating an instability of the vacuum against domain formation. It would make the coupling constant stop running at the domain size, and might explain an electric flux string as "beads" of domains with one unit of electric flux[2]. If we suppose these domains to have a size corresponding to $z_{E^+} \sim 5$ and we set the scale by a string tension of $(420\ \mathrm{MeV})^2$, one easily extracts a width of 0.55 fm and an energy density in the string of 2.9 $\mathrm{GeV/fm}^3$. This is sufficiently close to Monte Carlo results[14] to encourage making these vague ideas more precise.

There is a more direct indication that non-zero action instantons are relevant, namely the observation that the topological susceptibility (χ_t) shows a sudden impression in the deconfined region, when using the cooling method[15]. Unfortunately there is disagreement with the geometric methods[16] on the value of χ_t in the confined region and the suppression of χ_t in the deconfined region. We hope this discrepancy can be resolved in the near future.

To conclude, we feel confident of our understanding of the low energy dynamics of pure gauge theory in a cubic volume with periodic boundary conditions, up to $z_{E^+} \sim 5$, or even a little beyond. We hope that Monte Carlo results can guide us on how to extend our analytic understanding to larger, and hopefully infinite, volumes.

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